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A NEW INDEX OF FINANCIAL CONDITIONS

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A New Index of Financial Conditions

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Abstract

We use factor augmented vector autoregressive models with time-varying coefficients to construct a financial conditions index. The time-variation in the parameters allows for the weights attached to each financial variable in the index to evolve over time. Furthermore, we develop methods for dynamic model averaging or selection which allow the financial variables entering into the FCI to change over time. We discuss why such extensions of the existing literature are important and show them to be so in an empirical application involving a wide range of financial variables.

Keywords: financial stress; dynamic model averaging; forecasting

JEL Classification: C11, C32, C52, C53, C66

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1 Introduction

The recent financial crisis has sparked an interested in the accurate measurement of financial shocks to the real economy. An important lesson of recent events is that financial developments, not necessarily driven by monetary policy actions or fundamentals, may have a strong impact on the economy. The need for policymakers to closely monitor financial conditions is clear. In response to this need, a recent literature has developed several methods for constructing financial conditions indexes (FCIs). These indexes contain information from many financial variables, and the aim is for policymakers to use them to provide early warning of future financial crises. These FCIs range from simple weighted averages of financial variables through sophisticated econometric estimates. Many financial institutions (e.g. Goldman Sachs, Deutsche Bank and Bloomberg) and policymakers (e.g. the Federal Reserve Bank of Kansas City) produce closely-watched FCIs. An important recent contribution is Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010) which surveys and compares a variety of different approaches. The new FCI it proposes uses principal components methods to extract an FCI from a large number of quarterly financial variables. Other recent notable studies in this literature include English, Tsatsaronis and Zoli (2005), Balakrishnan, Danninger, Elekdag and Tytell (2008), Beaton, Lalonde and Luu (2009), Brave and Butters (2011), Gomez, Murcia and Zamudio (2011) and Matheson (2011).

The construction and use of an FCI involves three issues: i) selection of financial variables to enter into the FCI, ii) the weights used to average these financial variables into an index and iii) the relationship between the FCI and the macroeconomy. There is good reason for thinking all of these may be changing over time. Indeed, Hatzius et al (2010) discuss at length why such change might be occurring and document statistical instability in their results. For instance, the role of the sub-prime housing market in the financial crisis provides a clear reason for the increasing importance of variables reflecting the housing market in an FCI. A myriad of other changes may also impact on the way an FCI is constructed, including the change in structure of the financial industry (e.g. the growth of the shadow banking system), changes in the response of financial variables to changes in monetary policy (e.g. monetary policy works differently with interest rates near the zero bound), the changing impact of financial variables on real activity (e.g. the role of financial variables in the recent recession is commonly considered to have been larger than in other recessions) and many other things.

Despite such concerns about time-variation, the existing literature does little to statistically model it. Constant coefficient models are used with, at most, rolling methods to account for time-variation. Furthermore, many FCI's are estimated ex post, using the entire data set. So, for instance, at the time of the financial crisis, some FCIs will be based on financial variables which are selected after observing the financial crisis and the econometric model will be estimated using financial crisis data. The purpose of the present paper is to develop an econometric approach which estimates the FCI online (i.e. in real time) and allows for a time-varying treatment of the three issues described in the preceding paragraph. Following a common practice in constructing indexes, we use factor methods. To be precise, we use Factor-augmented VARs (FAVARs) which jointly model a large number of financial variables (used to construct the FCI) with key macroeconomic variables. However, we do not work with a single FAVAR, but rather work with a large set of FAVARs which differ in which financial variables are included. Faced with a large model space and the desire to allow for model change, we use dynamic model averaging (DMA) and selection (DMS) methods developed in Raftery et al (2010). DMS chooses different financial variables to make up the FCI at different points in time. DMA constructs an FCI by averaging over many individual FCIs constructed using different financial variables. The weights in this average vary over time. Such approaches can help address the three issues highlighted above in a dynamic fashion. Further flexibility can be attained through the use of time-varying parameter (TVP) FAVARs. Accordingly, we investigate constructing FCIs using DMA and DMS methods with various TVP-FAVARs as well as constant coefficient FAVARs.

Econometric methods for estimating FAVARs and TVP-FAVARs are well-established (see, e.g., Bernanke, Boivin and Eliasz, 2005, and Korobilis, 2013). However, these estimation methods (e.g. Bayesian methods using Markov chain Monte Carlo algorithms) are computationally demanding. With our large model space, it is computationally infeasible to use such methods. Accordingly, we use methods which require only the use of the Kalman filter or other filtering algorithms. In the FAVAR, an approach which meets these requirements is outlined in Doz, Giannone and Reichlin (2011) and we adopt their methods (with slight modifications). For the TVP-FAVAR we develop a new, computationally efficient algorithm, which is an extension of Doz, Giannone and Reichlin (2011).

We use our many FAVARs and TVP-FAVARs for three main purposes: i) to estimate an FCI (and compare our estimate to alternatives), ii) to investigate how well the FCI can be used to forecast macroeconomic variables, and iii) to calculate impulse responses in a time-varying fashion. The preceding discussion motivates why i) and ii) are important. With regards to iii), it is worth noting that the paper of Hatzius et al (2010) is followed by discussants' comments by several important policymakers. William Dudley (President of the Federal Reserve Bank of New York) highlights the importance of understanding the implications of the FCI for the conduct of monetary policy. Impulse response analysis using our FCI should represent a useful addition to the literature which offers insight in this regard.

Our empirical results indicate DMA and DMS methods do lead to FCIs which better track the macroeconomy and differ from conventional FCIs in some aspects. The version of our approach which performs best involves using DMA or DMS methods on a restricted version of a TVP-FAVAR which we call a factor augmented TVP-VAR (FA-TVP-VAR). This involves extracting the FCI using a constant coefficient approach (i.e. the factor loadings are constant over time), but modelling the FCI jointly with the macroeconomic variables in a TVP-VAR. The financial variables which make up our FCI change substantially over time. We discuss this variation and present impulse responses to shocks to the FCI.

2 Dynamic Model Averaging with FAVARs and TVP-FAVARs

2.1 Estimation and Forecasting FAVARs and TVP-FAVARs

In general, dynamic factor methods are popular in empirical macroeconomics and finance (e.g., a recent application is Bagliano and Morana, 2012) and extensions such as FAVARs are increasingly popular (e.g. a pioneering paper is Bernanke, Boivin and Eliasz, 2005). In the FCI literature in particular, factor methods are also common. For instance, Hatzius et al (2010) use factor methods to estimate their financial conditions index. In a similar spirit, we work with factor models. Furthermore, many authors, working with a range of data sets (e.g. Del Negro and Otrok, 2008, Meligotsidou and Vrontos, 2008, Eickmeier, Lemke and Marcellino, 2011, Felices and Wieladek, 2012 and Korobilis, 2013) have found it important to extend factor models to allow for time-variation in coefficients. We follow in this tradition.

Let x_t (for t = 1, ..., T) be an $n \times 1$ vector of financial variables to be used in constructing the FCI. It is important for the FCI to reflect information solely associated with the financial sector, rather than reflecting feedback from general macroeconomic conditions. Accordingly, we wish to purge macroeconomic effects from our FCI by including macroeconomic variables in the equations used to calculate the factors. Let y_t be an $s \times 1$ vector of macroeconomic variables of interest. In our empirical work, $y_t = (\pi_t, u_t, r_t)'$ where π_t is the inflation rate, u_t is the unemployment rate, and r_t is the interest rate. These macroeconomic variables also serve a second purpose in that we are interested in forecasting them using the FCI and this plays an important role in our DMA algorithm (to be described shortly).

The TVP-FAVAR takes the form:

$$\begin{aligned} x_t &= \lambda_t^y y_t + \lambda_t^J f_t + u_t \\ \begin{bmatrix} y_t \\ f_t \end{bmatrix} &= c_t + B_t \begin{bmatrix} y_{t-1} \\ f_{t-1} \end{bmatrix} + \varepsilon_t \end{aligned} ,$$
 (1)

with

$$\lambda_t = \lambda_{t-1} + v_t \beta_t = \beta_{t-1} + \eta_t$$
(2)

where $\lambda_t = \left((\lambda_t^y)', (\lambda_t^f)' \right)', \beta_t = (c'_t, vec(B_t)')'$ and f_t is the latent factor which we interpret as the FCI. In our empirical work, f_t is a scalar and we are estimating a single FCI. Note that this model allows factor loadings, regression coefficients and VAR coefficients to evolve over time according to a random walk.¹ All errors in the equations above are uncorrelated over time and with each other, thus having the following structure

¹Note that we have written second of equation (1) as a VAR(1) model But this does not restrict us since every VAR(p) admits a VAR(1) representation; see Lutkepohl (2005).

$$\begin{pmatrix} u_t \\ \varepsilon_t \\ v_t \\ \eta_t \end{pmatrix} \sim N \left(0, \begin{bmatrix} V_t & 0 & 0 & 0 \\ 0 & Q_t & 0 & 0 \\ 0 & 0 & W_t & 0 \\ 0 & 0 & 0 & R_t \end{bmatrix} \right).$$

Note that the TVP-VAR allows for all of error covariance matrices to be timevarying. We use exponentially weighted moving average (EWMA) methods. EWMA estimates are popularly used to model volatilities in many financial applications and their properties are familiar and well-established (see, among many others, RiskMetrics, 1996 and Brockwell and Davis, 2009, Section 1.4). Koop and Korobilis (2013) uses a similar approach and additional motivation for use of EWMA estimates is provided there. The Technical Appendix provides precise details on how they are estimated.

Identification in the FAVAR is achieved in a standard fashion by restricting V_t to be a diagonal matrix and the first element of λ_t^f to be one. The former restriction ensures that the factors, f_t , capture movements that are common to the financial variables, x_t , after removing the effect of current macroeconomic conditions through inclusion of the $\lambda_t^y y_t$ term.

Bayesian estimation of TVP-VARs and TVP-FAVARs is typically done using Markov Chain Monte Carlo (MCMC) methods (see, e.g., Primiceri, 2005 or Del Negro and Otrock, 2008). Such Bayesian simulation methods are computationally expensive even when the researcher is estimating a single TVP-FAVAR. When faced with multiple TVP-FAVARs and when doing recursive forecasting (which requires repeatedly doing MCMC on an expanding window of data), the use of MCMC methods is prohibitive.²

In this paper, we use fast, approximate, estimation methods which vastly reduce the computational burden. Similar to the approximate methods for TVP-VARs used in Koop and Korobilis (2013), we estimate all TVP-FAVAR coefficients using fast updating schemes based on one-sided exponentially weighted moving average (EWMA) filters combined with Kalman filter recursions. Complete details are provided in the Technical Appendix. Suffice it to note here that, for the constant coefficient dynamic factor model, an approximate two-step estimation approach is developed in Doz, Giannone and Reichlin (2011). It is straightforward to adapt this algorithm to estimate the constant coefficient FAVAR and our results using FAVARs are calculated using such an approach. The extension of the algorithm of Doz, Giannone and Reichlin (2011) to the TVP-FAVAR requires an additional step where the time-varying coefficients are drawn using the Kalman filter. For forecasting, the Kalman filter provides us with a one-step ahead predictive density. When we present results for forecast horizons greater than one we use iterative methods. With these approximations, it takes only a few seconds to carry out a full recursive forecasting exercise for a single TVP-FAVAR model given in (1).

²To provide the reader with an idea of approximate computer time, consider the three variable TVP-VAR of Primiceri (2005). Taking 10,000 MCMC draws (which may not be enough to ensure convergence of the algorithm) takes approximately 1 hour on a good personal computer. Thus, forecasting at 100 points in time takes roughly 100 hours. These numbers hold for a single small TVP-VAR, and would be much larger for the 65,536 larger TVP-FAVARs we use in this paper.

Our impulse responses are presented based on the TVP-VAR for $(y'_t, f_t)'$ (i.e. the second equation in (1)). We use a standard Cholesky factorization of V_t to identify the structural shocks (see., e.g., Primiceri, 2005, Castelnuovo, 2012 and Korobilis, 2013). Such a triangular identification scheme implies macroeconomic variables respond with a lag to changes in financial conditions, while financial conditions can respond contemporaneously to shocks in macroeconomic conditions. See the Technical Appendix for complete details about estimation, forecasting and impulse response analysis.

In addition to the unrestricted TVP-FAVAR given in (1) and (2), we consider several restricted versions. If we restrict $\lambda_t = \lambda$, then the factor equation will have constant factor loadings, but the VAR part of the model will still have time-varying parameters. Note that λ_t contains many parameters (i.e. $n \times (s + 1) = 80$ in our application) and, hence, restricting it to be constant may be important in reducing over-parameterization concerns. We refer to such a model as a factor-augmented TVP-VAR or FA-TVP-VAR to distinguish it from the unrestricted TVP-FAVAR. We consider such a model in our empirical work. All specification choices except relating to λ are identical in the TVP-FAVAR and FA-TVP-VAR.

The FCI constructed by Hatzius et al (2011) uses a FAVAR with homoskedastic errors (although the financial and macroeconomic variables they use differ somewhat from ours). To obtain something similar, we also use present results using a FAVAR which is obtained as a special case of our TVP-FAVAR with $V_t = V$, $\lambda_t = \lambda$ and $\beta_t = \beta$ for all t. Similar to the FA-TVP-VAR, all modelling choices except those relating to V, λ and β are identical to those for the TVP-FAVAR.

Some authors (e.g. Eickmeier, Lemke and Marcellino, 2011) use existing FCIs (i.e. estimated by others) in the context of a VAR or FAVAR model. In this spirit, we also present results for VARs (i.e. the second equation in (1) with $V_t = V$ and $\beta_t = \beta$ for all t) where the factors are replaced with an estimate. To be precise, our VARs use $(y'_t, \hat{f}_t)'$

as dependent variables for different choices of \hat{f}_t . Table 1 lists these choices. Again, these models are a restricted special case of our TVP-FAVAR and estimation proceeds accordingly. The error covariance matrix is modelled in the same manner as the FAVAR. We use an acronym for these VARs such that, e.g., VAR(FCI 5), is the VAR involving the macroeconomic variables and the Bank of America Merrill Lynch Global Financial Stress Index.³

³Note that the sample period of the indexes in Table 1 differ and are shorter than the sample period for the macroeconomic variables. We treat this issue by forecasting using a VAR for the macroeconomic variables up to time τ . Subsequently a VAR including macroeconomic variables plus the FCI is used. We set τ to be the time that the sample for the FCI begins plus 10 months.

Name	Acronym	Source	Sample
St. Louis Financial Stress Index	FCI 1	St Louis Fed	1993Q4 - 2012Q1
Kansas City Fed Financial Stress Index	FCI 2	Kansas Fed	1990Q1 - 2012Q1
Cleveland Fed Financial Stress Index	FCI 3	Cleveland Fed	1991Q3 - 2012Q1
Westpac US Financial Stress Index	FCI 4	Bloomberg	1998Q1 - 2012Q1
BofA Merrill Lynch Global FSI	FCI 5	Bloomberg	2000Q1 - 2012Q1
Bloomberg US FCI	FCI 6	Bloomberg	1994Q1 - 2012Q1
Bloomberg US FCI Plus	FCI 7	Bloomberg	1994Q1 - 2012Q1
Chicago Fed National FCI	FCI 8	Chicago Fed	1973Q1 - 2012Q1

Table 1. Financial Conditions and Stress Indexes

In summary, our empirical work involves:

- 1. TVP-FAVARs where all of the model coefficients change over time.
- 2. FA-TVP-VARs where only the VAR coefficients change over time.
- 3. FAVARs where coefficients are constant.
- 4. VAR (FCI) models which are VARs augmented with an FCI.

Complete details of the specification of all models and how they are estimated is provided in the Technical Appendix.

3 Dynamic Model Averaging and Selection

The preceding section discussed the econometrics of a single TVP-FAVAR which imposes the restriction that exactly the same financial variables are used to construct the factors in every time period. Simply using a single TVP-FAVAR can suffer from two sorts of problem. First, it can be over-parameterized. Working with all the possibly relevant financial variables can lead to very parameter rich models.⁴ Second, the best variables to include in an FCI may be changing over time. In this paper, we are interested in the ability of the FCI to forecast the real economy. For the reasons given in the introduction, the financial variables relevant for this may change over time.⁵ Accordingly, we want an approach which allows for such change and develop one in this section. It involves multiple models which are defined by which financial variables are included. To be precise, the TVP-FAVAR defined in (1) contains a n-vector of financial variables, x_t . There are up to $2^n - 1$ restricted versions of this TVP-FAVAR which contain one or more of the n financial variables. These are the types of models we consider in this paper.

⁴There are also statistical reasons for thinking that a strategy of always constructing the factors using all of the elements of x_t is not necessarily optimal, see Boivin and Ng (2006).

⁵A clever approach for deciding which variables should be used to construct a factor is given in Kaufmann and Schumacher (2012). However, this approach does not allow for this decision to be made in a time-varying manner (i.e. it does not allow for different variables to be selected at different points in time).

When faced with multiple models, it is common to use model selection or model averaging techniques. However, in the present context we wish such techniques to be dynamic. That is, in a model selection exercise, we want to allow for the selected model to change over time, thus doing dynamic model selection (DMS). In a model averaging exercise, we want to allow for the weights used in the averaging process to change over time, thus leading to dynamic model averaging (DMA). In this paper, we do DMA and DMS using an approach developed in Raftery et al (2010) in an application involving many TVP regression models. The reader is referred to Raftery et al (2010) for a complete derivation and motivation of DMA. Here we provide a general description of what it does.

Suppose the researcher is working with j = 1, ..., J models and the goal is to calculate $\pi_{t|t-1,j}$ which is the probability that model j applies at time t, given information through time t-1. Once $\pi_{t|t-1,j}$ for j = 1, ..., J are obtained they can either be used to do model averaging or model selection. DMS arises if, at each point in time, the model with the highest value for $\pi_{t|t-1,j}$ is used. Note that $\pi_{t|t-1,j}$ will vary over time and, hence, the selected model can switch over time. DMA arises if model averaging is done in period t using $\pi_{t|t-1,j}$ for j = 1, ..., J as weights. The contribution of Raftery et al (2010) is to develop a fast recursive algorithm for calculating $\pi_{t|t-1,j}$.

To explain this algorithm, let $w_t = (x'_t, y'_t)'$ denote time t data and $w_{1:s} = (w'_1, ..., w'_s)'$ denote all the data up to and including time s. In an online exercise, we wish to use $w_{1:t-1}$ to calculate the time t value of the FCI or to forecast. Given an initial condition, $\pi_{0|0,j}$ for j = 1..., ...J⁶ Raftery et al (2010) derive a model prediction equation using a so-called forgetting factor α :

$$\pi_{t|t-1,j} = \frac{\pi_{t-1|t-1,j}^{\alpha}}{\sum_{l=1}^{J} \pi_{t-1|t-1,l}^{\alpha}},$$
(3)

and a model updating equation of:

$$\pi_{t|t,j} = \frac{\pi_{t|t-1,j} f_j \left(w_t | w_{1:t-1} \right)}{\sum_{l=1}^J \pi_{t|t-1,l} f_l \left(w_t | w_{1:t-1} \right)},\tag{4}$$

where $f_j(w_t|w_{1:t-1})$ is a measure of fit for model j. Many possible measures of fit can be used. Since our focus is on the ability of the FCI to forecast y_t , we set as a measure of fit the predictive likelihood for the macroeconomic variables, $p_j(y_t|w_{1:t-1})$.

We refer the reader to Raftery et al (2010) for additional details, but note here that the calculation of $\pi_{t|t,j}$ and $\pi_{t|t-1,j}$ is simple and fast, involving only recursive evaluation of formulae beginning with $\pi_{0|0,j}$ and not involving the use of simulation methods. To help understand the implication of the choice of α , note that $\pi_{t|t-1,j}$ can be written as:

$$\pi_{t|t-1,j} \propto \prod_{i=1}^{t-1} \left[p_j \left(y_{t-i} | w_{1:t-i-1} \right) \right]^{\alpha^i}.$$

⁶In our empirical work, we make the standard noninformative choice of $\pi_{0|0,j} = \frac{1}{I}$.

Thus, model j will receive more weight at time t if it has forecast well in the recent past (where forecast performance is measured by the predictive density, $p_j(y_{t-i}|w_{1:t-i-1})$). The interpretation of "recent past" is controlled by the forgetting factor, α . For instance, with quarterly data, if $\alpha = 0.99$, forecast performance five years ago receives 80% as much weight as forecast performance last period whereas if $\alpha = 0.95$, it only receives 35% as much weight. $\alpha = 1$ leads to conventional Bayesian model averaging implemented one time period at a time on an expanding window of data. Lower values of α allow for more rapid switching between models.

In the present paper, our set of models is potentially huge. We have n = 20 financial variables (listed in Table A1 in the Data Appendix) and, thus, up to $2^{20}-1$ models. Even with our use of computationally efficient approximations, doing DMA or DMS with large n is very computationally demanding. Accordingly, we take a core set of four variables which are always included in the construction of the FCI: the S&P 500 stock return, the exchange rate, the Household Credit Market Debt Outstanding and the 30 year mortgage rate spread. This means that the factor, f_t , comprises these four financial variables plus any combination of the remaining 16 variables leading to 65,536 TVP-FAVARs (and the same number of FA-TVP-VARs and FAVARs). Using our methods, we are able to estimate all these factor models in about the same computing time required to estimate a single factor model using MCMC methods. Note that the identification restriction on the factor loading vector plus the fact that we are ordering the S&P 500 stock return first, means that our estimated FCI is such that positive (negative) values indicate an improvement (deterioration) in financial conditions.

4 Empirical Results

4.1 Data and Models

We use 20 financial variables which cover a wide variety of financial considerations (e.g. asset prices, volatilities, credit, liquidity, etc.). These are gathered from several sources. Our macroeconomic variables are inflation, GDP growth and the interest rate. All of the variables (i.e. both macroeconomic and financial variables) are transformed to stationarity following Hatzius et al (2010) and many others. The Data Appendix provides precise definitions, acronyms, data sources, sample spans and details about the transformations. Our data sample runs from 1959q1 to 2012q1. All of our models use four lags and, hence, our estimation period begins in 1960Q1. However, data for many of our financial variables begins later than 1959. Treatment of missing values for these variables is discussed in the Technical Appendix.

We remind the reader that a list of the models used (and their acronyms) is given at the end of Section 2 and that complete specification details of all models are presented in the Technical Appendix. Our models are TVP-FAVARs, FA-TVP-VARs, FAVARs and VARs and our methods include DMS, DMA and simply using a single model which includes all 20 of the financial variables. For DMS and DMA, we use differing values for the forgetting factor, α . We distinguish between methods through parenthetical comments so that, e.g., TVP-FAVAR (DMS, $\alpha = 0.95$) does DMS over the 65,536 TVP-FAVARs defined in Section 3 using a forgetting factor of $\alpha = 0.95$.

4.2 Estimating the Financial Conditions Index

Figure 1 plots three different estimates of the FCI (standardized so as to have zero mean and standard deviation of one) using factor methods based on: i) a single FAVAR using all of the financial variables so that the factor has a similar interpretation to that used by other researchers such as Hatzius et al (2010); ii) a single TVP-FAVAR version of i); and iii) dynamic model averaging of the FA-TVP-VARs using $\alpha = 0.95$. Note that, as we shall see in the next sub-section, iii) is the approach which forecasts best.

In general, the three FCI's in Figure 1 exhibit similar patterns. However, the TVP-FAVAR is producing a noticeably more volatile FCI for most of the sample. Using DMA on the FA-TVP-VAR yields the smoothest FCI for most of the sample. However, at the time of the financial crisis, the three FCIs show an equally sharp deterioration in financial conditions. This suggests that using DMA with the FA-TVP-VARs is capable of producing large swings in the FCI, but that it does not choose to do so throughout most of the sample period. The FAVAR is producing results which are similar to, but slightly more volatile than, the FA-TVP-VAR with DMA. However, there are some periods (e.g. 1974) when these two FCI estimates diverge more.



Figure 1: FCI Estimates

Figure 2 compares our FCI, estimated using DMA on the FA-TVP-VARs, to a few existing FCIs selected from the list in Table 1. They are all standardized to have mean zero and standard deviation one. It can be seen that the, although the various FCIs are exhibiting broadly similar patterns, there are periods where they differ substantially. Our FCI is most similar to the St. Louis FSI, although they differ substantially in 2004-2005 in the run-up to the financial crisis. The Bloomberg US FCI is similar to the St. Louis FCI in most periods, but is substantially more volatile in the late 1990s. The Chicago Fed NFCI differs the most from the others. Unlike the others, it does not signal a deterioration in financial conditions in the early 2000s. Furthermore, in the late 1970s and early 1980s it is much more volatile than our FCI.

Figures 1 and 2, of course, do not establish that one FCI is better than another. That issue is addressed in the following sub-section where we compare FCIs in terms of how well they perform in forecasting three major macroeconomic variables. The key point of this sub-section is that we have established that FCIs estimated using different methods can be substantially different from one another. This suggests that care should be taken with the econometric methods used to estimate the FCI.



Figure 2: Comparing our FCI Estimate to Other Available FCIs

Before we turn to forecasting, it is interesting to see if DMA (with the FA-TVP-VARs) is allocating different weight to different financial variables at different points in time and, if so, which financial variables receive more weight at each point in time. Figure 3 sheds light on this issue. It plots posterior inclusion probabilities for the 16 financial variables which are given the DMA treatment (see the end of Section 3 and Table A1 in the Data Appendix for exact definitions and acronyms). These inclusion probabilities are calculated using the model probabilities, $\pi_{t|t-1,j}$ for j = 1, ...J models, defined in Section 3. In particular, the inclusion probability associated with a particular financial variable is the total probability attached to models which include that financial variable. Note that some of the financial variables are not available for our entire sample span and, for these variables, inclusion probabilities are zero for times where data is not available.

The main point worth noting is that the inclusion probabilities do vary over time, indicating that DMA is attaching different weights to different financial variables over time. And DMS will be choosing different variables to construct its FCI.

Remember that our model space is defined so that four financial variables are always included. These four variables are chosen to reflect different aspects of the financial situation: the stock market (S&P500), exchange rates (TWEXMMTH), household debt (CMDEBT) and interest rates (30y Mortgage spread). Thus, interpretations of the inclusion probabilities for the remaining 16 financial variables relate to whether they contain information useful for forecasting macroeconomic variables beyond that provided by these four variables.

Several variables have inclusion probabilities which abruptly rise around 1990 and remain high for most of the remainder of the sample. This partly arises due to the fact that data only becomes available for several of our variables in the late 1980s or early 1990s. But, even for variables with longer data spans (i.e. the LOANHPI index, VXO+VIX, WILL5000PR and Mich), inclusion probabilities often rise substantially around this time. In relation to the housing market, it is interesting to note the growing importance of the LOANHPI index around this time. Another variable relating to the housing market, ABS Issuers (Mortgage), has increasing inclusion probability throughout the 1990s, peaking at the time of the financial crisis. STDSCOM, which relates to bank credit standards for real estate loans, has an inclusion probability with jumps to one as soon as data is available for this variable. Thus, our FCI will have an increasing role for variables relating to housing finance throughout the run-up to the financial crisis. However, after the initial impact of the financial crisis, DMA greatly down-weights several variables, including two relating to the housing market (i.e. the LOANHPI index and ABS Issuers (Mortgage)). That is, their inclusion probabilities drop dramatically in early 2009. Presumably the actions taken by the Treasury and TARP and QE1), which involved the purchase of the Fed around this time (e.g. hundreds of billions of dollars in mortgage backed securities, diminished the usefulness of these financial variables relating to bank provision of mortgage finance for forecasting macroeconomic variables.

Measures of financial volatility (e.g. the MOVE index and VXO+VIX) remain important from the early 1990s until the end of the sample and their inclusion probabilities do not drop after 2009. The Michigan survey of the expected change in the financial situation, which could also be an indicator of financial volatility, also exhibits this pattern.

With regards to the various interest rate spread variables we are including, no clear patterns emerge. The commercial paper spread is unimportant for most of the sample, but as of 2009 its inclusion probability suddenly increases to one. The 2y/3m spread variable, which has an inclusion probability of roughly 0.5 for most of the sample, suddenly sees its important increase in 2008Q4 before falling to zero in early 2009. In contrast, the longer term spread (10/2yr spread) exhibits the opposite pattern, with its importance collapsing in late 2008 before rising in 2009. And the TED spread, which is often considered to be an important indicator of financial conditions, is allocated relatively low probability for most of the sample (and this probability reaches zero at the peak of the recent financial crisis).

A very broad measure of stock market performance, WILL5000PR, does have a very high inclusion probability from 1990 through 2008, but this collapses to zero in 2009. Our commodity price index, CRY index, is never important and DMA never attaches appreciable weight to it when constructing the FCI.



Figure 3: Probability of Inclusion for Each Financial Variable

4.3 Forecasting

In this section, we investigate the performance of a wide range of models and methods for forecasting the macroeconomic variables (inflation, GDP and the interest rate). Our main measure of forecast performance is the square root of mean squared forecast errors (RMSFE) which is evaluated over the period 1974Q1 to 2012Q1-h for h=1,...,8 forecast horizons. Table 2 presents RMSFEs relative to a VAR for y_t (i.e. one which does not include any FCI).

As commonly happens with macroeconomic forecasting, it is difficult to beat a simple benchmark VAR by a large amount. Nevertheless, Table 2 shows some appreciable improvements in forecast performance and some interesting patterns.

First, with some exceptions, we are finding that DMS or DMA do lead to improved forecast performance over approaches involving a single model. The single model cases (TVP-FAVAR, FA-TVP-VAR and FAVAR) which include all of the financial variables typically forecast worse than the benchmark VAR. In contrast, when DMA or DMS is used with these approaches, they forecast better than the benchmark VAR. In sum, we are finding evidence that DMA and DMS techniques are useful in our application. Model switching is occurring and methods which ignore this tend to forecast poorly even if they allow for time-variation in parameters.

Second (and, again, with some exceptions), FA-TVP-VARs forecast better than either the more parameter-rich TVP-FAVARs or the more restrictive FAVARs which do not allow for time-variation in parameters. Presumably the fact that the TVP-FAVAR has time-variation in the high-dimensional factor loading vector leads to an overparameterized model which sometimes forecasts poorly. The best forecast performance is produced by the FA-TVP-VAR with $\alpha = 0.95$. Both DMA and DMS forecast well for this case. For every forecast horizon and macroeconomic variable, the RMSFE is lower than the benchmark VAR. Typically, the improvements in RMSFE are of the order of 5 or 10%, but occasionally they are even better than this (see the short-horizon results for forecasting interest rates). Doing DMA or DMS with the FAVARs with $\alpha = 0.95$ also leads to substantial forecast improvements relative to the benchmark VAR, but these gains are not as great as when doing DMA or DMS with the FA-TVP-VARs.

Third, working with a VAR augmented to include one of the FCIs listed in Table 1 does not lead to good forecast performance. With only a few exceptions, such a strategy actually leads to a decrease in forecast performance relative to the benchmark VAR.

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4.4 Impulse Response Analysis

Figure 4 presents impulse responses of the macroeconomic variables to a negative shock to the FCI (i.e. a deterioration in financial conditions), where the FCI is constructed using DMA methods on the FA-TVP-VARs. Care must be taken in interpreting such a financial shock since the variables used to construct the FCI (and thus the nature of the financial shock) are changing over time. Nevertheless, with this qualification in mind, a study of impulse responses is informative. They are calculated at every time period and for horizons of up to 21 quarters. We can also present the response of any of the financial variables to this shock. For the sake of brevity, we only choose one of these variables: the S&P500. Remember that we use a standard identification scheme to identify this shock (see Section 2.1) and details about estimation of impulse responses are given in the Technical Appendix.

In general, Figure 4 indicates that impulse responses are changing over time, indicating that our use of DMA methods and TVP models is important and constant coefficient FAVARs are mis-specified. Our impulse responses tend to vary a bit more than others in the literature (e.g. Primiceri, 2005). This is due to the fact that our estimates (like our forecasts) are done online and not smoothed. That is, the impulse response at time t is estimated using data available at time t, not T as is done in many papers. We also do not impose a stationarity condition on the time-varying VAR coefficients.

Figure 4 reveals that the financial shock during the recent financial crisis is large and persistent, although at the very end of our sample this effect disappears. But especially in late 2008 and early 2009, impulse responses of all variables fall and do not bounce back to zero. This effect is particularly notable for GDP growth.

However, it is in the late 1970s and early 1980s that we are finding the effects of negative financial shocks to be greatest. This is sensible if we remember that, at each time period, the impulse responses measure the impact of a one standard deviation shock. Our estimated FCI, plotted in Figure 1, suggests that the financial shock which hit in the recent financial crisis was much more than a one standard deviation shock. Our impulse responses are indicating that the 1970s and early 1980s was a time of smaller financial shocks which had large effects. In contrast, the recent financial crisis was a time of a larger financial shocks having a proportionally smaller effect. Nevertheless, if we compare the recent financial crisis to the period preceding it, we see that impulse response functions increased in magnitude.



Figure 4: Impulse Responses to a Negative Financial Shock

5 Conclusions

In this paper, we have argued for the desirability of constructing a dynamic financial conditions index which takes into account changes in the financial sector, its interaction with the macroeconomy and data availability. In particular, we want a methodology which can choose different financial variables at different points in time and weight them differently. We develop DMS and DMA methods, adapted from Raftery et al (2010), to achieve this aim.

Working with a large model space involving many TVP-FAVARs (and restricted variants) which make different choices of financial variables, we find DMA and DMS methods lead to improve forecasts of macroeconomic variables, relative to methods which use a single model. This holds true regardless of whether the single model is parsimonious (e.g. a VAR for the macroeconomic variables) or parameter-rich (e.g. an unrestricted TVP-FAVAR which includes the same large set of financial variables at every point in time). The dynamic FCIs we construct are mostly similar to those constructed using conventional methods. However, particularly at times of great financial stress (e.g. the late 1970s and early 1980s and the recent financial crisis), our FCI can be quite different from conventional benchmarks. The DMA and DMS algorithm also indicates substantial inter-temporal variation in terms of which financial variables are used to construct it.

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A. Data Appendix

The following table describes the series we used to extract our Financial Conditions Index. The fourth column describes the stationarity transformation codes (Tcodes) which have been applied to each variable. Tcode shows the stationarity - Board of Governors of the Federal Reserve System (http://www.federalreserve.gov/); U - University of Michigan Reserve Economic Data (http://research.stlouisfed.org/fred2/); G - Amit Goyal (http://www.hec.unil.ch/agoyal/); R mwatson/).transformation for each variable: Tcode=1, variable remains untransformed (levels) and Tcode=5, use first log differences. The fifth column describes the source of each variable. The codes are: B - Bloomberg; D - Datastream; F - Federal Watson (http://www.princeton.edu/ W. W - Mark (http://www.sca.isr.umich.edu/);

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N_0	Mnemonic	Description	Tcode	Source	Sample
-	SP500	S&P 500 Stock Price Index	ъ	ſщ	1959Q1 - 2012Q1
2	TWEXMMTH	FRB Nominal Major Currencies Dollar Index (Linked To EXRUS In 1973:1)	1	Μ	1959Q1 - 2012Q1
n	CMDEBT	Household Sector: Liabilities: Household Credit Market Debt Outstanding	ъ	Ĺт	1959Q1 - 2012Q1
4	30y Mortgage Spread	30y Conventional Mortgage Rate - 10y Treasury Rate	1	Ĺц	1959Q1 - 2012Q1
ю	ABS Issuers (Mortgage)	Issuers Of Asset-Backed Securities; Total Mortgages	1	Ĺт	1984Q4 - 2012Q1
9	TERMCBAUT048NS	Finance Rate On Consumer Installment Loans, New Autos 48 Month Loan	1	Ĺт	1972Q1 - 2012Q1
2	TED spread	3m LIBOR - 3m Treasury Bill Rate	1	Ĺц	1981Q4 - 2012Q1
x	10/2 y spread	10-Year/2-Year Treasury Yield Spread	1	Ĺц	1976Q3 - 2012Q1
6	2y/3m spread	2-Year/3-Month Treasury Yield Spread	1	Ĺц	1976Q3 - 2012Q1
10	Commercial Paper spread	3-Month Financial Commercial Paper/Treasury Bill Spread	1	В	1971Q2 - 2012Q1
11	LOANHPI Index	Home Loan Performance Index U.S. Index Level	5	В	1976Q1 - 2012Q1
12	High yield spread	BofA Merrill Lynch US High Yield Master II Effective Yield - Moody's BAA	1	Ĺц	1992Q1 - 2012Q1
13	WILL5000PR	Wilshire 5000 Price Index	5	Ĺт	1971Q1 - 2012Q1
14	CRY Index	Thomson Reuters/Jefferies CRB Commodity Index	1	В	1994Q1 - 2012Q1
15	MOVE Index	Merrill Lynch One-Month Treasury Options Volatility Index (MOVE)	1	В	1988Q2 - 2012Q1
16	VX0+VIX	CBOE (S&P100 + S&P500) Volatility Index	1	В	1986Q3 - 2012Q1
17	USBANCD	US Banks Sector CDS Index 5Y - CDS Prem. Mid	1	D	2004Q1 - 2012Q1
18	TOTALSL	Total Consumer Credit Owned And Securitized, Outstanding	5 C	Ĺщ	1959Q1 - 2012Q1
19	STDSCOM	SLOOS: Percentage of banks tightening standards for real estate loans	1	Ч	1990Q3 - 2012Q1
20	Mich	Michigan Survey: Expected Change In Financial Situation	-	11	197801 - 201201

B: Technical Appendix

In this appendix, we describe the econometric methods we use to estimate and forecast with a TVP-FAVAR (and restricted versions of it) along with various specification details.

The Algorithm

We write the TVP-FAVAR compactly as:

 $\begin{aligned} x_t &= z_t \lambda_t + u_t, \quad u_t \sim N\left(0, V_t\right) \\ z_t &= z_{t-1} \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, Q_t\right) \\ \beta_t &= \beta_{t-1} + \eta_t, \quad \eta_t \sim N\left(0, R_t\right) \\ \lambda_t &= \lambda_{t-1} + v_t, \quad v_t \sim N\left(0, W_t\right) \end{aligned}$

where $\lambda_t = (\lambda_t^y, \lambda_t^f)'$. Note that, as specified in the body of the paper, we identify λ_t^f by setting its first element to 1 and this restriction is maintained at all time periods. We also use notation where \tilde{f}_t is the standard principal components estimate of f_t based on x_t (using data up to time t), $z_t = \begin{bmatrix} y_t \\ f_t \end{bmatrix}$, $\tilde{z}_t = \begin{bmatrix} y_t \\ \tilde{f}_t \end{bmatrix}$ and, if a_t is a vector then $a_{i,t}$ is the i^{th} element of that vector and if A_t is a matrix $A_{ii,t}$ is its $(i, i)^{th}$ element. The algorithm below require priors for the initial states. We make the relatively diffuse choices: $f_0 \sim$ N(0, 100). $f_0 \sim N(0, 10)$, $\lambda_0 \sim N(0, I)$, $\beta_0 \sim N(0, I)$. For the EWMA estimates of the error covariance matrices, we initialize them with $\hat{V}_0 = 0.1 \times I$, $\hat{Q}_0 = 0.1 \times I$, $\hat{R}_0 = 10^{-5} \times I$ and $\hat{W}_0 = 10^{-5} \times I$. Note that setting \hat{R}_0 and \hat{W}_0 to small values is motivated by the fact that R_t and Q_t control the degree of evolution in the coefficients. Even apparently small variances such as 10^{-5} will allow for a large degree of coefficient variation in a relatively short time period.

The algorithm extends the one derived in Doz, Giannone and Reichlin (2011) to the TVP-FAVAR and involves two main steps which are repeated for t = 1, ..., T.

Step 1: Conditional on f_t , estimate the parameters in the TVP-FAVAR.

Step 2: Conditional on the estimated TVP-FAVAR parameters from Step 1, use the Kalman filter to produce an estimate of f_t which is used as our FCI.

Step 2 requires no additional explanation, being just a standard application of Kalman filtering in a state space model. We provide exact details of Step 1 here.

Step 1 involves using the priors/initial conditions above for t = 0 values and then, for t = 1, ..., T, proceeding as follows:

1. Calculate the residuals for the state equations, $\hat{\eta}_{t-1}$ and \hat{v}_{t-1} , using the formulas

$$\widehat{v}_{t-1} = \widehat{\lambda}_{t-1} - \widehat{\lambda}_{t-2}, \widehat{\eta}_{t-1} = \widehat{\beta}_{t-1} - \widehat{\beta}_{t-2}.$$

2. Estimate the state covariances R_t and W_t using:

$$\widehat{R}_t = \kappa_3 \widehat{R}_{t-1} + (1 - \kappa_3) \widehat{\eta}_{t-1} \widehat{\eta}'_{t-1}, \widehat{W}_t = \kappa_4 \widehat{W}_{t-1} + (1 - \kappa_4) \widehat{v}_{t-1} \widehat{v}'_{t-1}.$$

3. Calculate the quantities in the Kalman filter prediction equations for λ_t and β_t given information at t-1:

$$\begin{aligned} \lambda_t &\sim N\left(\lambda_{t|t-1}, \Sigma_{t|t-1}^{\lambda}\right), \\ \beta_t &\sim N\left(\beta_{t|t-1}, \Sigma_{t|t-1}^{\beta}\right), \end{aligned}$$

where $\lambda_{t|t-1} = \lambda_{t-1|t-1}$ and $\Sigma_{t|t-1}^{\lambda} = \Sigma_{t-1|t-1}^{\lambda} + \widehat{W}_t$ and $\beta_{t|t-1} = \beta_{t-1|t-1}$ and $\Sigma_{t|t-1}^{\beta} = \Sigma_{t-1|t-1}^{\beta} + \widehat{R}_t$.

4. Compute the measurement equation prediction errors as

$$\widehat{u}_t = x_t - \widehat{x}_{t|t-1}, \widehat{\varepsilon}_t = z_t - \widehat{z}_{t|t-1},$$

where $\widehat{x}_{t|t-1} = \widetilde{z}_t \lambda_{t|t-1}$ and $\widehat{z}_{t|t-1} = z_{t-1}\beta_{t|t-1}$.

5. Estimate the measurement equation error covariance matrices, Q_t and V_t using EWMA specifications:

$$\widehat{V}_t = \kappa_1 \widehat{V}_{t-1} + (1 - \kappa_1) \,\widehat{u}_t \,\widehat{u}_t' \widehat{Q}_t = \kappa_2 \widehat{Q}_{t-1} + (1 - \kappa_2) \,\widehat{\varepsilon}_t \widehat{\varepsilon}_t'$$

6. Update $\lambda_{i,t}$ for each i = 1, ..., n from

$$\lambda_{it} \sim N\left(\lambda_{i,t|t}, \Sigma_{ii,t|t}^{\lambda}\right),$$

where $\lambda_{i,t|t} = \lambda_{i,t|t-1} + \Sigma_{ii,t|t-1}^{\lambda} \widetilde{z}'_t \left(\widehat{V}_t + \widetilde{z}_t \Sigma_{ii,t|t-1}^{\lambda} \widetilde{f}'_t \right)^{-1} \widehat{\varepsilon}_{it}$ and $\Sigma_{ii,t|t}^{\lambda} = \Sigma_{ii,t|t-1}^{\lambda} - \Sigma_{ii,t|t-1}^{\lambda} \widetilde{z}'_t \left(\widehat{V}_t + \widetilde{z}_t \Sigma_{ii,t|t-1}^{\lambda} \widetilde{z}'_t \right)^{-1} \widetilde{z}_t \Sigma_{ii,t|t-1}^{\lambda}$.

7. Update the estimate of β_t given information at time t using:

$$\beta_t \sim N\left(\beta_{t|t}, \Sigma_{t|t}^{\lambda}\right),$$

where
$$\beta_{t|t} = \beta_{t|t-1} + \Sigma_{t|t-1}^{\beta} \widetilde{z}'_{t-1} \left(\widehat{Q}_t + \widetilde{z}_{t-1} \Sigma_{t|t-1}^{\beta} \widetilde{z}'_{t-1} \right)^{-1} \left(\widetilde{z}_t - \widetilde{z}_{t-1} \widehat{\beta}_{t-1} \right)$$
 and $\Sigma_{t|t}^{\beta} = \Sigma_{t|t-1}^{\beta} - \Sigma_{t|t-1}^{\beta} \widetilde{z}'_{t-1} \left(\widehat{Q}_t + \widetilde{z}_{t-1} \Sigma_{t|t-1}^{\beta} \widetilde{z}'_{t-1} \right)^{-1} \widetilde{z}_{t-1} \Sigma_{t|t-1}^{\beta}$.

8. The time t parameter estimates produced are \widehat{V}_t , \widehat{Q}_t , \widehat{R}_t , \widehat{W}_t , $\widehat{\beta}_t = \beta_{t|t}$ and $\widehat{\lambda}_t = \lambda_{t|t}$.

Choice of Decay Factors

For the time-varying error covariances, V_t , Q_t , W_t and R_t , we use EWMA methods which involve decay factors, κ_1 , κ_2 , κ_3 and κ_4 which are used to discount observations in the more distant past and put more weight on the most recent observations. We set $\kappa_1 = \kappa_2 = 0.94$ and $\kappa_3 = \kappa_4 = 0.98$. These values were chosen based on our previous experience (Koop and Korobilis, 2012, 2013). The choices $\kappa_1 = \kappa_2 = 0.94$ are fairly low, allowing for a fair degree of heteroskedasticity in the equations for x_t and $(y'_t, f_t)'$. For fast-moving financial variables and macroeconomic VARs, there is potentially a fairly large amount of volatility and we allow for this. The choices $\kappa_3 = \kappa_4 = 0.98$ relate to the error covariance in the state equations for λ_t and β_t and are inspired by the consideration that we them to be small and varying little over time. In many applications, errors in state equations are assumed to be homoskedastic and we are near to this case. Furthermore, we want to avoid explosive behavior in the VAR coefficients.

Treatment of missing values

In our application our sample is unbalanced, since it contains many financial variables which have been collected only after the 1970s or the 1980s or later. Similar issues are faced by organizations which monitor FCIs, for instance, the Chicago Fed National FCI comprises 100 series where most of them have different starting dates. Although specific computational methods for dealing with such issues exist (e.g. the EM algorithm, or Gibbs sampler with data augmentation), our focus is on averaging over many models which means such methods are computationally infeasible. Accordingly, similar to our purpose of developing a simulation-free and fast algorithm for parameter estimation, we want to avoid simulation methods for estimating the missing data in x_t . Additionally, methods such as interpolation work poorly when missing values are at the beginning of the sample.

Since the missing data in x_t are in the beginning, we make the assumption that the factor (FCI) is estimated using only the observed series. The estimation algorithm above allows for such an approach in a straightforward manner by just replacing missing values with zeros. The loadings λ (whether time-varying, or constant) will become equal to 0, thus removing from the estimate of f_t the effect of the variables in x_t which have missing values at time t. This feature holds both for the initial principal components estimate \tilde{f}_t , as well as the final Kalman filter estimate.

Forecasting and Impulse Responses

One step ahead forecasting is easily done since the Kalman filter referred to in Step 2 provides us with the one-step ahead predictive density. The mean of this density can be used as a point forecast. For h > 1 step ahead forecasts, the predictive density can be produced by using simulation methods. However, this is computationally costly and we do not follow this strategy in this paper. Instead, when forecasting $y_{\tau+h}$ at time τ , we use the estimated VAR coefficients at time τ (i.e. $\hat{\beta}_{\tau}$) and calculate iterated h-step ahead point forecasts using the standard formula for VAR forecasts (see, e.g., Lutkepohl, 2005, page 37).

The identification scheme for calculating impulse responses is described in the paper. Note that this involves an ordering of the variables in the TVP-VAR part of the model and this ordering is $(\pi_t, u_t, r_t, f_t)'$. We use the estimated VAR coefficients and error covariance matrix at time τ to calculate the impulse responses at time τ and assume they are unchanged over the horizon of 21 quarters for which we calculate impulse responses.

Restricted models:

The FA-TVP-VAR is obtained by setting $W_t = 0$ in the TVP-FAVAR, thus producing OLS estimates (on an expanding window of data) for λ in this model. The FAVAR is treated in a similar fashion, except that OLS methods are used on both λ and β . The FAVAR is homoskedastic and the error covariance matrices are estimated using OLS methods on an expanding window of data.

The VAR models are also homoskedastic and their error covariance matrices are estimated as for the FAVARs. We use a N(0, I) prior for the VAR coefficients. Note that this is the same prior as we use to initialize the time-varying VAR coefficients in the TVP-VAR.