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Investigating Economic Uncertainty Using Stochastic Volatility in Mean VARs: The Importance of Model Size, Order-Invariance and Classification∗

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Investigating Economic Uncertainty Using Stochastic Volatility in Mean VARs: The Importance of Model Size, Order-Invariance and Classification*

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Abstract

Stochastic Volatility in Mean Vector Autoregressions (SVMVARs) are popularly used to jointly estimate macroeconomic and financial uncertainty and their economic effects. However, SVMVARs are computationally demanding. To ease the computational burden, existing approaches limit the number of variables included, adopt a specification which is not invariant to the way the variables are ordered and require the researcher to classify each variable as macroeconomic or financial before estimation. To overcome these limitations, we develop an efficient Markov Chain Monte Carlo (MCMC) algorithm for SVMVARs which are large, order-invariant and have unclassified variables. For each unclassified variable, the algorithm determines the appropriate classification at each point in time. We demonstrate the importance of these extensions using a large SVMVAR with over 40 U.S. variables, 16 of which are treated as unclassified. We show that smaller SVMVARs overestimate the economic effects of macroeconomic uncertainty, failing to reveal that financial uncertainty plays a larger role. When using large SVMVARs, however, different orderings yield conflicting results and it becomes critical to use an order-invariant specification. We also find that most unclassified variables change classification over time with changes often occurring during crisis periods.

Keywords: Large VAR, Uncertainty, Stochastic Volatility, Order Invariance

JEL Codes: C11, C52, C55, E32, E44.

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1 Introduction

Economists and policymakers have begun to investigate the different roles played by macroeconomic and financial uncertainty. But can we easily measure macroeconomic and financial uncertainty? Many studies attempt to do so. Financial uncertainty is regularly proxied using financial market volatility (Bloom, 2009) while disagreement among forecasters (Bachmann, Elstner and Sims, 2013), the magnitude of surprises when economic data is released (Scotti, 2016) and the number of terms relating to uncertainty in newspapers (Baker, Bloom and Davis, 2016) can be used to capture different dimensions of macroeconomic uncertainty.

Producing comparable measures of macroeconomic and financial uncertainty is even more challenging. Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma and Ng (2021) extract the common variation in the unforecastable component of a large number of macroeconomic and financial time series. However, a growing literature instead equates uncertainty with Stochastic Volatility (SV), the time-varying second moments of time series variables (see Carriero, Clark and Marcellino, 2018, 2020; Carriero, Clark, Marcellino and Mertens, 2021; Berger, Grabert and Kempa, 2016; Mumtaz and Theodoridis, 2017; and Mumtaz and Musso, 2021 among many others). Within this literature, the Stochastic Volatility in Mean Vector Autoregression (SVMVAR) is a popular model. In the SVMVAR, macroeconomic (financial) uncertainty is modeled as the common component driving the volatilities of all macroeconomic (financial) variables. The SVMVAR is attractive since it jointly estimates uncertainty (through the SV in the errors) and produces an estimate of its impact on the economy (by allowing the SVs to enter the conditional mean of the VAR).

Despite its popularity, the SVMVAR faces three major challenges. First, efficient MCMC algorithms typically used to estimate VARs with SV cannot be applied once SV is added to the mean. Computationally demanding alternatives involving a Metropolis step (see Jacquier, Polson and Rossi, 2002) or particle Gibbs step (see Andrieu, Doucet and Holenstein, 2010 and Lindsten, Jordan and Schon, 2014) must instead be used. This limits the number of variables which can be included in the model. This is an unattractive property given the literature's recent focus on using large VARs (see, among many others, Banbura, Giannone and Reichlin, 2010; Koop and Korobilis, 2013; Korobilis and Pettenuzzo, 2019; Carriero, Clark and Marcellino, 2019; Gefang, Koop and Poon, 2022; and Chan, 2022a).

Second, existing SVMVAR specifications rely on the use of a lower triangular parameterization for modeling the reduced-form error covariance matrix. This does not relate to structural identification. Rather, it facilitates model estimation and improves computational efficiency (Chan, 2022b). However, recent studies have highlighted that this lower triangular structure results in an order-dependence issue where the results can change depending on the way variables are ordered. This effects both structural analysis and forecasting, becoming an acute problem when large models are considered (Chan, Koop and Yu, 2021, Hartwig, 2019, Arias, Rubio-Ramirez and Shin, 2021, Chan, Doucet, Len-Gonzlez and Strachan, 2018).

Third, it remains unclear how dozens of series should be classified prior to estimation. Should variables relating to money supply, credit, exchange rates, interest rates and stock prices be classified as macroeconomic or financial? If 10 such variables are included in a model, this means there are 10^2 possible classification schemes which the researcher must choose between. This has led to different studies (for example, Carriero, Clark and Marcellino, 2018, Ludvigson, Ma and Ng, 2021 and Redl, 2020) classifying key variables such as the federal funds rate and stock price in different ways.

To circumvent these challenges, this paper develops a novel model and MCMC algorithm for SVMVARs which are large, order-invariant and have unclassified variables. Our computationally efficient algorithm builds on Cross, Hou, Koop and Poon (2022), exploiting band and sparse matrix algorithms. This scalable approach allows us to consider the estimation of very large SVMVARs. Following Bertsche and Braun (2022) and Chan, Koop and Yu (2021), we also use a specification which does not rely on a lower triangular structure for the reduced-form error covariance matrix and is, thus, order invariant. Our order-invariant approach instead identifies the model using multivariate SV. Using our model, we can also distinguish between macroeconomic, financial and unclassified variables. Specifically, we allow the algorithm to determine whether each unclassified variable should be included in the macroeconomic or financial block. As the structure of the economy evolves or experiences major crises, a variable's classification may change over time. Consequently, we allow for Markov switching time-varying classification.

Other studies which deploy SVMVAR methods include Cross, Hou and Poon (2018) who examine the effects of domestic and foreign uncertainty in three small open economies and Carriero, Clark and Marcellino (2020) who examine comovements in macroeconomic uncertainty across advanced economies. The study most closely related to ours is Carriero, Clark and Marcellino (2018), hereafter CCM. CCM develop Bayesian methods to estimate a 30 variable SVMVAR and measure the effects of macroeconomic and financial uncertainty on the US economy. The forthcoming corrigendum corrects a mistake in the algorithm but this does not significantly affect the overarching results. However, the authors note that issues with the mixing and convergence of the MCMC chain are heightened and a shorter sample period is needed to improve model stability.

In our empirical work, we use U.S. data to contrast results obtained from six different SVMVARs including a 43 variable order-invariant SVMVAR (OI-SVMVAR) with 16 unclassified variables. This allows us to assess the importance of model size, order invariance and time-varying classification when estimating macroeconomic and financial uncertainty and their impact on the economy. When a larger 43 variable model is used, model misspecification issues resulting from omitted variables bias are likely to be alleviated. We show that smaller 30 variable SVMVARs tend to overestimate the effects of macroeconomic uncertainty but the effects of financial uncertainty are less sensitive to the size of our dataset. Using our larger models it therefore becomes clear that with macroeconomic uncertainty playing a smaller role, financial uncertainty has a more pronounced effect on economic activity. This aligns with Ludvigson, Ma and Ng (2021) who also find that financial rather than macroeconomic uncertainty plays a larger role in lowering economic activity.

We also find that to produce robust estimation results it is critical to use an order invariant specification. Many studies using the lower triangular parameterization order the macroeconomic variables before the financial variables. This implicitly assumes that the variation in financial variables is partly explained by the volatility of macroeconomic variables with the remaining variation explained by the volatility of financial variables. We show that this leads to underestimation of the common SV of financial variables, our measure of financial uncertainty. This means that order-dependent version of our model (i.e. one which uses the lower triangular parameterization) fails to detect the substantive effect financial uncertainty has on the economy. Conversely, when the ordering of variables is reversed (i.e. an upper triangular parameterization is used) then the magnitude of the financial uncertainty estimate increases substantially. The order-invariant specification adopted in this paper does not suffer from these order dependence issues and produces robust results in larger models.

Last, we show that most of our unclassified variables change classification at some point during the sample with important shifts often occurring during crises. Allowing for unclassified variables therefore ensures that variables are assigned to the appropriate block when our uncertainty measures and their impacts are estimated.

The remainder of this paper is organized as follows. Section 2 introduces our OI-SVMVAR model with time-varying classification. Section 3 outlines the data used and different model specifications. Section 4 discusses our empirical results. Section 5 concludes. Our Appendix contains a Data Appendix, Technical Appendix and supplementary figures.

2 A Model to Distinguish between Macroeconomic and Financial Uncertainty

In this section, we describe a new SVMVAR which is order-invariant and allows for uncertainty in the way variables are classified. We then provide an informal description of the MCMC algorithm which allows for efficient Bayesian inference in our large OI-SVMVAR with time-varying classification. Full details of the the priors and algorithm are provided in the Technical Appendix.

2.1 The Order-Invariant Stochastic Volatility in Mean VAR

Let $y_t^m = (y_{1,t}^m, \ldots, y_{n_m,t}^m)$ be an $n_m \times 1$ vector of macroeconomic variables, $y_t^f = (y_{1,t}^f, \ldots, y_{n_f,t}^f)$ be an $n_f \times 1$ vector of financial variables, and $y_t^u = (y_{1,t}^u, \ldots, y_{n_u,t}^u)$ be an $n_u \times 1$ vector of unclassified variables that could belong to either the macro or financial block. One contribution of this paper lies in the treatment of these unclassified variables.

We consider the following SVMVAR model, denoting $\mathbf{y}_t = (y_t^{m'}, y_t^{u'}, y_t^{f'})'$ and $n = n_m + n_f + n_u$:

$$\mathbf{y}_t = \sum_{i=1}^{p} \mathbf{B}_i \mathbf{y}_{t-i} + \sum_{j=0}^{q} \mathbf{A}_j \mathbf{h}_{t-j} + \mathbf{B}_0^{-1} \boldsymbol{\epsilon}_t^y, \quad \boldsymbol{\epsilon}_t^y \sim \mathcal{N}(\mathbf{0}, \mathbf{U}_t), \tag{1}$$

where $\mathbf{B}_0$ is a non-singular unrestricted $n \times n$ matrix, $\mathbf{B}_1, \ldots, \mathbf{B}_p$ are $n \times n$ VAR coefficient matrices and $\mathbf{h}_t = (h_{m,t}, h_{f,t})'$ is a 2×1 vector of common log-volatilities which capture the co-movement in the time-varying variances of the macro and financial variables. We use $e^{\frac{1}{2}h_{m,t}}$ and $e^{\frac{1}{2}h_{f,t}}$ as our measures of macro and financial uncertainty - these will be described in more detail shortly. The coefficient matrices associated with the common log-volatilities $\mathbf{A}_0, \ldots, \mathbf{A}_q$ are of dimension $n \times 2$ and capture the effects of the contemporaneous and lagged common log-volatilities on the VAR variables.

It is worth emphasizing that we do not assume that the contemporaneous coefficient matrix $\mathbf{B}_0$ has a lower triangular structure. The latter assumption is widely applied in macroeconomic analyses using multivariate SV models. However, recent studies have uncovered that order dependence issues resulting from this modeling strategy tend to become serious in large models. To this end, we adopt the order invariant approaches of Bertsche and Braun (2022) and Chan, Koop and Yu (2021) by specifying an unrestricted $\mathbf{B}_0$ in our proposed SVMVAR model. It has been shown in these papers that the contemporaneous coefficient matrix $\mathbf{B}_0$ is identified up to permutations and sign changes given that the volatility is changing over time.

Turning back to our uncertainty measures, the matrix $\mathbf{U}_t$ is assumed to be diagonal:

$$\mathbf{U}_t = \begin{pmatrix} \Omega_{m,t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Omega_{u,t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Omega_{f,t} \end{pmatrix}, \tag{2}$$

where the volatilities of the macro, financial and unclassified variables are respectively defined as $\Omega_{m,t} = \mathrm{diag}(e^{\omega^m_{1,t}}, \ldots, e^{\omega^m_{n_m,t}})$, $\Omega_{f,t} = \mathrm{diag}(e^{\omega^f_{1,t}}, \ldots, e^{\omega^f_{n_f,t}})$ and $\Omega_{u,t} = \mathrm{diag}(e^{\omega^u_{1,t}}, \ldots, e^{\omega^u_{n_u,t}})$.

For variables in the macro and financial blocks, their log-volatilities are specified as:

$$\omega^m_{i,t} = \eta^m_{i,t} + h_{m,t}, \quad i = 1, \ldots, n_m, \tag{3}$$

$$\omega^f_{i,t} = \eta^f_{i,t} + h_{f,t}, \quad i = 1, \ldots, n_f, \tag{4}$$

where $\eta^m_{i,t}$ and $\eta^f_{i,t}$ are the idiosyncratic volatility components associated with the ith macro and financial variables respectively. Equations (3) and (4) indicate that the volatility of each variable in the macro (financial) block is defined as the sum of its idiosyncratic volatility and the common log-volatility of the macro (financial) variables.

The log-volatilities for the unclassified variables are specified as:

$$\omega^u_{i,t} = \eta^u_{i,t} + h_{s_{i,t},t}, \quad i = 1, \ldots, n_u, \tag{5}$$

where $s_{i,t} \in \{m, f\}$ is the indicator variable for the ith unclassified variable, which is assumed to follow a Markov switching process with transition probability $p(s_{i,t} = k | s_{i,t-1} = l) = p^i_{l,k}$, $k, l \in \{m, f\}$. Note that the volatility of the ith unclassified variable is again defined as the sum of two components. The first component is the idiosyncratic component denoted as $\eta^u_{i,t}$, and the second component is determined by the indicator variable $s_{i,t}$ as either the common log-volatility of the macro block or of the financial block. For example, if $s_{i,t} = m$, then

$h_{s_{i,t},t} = h_{m,t}$, which indicates that the ith unclassified variable belongs to the macro block at time t. This specification not only allows each unclassified variable to be assigned to either the macro or financial block, but does so in a time-varying fashion. So it is possible that a variable switches from the financial block to the macro block (or vice versa). This allows us to investigate a range of interesting possibilities. For instance, the volatility of a variable may appear like financial volatility in normal times but like macro volatility in times of crisis.

We follow CCM in assuming that our measures of uncertainty depend not only on past uncertainty but also past values of the variables themselves. That is, we assume the common log-volatilities evolve as the following VAR process:

$$\mathbf{h}_t = \sum_{i=1}^{p_h} \mathbf{\Phi}_i \mathbf{h}_{t-i} + \sum_{j=1}^{p_y} \mathbf{\Psi}_j \mathbf{y}_{t-j} + \boldsymbol{\epsilon}_t^h, \quad \boldsymbol{\epsilon}_t^h \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}_h\right), \tag{6}$$

where the initial state is specified as $\mathbf{h}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_h)$. The idiosyncratic log-volatilities are assumed to follow stationary AR(1) processes:

$$\eta_{i,t}^k = \rho_{k,i}\eta_{i,t-1}^k + \epsilon_{i,t}^k, \quad \epsilon_{i,t}^k \sim \mathcal{N}(0, \sigma_{k,i}^2), \tag{7}$$

for $i = 1, \ldots, n_k,\ k \in \{m, f, u\}$ and $|\rho_{k,i}| < 1$. The initial state is specified as $\eta_{i,1}^k \sim \mathcal{N}\left(0, \frac{\sigma_{k,i}^2}{1-\rho_{k,i}^2}\right)$. We also note that in (1) and (6) we follow CCM in setting the lag orders of our models to $p = 6$, $q = 2$ and $p_h = 2$ and $p_y = 1$ respectively.

It is worth mentioning that Bertschke and Braun (2022) and Chan, Koop and Yu (2021) show that $\mathbf{B}_0$ is identified up to permutation of its columns. With regards to the common stochastic volatilities that we interpret as macroeconomic and financial uncertainties, in general, SVMVARs with unrestricted $\mathbf{B}_0$ matrix would result in a potential label switching problem. However, it can be shown that this label switching problem would not influence our empirical results (see Technical Appendix for details).

2.2 Posterior Inference in the OI-SVMVAR

Bayesian inference in VARs with SV is typically undertaken using MCMC methods involving the auxiliary mixture sampler of Kim, Shephard and Chib (1998). However, once SV is added to the mean the auxiliary mixture sampler can no longer be used. Instead, papers such as CCM use particle filtering and, in particular, the particle Gibbs step proposed by Andrieu, Doucet and Holdenstein (2010). However, particle filtering can be computationally burdensome in large models and can suffer from particle degeneracy problems. These points are demonstrated in Cross, Hou, Koop and Poon (2022) who develop an MCMC algorithm which uses a Metropolis-Hastings step.[1] This involves a Gaussian candidate generating density with variance depending on the Hessian of the conditional posterior of the log-volatilities. Crucially, this Hessian is block-banded. Band and sparse matrix algorithms can therefore be exploited to allow for efficient computation even in large SVMVARs. This opens the door to Bayesian estimation of very large SVMVARs such as those considered in this paper.

In the present paper, we extend the methods of Cross, Hou, Koop and Poon (2022) to allow for variables whose classification is uncertain. Conditional on knowing the way the variables are classified (i.e. conditional on $(s_{i,1}, \dots, s_{i,T})$ for $i = 1, \dots, n_u$), we can transform our model by first reordering the equations of the unclassified variables so as to group them appropriately with those predetermined macro and financial variables. After this transformation, the transformed model reduces to a standard SVMVAR and the methods of Cross, Hou, Koop and Poon (2022) can then be applied directly to sample the log-volatilities $(\mathbf{h}_1, \dots, \mathbf{h}_T)$. Next, it can be shown that, given the log-volatilities $(\mathbf{h}_1, \dots, \mathbf{h}_T)$, draws of the indicator variable $(s_{i,1}, \dots, s_{i,T}), i = 1, \dots, n_u$ and the Markov transition probabilities $(p^i_{m,m}, p^i_{m,f}, p^i_{f,m}, p^i_{f,f})$ can be directly obtained by using the algorithm of Chib (1996). More details about prior and the MCMC sampler are given in the Technical Appendix B.

[1] While Jacquier, Polson and Rossi, 2002 also use a Metropolis step to draw the log-volatilities this must be done date-by-date. In contrast, Cross, Hou, Koop and Poon (2022) sample the log-volatilities across all dates in a single step, significantly reducing computation time.

3 Large Dataset, Unclassified Variables and Models

In this section, we discuss our large dataset and unclassified variables. We also provide an overview of the six different SVMVARs estimated to analyze the importance of model size, order invariance and time-varying classification. The complete list of variables and their abbreviations and transformations are given in the Data Appendix. Like CCM and Jurado, Lugvigson and Ng (2015), all our models are estimated with standardized data.

CCM include 30 monthly U.S. variables in their analysis, classifying 18 variables as macroeconomic and 12 as financial. Originally, they used a sample from July 1960 to December 2014 but in their forthcoming corregendum, which corrects the algorithm used, the sample instead starts in January 1985 to minimize instabilities. The macroeconomic series and some financial series are obtained from the FRED-MD dataset described in McCracken and Ng (2016) with the remaining variables available from Kenneth French. Determining the classification of the federal funds rate, the S&P 500 and credit spread is particularly challenging since this "reflects some choice as to what constitutes a macroeconomic variable rather than a financial variable" (CCM, page 804). CCM suggest that the federal funds rate should be treated as a macro variable since it is the instrument of monetary policy. That said, studies as recent as Redl (2020) instead treat the policy rate as a financial variable. For the S&P 500 and credit spread "the distinction between macro and finance is admittedly less clear" (CCM, page 805) but ultimately CCM place them in the group of financial variables.

In our analysis, we use an updated and extended version of the dataset considered in CCM to estimate the six SVMVARs outlined in Table 1. The sample spans January 1960 to October 2021, allowing us to capture the coronavirus pandemic. Model 1 is our benchmark model - it is order dependent, includes the same 30 variables as CCM and classifies them in the same way with 18 variables treated as macroeconomic and the remaining 12 as financial. Model 2 is almost identical but utilizes our order invariant specification.

Models 3 and 4 are very large including 43 variables and introducing time-varying classification. In these models we treat the federal funds rate, credit spread and S&P 500 as unclassified variables. We also include data on an additional 13 unclassified variables spanning: money supply, credit, house prices, interest rates and exchange rates. These variables were obtained from the FRED-MD and FRED datasets with historical house price data available from Robert Shiller. The only difference between models 3 and 4 is that model 4 is order invariant. Models 5 and 6 mirror models 3 and 4 but the ordering of the variables is reversed to examine whether the results obtained are robust to this change.

As previously emphasized, when $\mathbf{B}_0$ is not left unrestricted, the ordering of the variables matters. For models 1 and 3 which involve a lower triangular parameterization, we order the macro variables before the financial variables as is common practice in the literature. This implies that the variation in financial variables is explained by the volatility of macroeconomic variables with the remaining variation explained by the volatility of financial variables. When assessing robustness to reversing the ordering of variables, an upper triangular parameterization is used and financial variables are then ordered before macro variables. Models 4 and 6 should produce the same resuilts, we include both just to confirm empirically that this is so.[2]

Table 1: Summary of SVMVAR Models

Main Empirical Analysis					
Models	30 variables	43 variables	Time-Varying Classification (TVC)	Order-Invariant (OI)	Specification of $\mathbf{B}_0$
1. CCM-30	•				lower triangular
2. OI-30	•			•	unrestricted
3. CCM-TVC-43		•	•		lower triangular
4. OI-TVC-43		•	•	•	unrestricted
Assessing Robustness to Reversing Variable Ordering					
5. TVC-RO-43		•	•		upper triangular
6. OI-TVC-RO-43		•	•	•	unrestricted

[2]It is worth stressing that these models, which adopt different assumptions about $\mathbf{B}_0$ and the ordering of variables, are reduced form models. Our empirical results include impulse responses which involve a structural identification assumption detailed below. This assumption is the same for all models.

4 Empirical Results

In this section, we contrast and present results from our six different SVMVARs. We consider the importance of model size, order invariance and time-varying classification when measuring uncertainty and its economic effects. We find that when using a larger SVMVAR, the economic effects of macroeconomic uncertainty are smaller with financial uncertainty now dominating. We will also demonstrate that in larger models the results obtained are sensitive to the ordering of the variables underscoring the importance of adopting an order-invariant specification. We also show that the majority of our unclassified variables exhibit time-varying classification.

4.1 The Importance of Model Size

We begin by contrasting the uncertainty estimates and impulse response functions obtained from our two smaller models, CCM-30 and OI-30, and our large order invariant model, OI-TVC-43. Throughout, we report the cumulated impulse response functions. Specifically, we transform the impulse responses of each variable by multiplying them with the standard deviations used to standardize the data prior to estimation. We then cumulate them to obtain the impulse responses in levels or log levels.

As demonstrated in CCM, in the SVMVAR uncertainty shocks are orthogonal to the VAR shocks by construction, providing structural identification. To separately identify the effects of macro and financial uncertainty, for all of our models we follow standard practice (see CCM and Banbura, Giannone and Reichlin, 2010 among many others), assuming that macroeconomic uncertainty affects financial uncertainty contemporaneously while financial uncertainty affects macroeconomic uncertainty with a lag.[3]

Our uncertainty estimates are presented in Figure 1. We can see that the estimates

[3]Specifically, we assume that the reduced-form errors in equation (6), which describes the evolution of our common log-volatilities, can be decomposed as $\boldsymbol{\epsilon}_t^h = \mathbf{L}\mathbf{e}_t^h$. In models 1 to 4 $\mathbf{L}$ is a lower triangular matrix such that $\mathbf{L}\mathbf{L}' = \boldsymbol{\Sigma}_h$ and $\mathbf{e}_t^h$ is a vector of uncorrelated structural shocks. When the ordering of our variables is reversed in models 5 and 6, $\mathbf{L}$ is instead an upper triangular matrix with $\mathbf{L}\mathbf{L}' = \boldsymbol{\Sigma}_h$.

produced by our smaller models are nearly identical and that the broad trends observed are similar across all three models. There are, however, some important differences in the magnitude of our uncertainty measures particularly during economic crises when uncertainty is high. We find that our small and large models produce similar estimates of macroeconomic uncertainty throughout the sample with the exception of the Great Recession and coronavirus pandemic where small models underestimate macro uncertainty. Without a larger information set and data-driven classification, however, financial uncertainty is underestimated throughout the 1980s, 1990s and early 2000s. More recently, financial uncertainty is overestimated when using smaller models. This is particularly notable during the pandemic where we would expect the increase in financial uncertainty to be more modest as captured by our large model, OI-TVC-43.

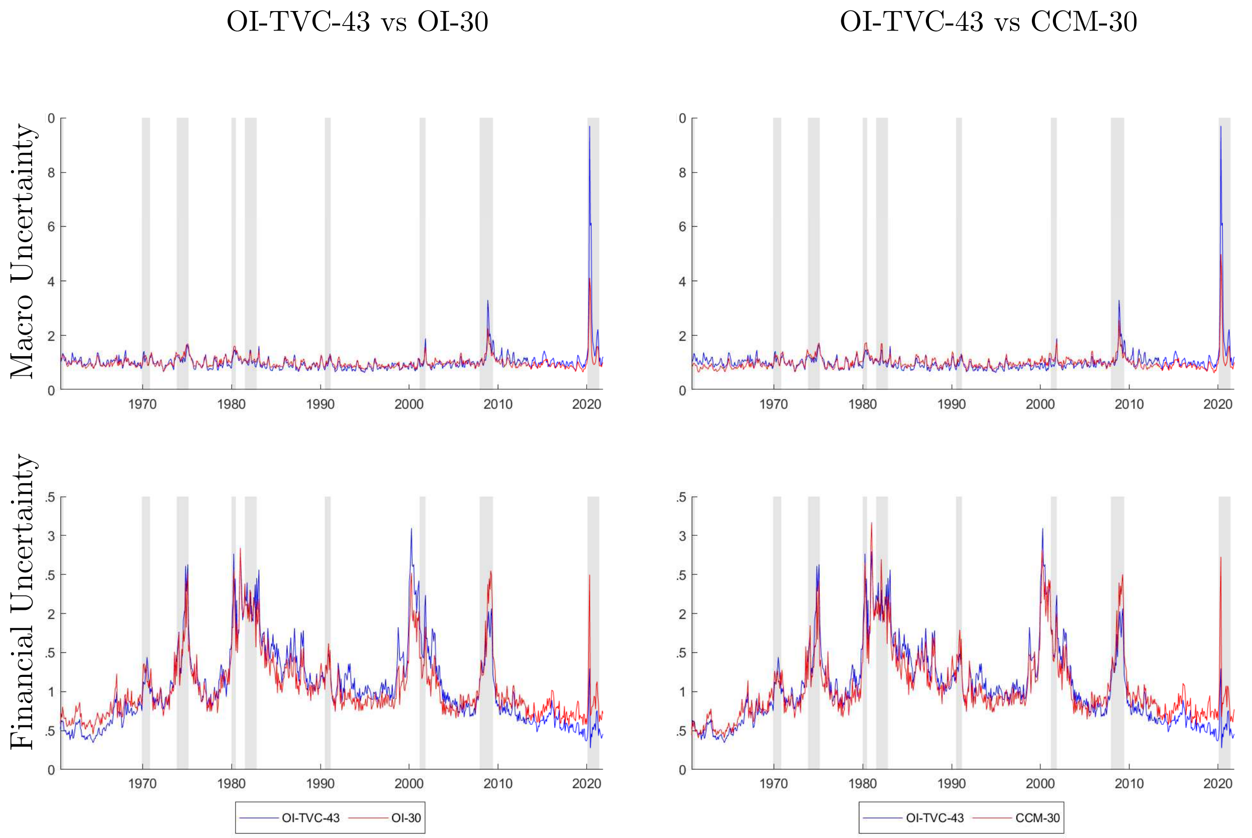


Figure 1: Uncertainty estimates: posterior medians of the macro ($e^{\frac{1}{2}h_{m,t}}$) and financial uncertainty ($e^{\frac{1}{2}h_{f,t}}$). Grey shading indicates NBER recession periods.

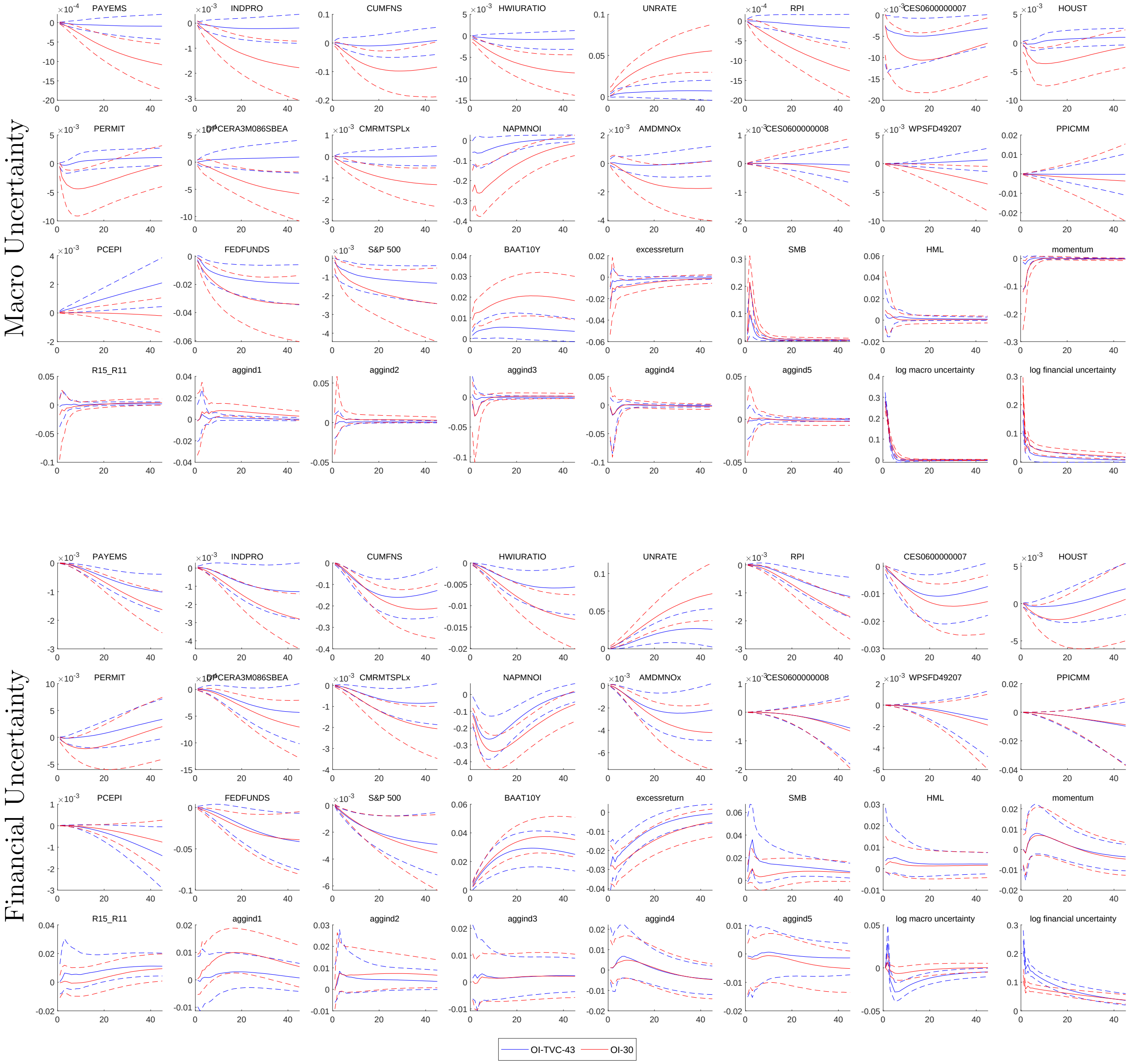


Figure 2: Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for OI-30 and OI-TVC-43

Focusing on the role of model size, we now contrast impulse responses generated by our two order-invariant models OI-TVC-43 and OI-30 in Figure 2. Before comparing results from our two models, it is worth noting that our OI-30 results are qualitatively similar to CCM. In response to a macroeconomic uncertainty shock, there is a decline in real economic activity, little movement in prices and a subsequent monetary policy easing. The responses of financial indicators are also muted with the exception of the credit spread. A financial uncertainty shock has similar effects but fails to dampen the housing sector and affects financial indicators including the S&P 500, credit spread and excess returns more strongly.

When a larger model is considered, however, we can shed further light on the relative importance of macroeconomic and financial uncertainty shocks. This is because issues around model misspecification arising from omitted variables bias are likely to be alleviated. In our case, we can clearly see that using our larger OI-TVC-43, the effects of macro uncertainty are far less pronounced with many credible intervals spanning zero. Where responses are non-zero, they are relatively modest. For example, we see a small increase in the unemployment rate and credit spread and a slight fall in the federal funds rate and the S&P 500.

In contrast, the effects of the financial uncertainty shock are similar for the OI-TVP-43 and OI-30 with a greater overlap of the impulse responses. Overall, our findings suggest that financial uncertainty has a stronger adverse impact on the economy than macroeconomic uncertainty. This aligns with Ludvigson, Ma and Ng (2021) who, using a small-scale trilateral VAR and a novel structural identification scheme, also uncover that financial uncertainty plays a larger role in lowering economic activity. They also find that macroeconomic uncertainty has a positive effect on output in the short-run. However, it is possible that this pattern is induced by omitted variables bias and that inclusion of other key variables in the model such as the federal funds rate eliminates this puzzle.

4.2 The Importance of Order Invariance

Having revealed the dominant role played by financial uncertainty in lowering economic activity, we now assess the severity of order dependence issues. If we contrast the results obtained from CCM-30 and its order invariant counterpart OI-30, we saw in Figure 1 that they produce similar measures of uncertainty. We can also see from Figure 3 that the impulse response functions look very similar. For brevity, we do not include a comparison of CCM-

30 and its counterpart with the ordering reversed but we again find very similar results.[4] Overall, this demonstrates that the ordering issue is not as severe when we use smaller models.

We next compare two order dependent models, CCM-TVC-43 and its smaller counterpart CCM-30. We can clearly see from Figure 4 (left panel) that when using a larger model with an order dependent specification, our financial uncertainty measure is much smaller in magnitude. This result arises directly from $\mathbf{B}_0$ having a lower triangular parameterization. Since our macro variables are ordered before our financial variables, the variation in financial variables will be explained not only by financial uncertainty (proxied by the common SV of financial variables) but also macroeconomic uncertainty (proxied by the common SV of macroeconomic variables). Unsurprisingly, this leads to financial uncertainty being underestimated when more variables are included in the model. If we consider Figure 5, this has important repercussions with the larger model incorrectly detecting that financial uncertainty as well as macroeconomic uncertainty do not have adverse effects on the economy.

If we then reverse the ordering of the variables in our large order dependent model CCM-TVC-43 and consider CCM-TVC-RO-43, our uncertainty measures are again shown in Figure 4 (right hand panel). The magnitude of our financial uncertainty measure has now increased since the variation in financial variables is now explained by the volatility of financial variables but not macro variables. The adverse response to a financial uncertainty shock consequently become much larger but is less precisely estimated as shown in Figure 6.

These findings illustrate that when we use a lower triangular parameterization in large SVMVARs the impulse response functions are highly sensitive to the ordering. In fact, the two sets of impulse responses shown in Figure 6 have contradictory findings regarding whether financial uncertainty affects the economy. Consequently, an order-invariant approach is required to obtain robust results in larger models. Using our order-invariant specification, we obtain very similar results even when we reverse the ordering of the variables (see Figure 8

[4] These are available upon request.

in the Supplementary Figures Appendix). The very slight differences are due to MCMC approximation error.

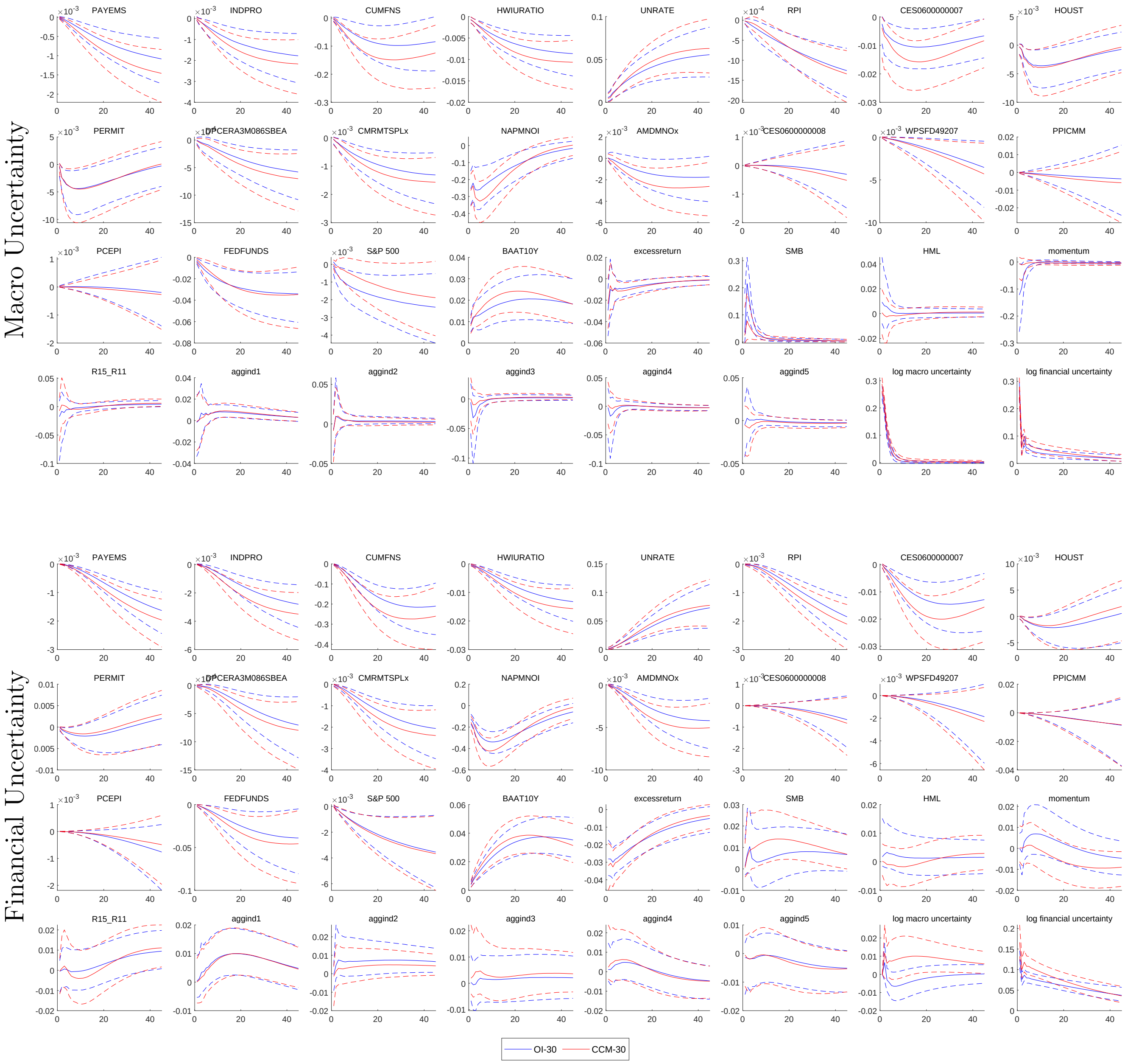


Figure 3: Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals for OI-30 and CCM-30

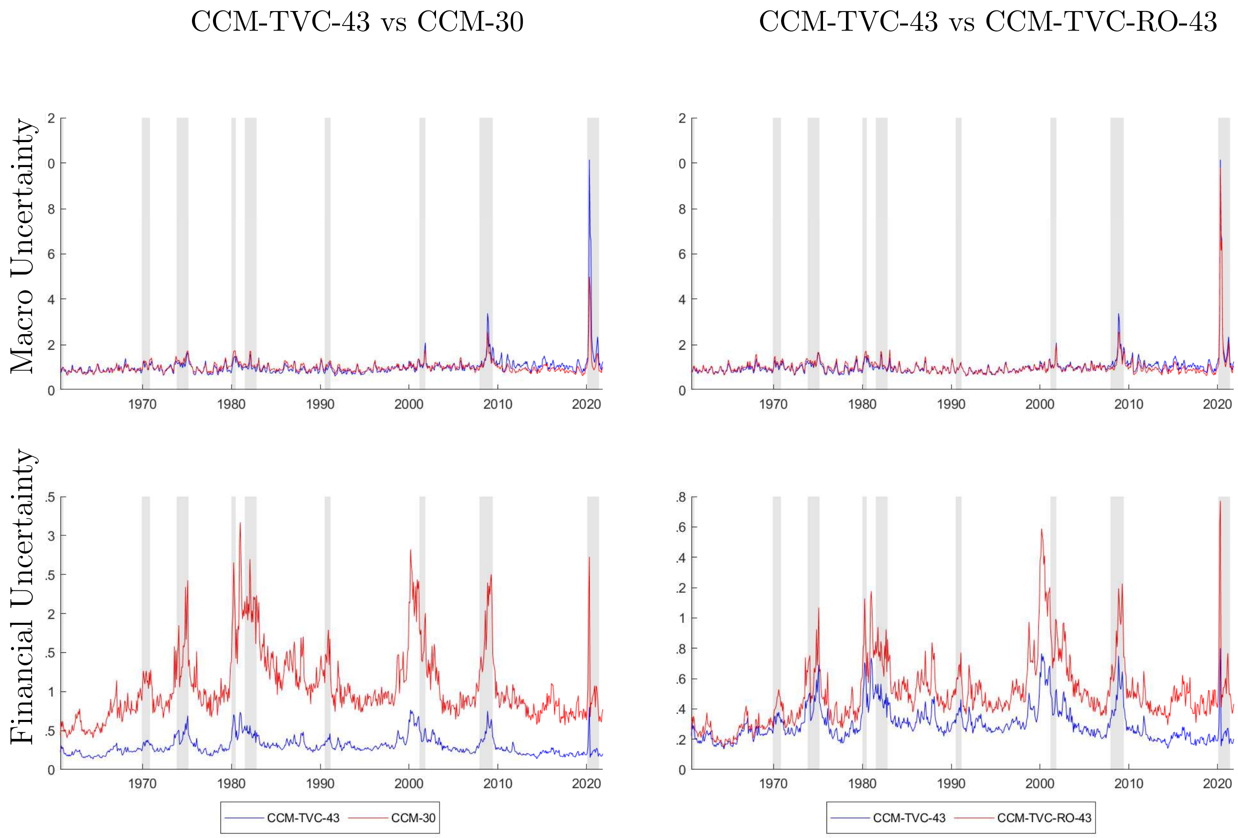
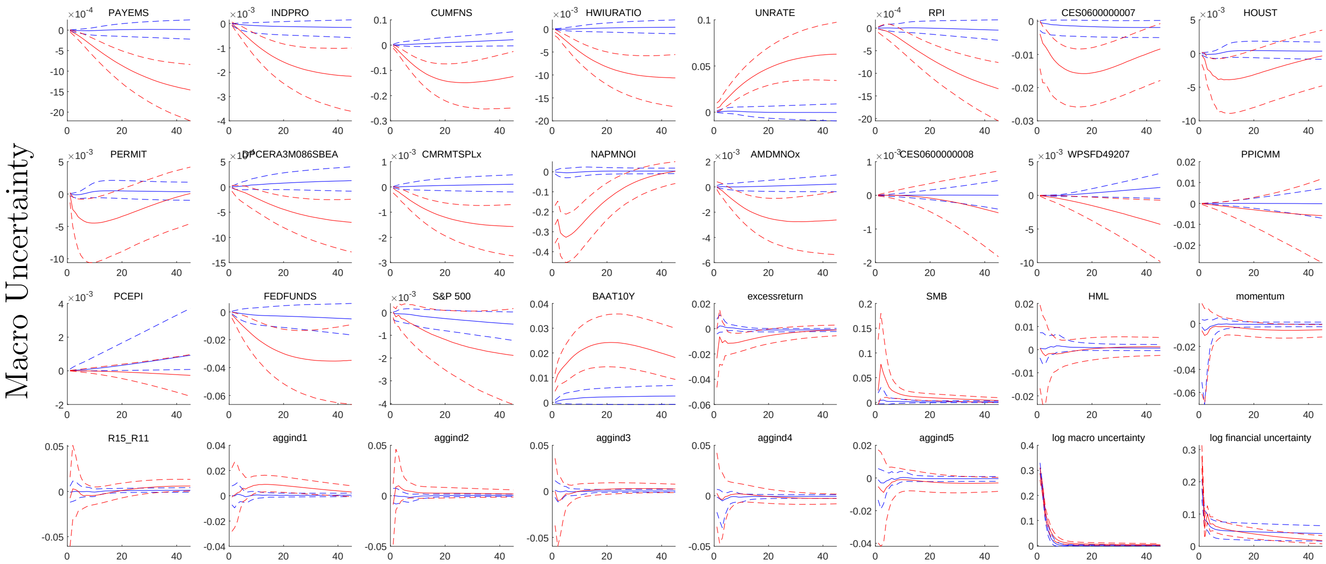


Figure 4: Uncertainty estimates: posterior medians of the macro ($e^{\frac{1}{2}h_{m,t}}$) and financial uncertainty ($e^{\frac{1}{2}h_{f,t}}$). Grey shading indicates NBER recession periods.



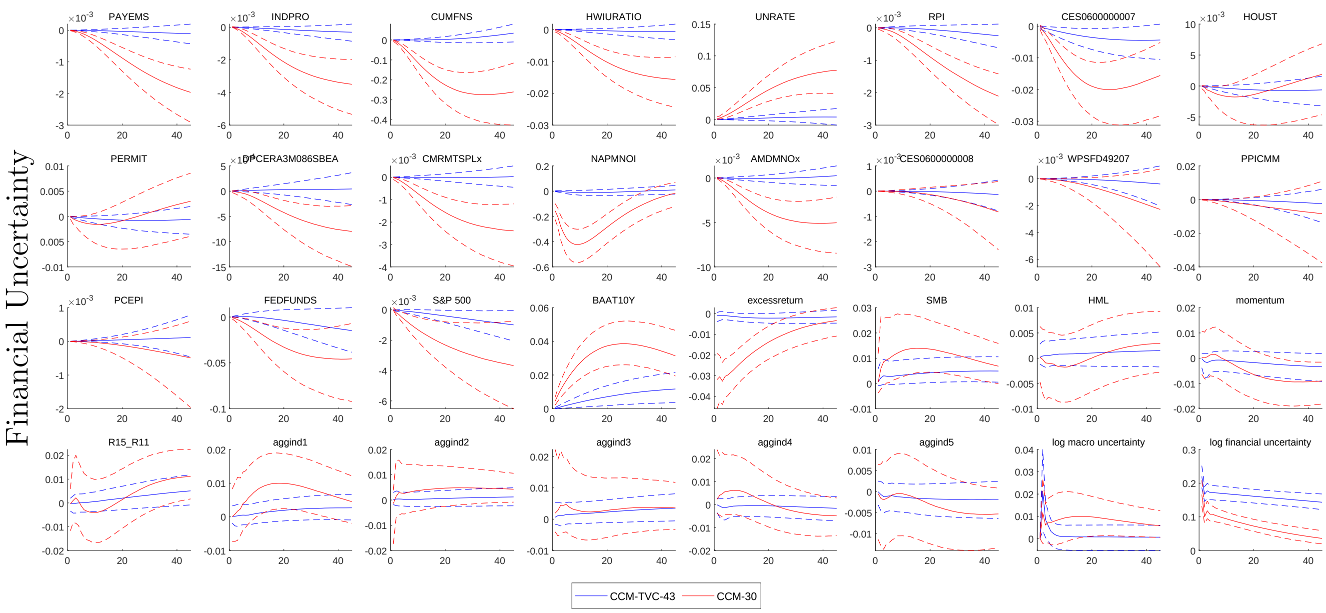


Figure 5: Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for CCM-TVC-43 and CCM-30

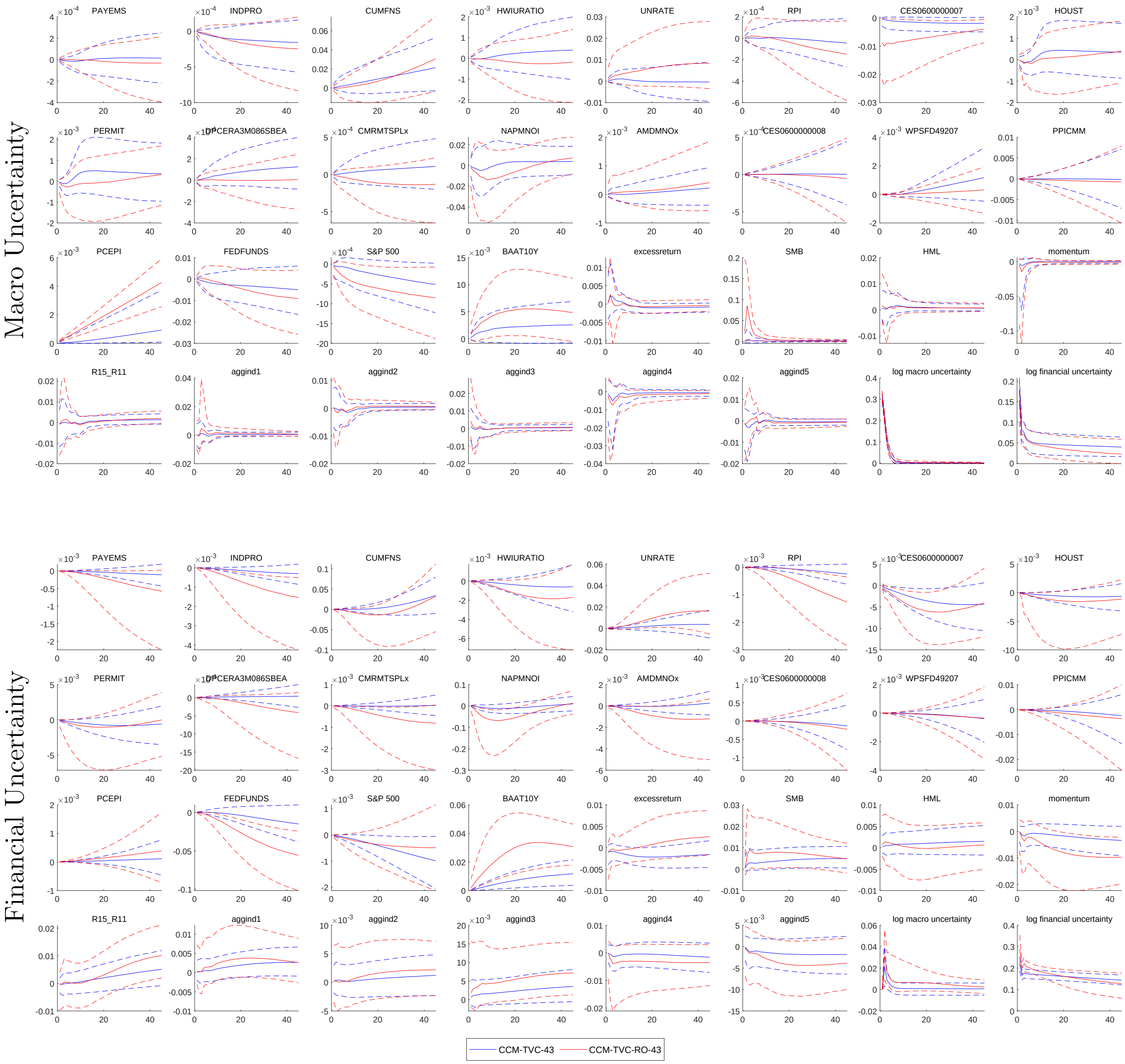


Figure 6: Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for CCM-TVC-43 and CCM-TVC-RO-43

4.3 The Importance of Time-Varying Classification

In addition to developing an efficient algorithm to estimate large order-invariant SVMVARs, another contribution of our paper lies in the treatment of unclassified variables. In the OI-

TVC-43 we allow 16 variables to be treated as unclassified. As shown in Figure 7, using the OI-TVC-43, we find that the federal funds rate and the credit spread are, on average, classified as financial variables. The classification of the S&P 500 varies considerably over time. These findings differ from the classification schemes selected by CCM and others in the literature including Jurado, Ludvigson and Ng (2015).

We also find evidence which suggests that key changes often occurring during crisis periods. To illustrate, we discuss the results from the three unclassified variables considered in CCM. Focusing first on the federal funds rate, we uncover that the probability that the federal funds rate should be included as a macroeconomic variable rises around recessions when there are significant periods of monetary policy loosening. For example, the probability that the federal funds rate should be classified as a macroeconomic variable rises significantly during the early 1980s and dot com bust. Similarly, during the global financial crisis and coronavirus pandemic, as monetary policy hit the zero lower bound following a period of normalization we again see stark changes.

If we now consider the S&P 500, as expected, the probability that it is classified as a financial variable is high during times of financial turmoil such as the 1973-1974 crash, the 2001 dot com bust and the global financial crisis. Last, if we turn to the credit spread, its importance as a macroeconomic variable surges during the global financial crisis and coronavirus pandemic. These episodes reflect businesses increased borrowing cost which, in turn, reduces investment and economic growth (Arrelano, Bai and Kehoe, 2019; Christiano, Motto and Rostagno, 2014; Gilchrist, Sim and Zakrajek, 2014).

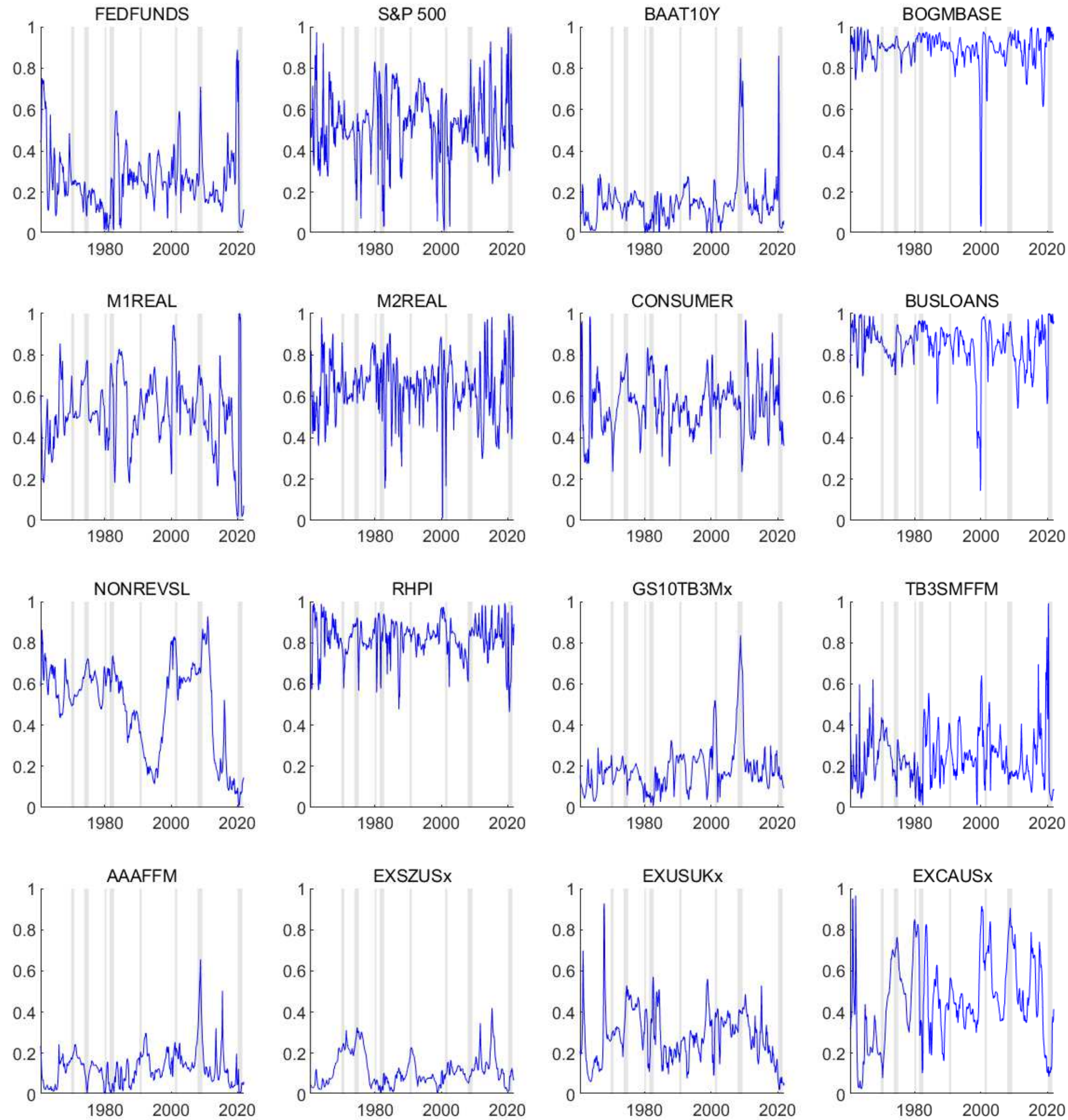


Figure 7: The estimated posterior probability $p(s_{i,t} = m|\mathbf{y})$ that each of unclassified variable should be treated as a macro variable for OI-TVC-43. Grey shading indicates NBER recession periods.

5 Conclusion

The SVMVAR has emerged as an attractive model, allowing for joint estimation of macroeconomic and financial uncertainty and their impact on the economy. However, SVMVARs are difficult to estimate and computationally demanding. To overcome these challenges, existing approaches make a number of assumptions. First, a limit is placed on the number of variables which can be included in the model. Second, a lower triangular parameterization is used for the reduced-form error covariance matrix — this improves computational efficiency but introduces an order dependence issue. Third, the researcher must use expert judgment to classify each variable as macroeconomic or financial prior to estimation. Each of these assumptions may have an impact on empirical results. The models and posterior simulation methods developed in this paper avoid making such assumptions. We introduce a new SVMVAR model in which the classification of some variables is uncertain. The algorithm can then decide whether to classify them as financial or macroeconomic variables in a time-varying fashion. Our model also relaxes the lower triangularity assumption, thus allowing for order invariant inference. A novel MCMC algorithm, which extends that of Cross, Hou, Koop and Poon (2022), is computationally efficient and exhibits good convergence properties, thus allowing us to include a large number of variables in the model.

We compare our very large OI-SVMVAR with unclassified variables with a range of alternative SVMVARs. This allows us to investigate the importance of model size, order-invariance and time-varying classification. We show that smaller SVMVARs overestimate the effects of macroeconomic uncertainty on the economy. However, using our larger model it becomes clear that financial not macroeconomic uncertainty has a larger negative effect on the economy. This finding aligns with Ludvigson, Ma and Ng (2021). We also show that using a lower triangular parameterization for the error covariance matrix yields results which are strongly influenced by the ordering of variables. It is therefore critical to use an order invariant specification such as the one proposed in this paper. Last, we show that most of our unclassified variables change classification at some point during the sample with important shifts often occurring during crises. Time-varying classification ensures that variables are assigned to the appropriate block when our uncertainty measures and their impacts are estimated.

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A Data Appendix

Table 2: Description of the Data

Abbreviation	Macroeconomic variable	Transformation
PAYEMS	All employees: total nonfarm	Δ log
INDPRO	Industrial production index	Δ log
CUMFNS	Capacity utilization: manufacturing	Δ
HWIURATIO	Help wanted-to-unemployed ratio	Δ
UNRATE	Unemployment rate	Δ
RPI	Real personal income	Δ log
CES0600000007	Weekly hours: goods producing	no transformation
HOUST	Housing starts	log
PERMIT	Housing permits	log
DPCERA3M086SBEA	Real consumer spending	Δ log
CMRMTSPLx	Real manufacturing trade sales	Δ log
NAPMNOI	ISM: new orders index	no transformation
AMDMNOx	Orders for durable goods	Δ log
CES0600000008	Avg. hourly earnings, goods producing	Δ^2 log
WPSFD49207	PPI, finished goods	Δ^2 log
PPICMM	PPI, commodities	Δ^2 log
PCEPI	PPI, price index	Δ^2 log
	Financial variable	
excessreturn	Excess return	no transformation
SMB	SMB FF factor	no transformation
HML	HML FF factor	no transformation
momentum	Momentum factor	no transformation
R15_R11	small stock value spread	no transformation
aggind1	Industry 1 return	no transformation
aggind2	Industry 2 return	no transformation
aggind3	Industry 3 return	no transformation
aggind4	Industry 4 return	no transformation
aggind5	Industry 5 return	no transformation
	Unclassified variable	
S&P 500	S&P 500	Δ log
BAAT10Y	Spread, Baa-10y Treasury	no transformation
FEDFUNDS	Federal funds rate	Δ
BOGMBASE	Monetary Base	Δ^2 log
M1 Real	Real M1 Money Stock	Δ^2 log
M2 Real	Real M2 Money Stock	Δ^2 log
CONSUMER	Consumer Loans	Δ^2 log
BUSLOANS	Commercial & Industrial Loans	Δ^2 log
NONREVSL	Total Nonrevolving Credit	Δ^2 log
RHPI	Real House Price Index	Δ log
GS10TB3Mx	10 Yr Treasury Constant Maturity - 3 Mnth TBill	no transformation
TB3SMFFM	3 Mnth Treasury Constant Maturity - Federal Funds Rate	no transformation
AAAFFM	Moody's Seasoned Aaa Corporate Bond - Federal Funds Rate	no transformation
EXSZUSx	Switzerland/US Foreign Exchange Rate	Δ log
EXUSUKx	UK/US Foreign Exchange Rate	Δ log
EXCAUSx	Canada/US Foreign Exchange Rate	Δ log

For our 30 variable models (models 1 and 2), we follow CCM classifying the federal funds rate as a macroeconomic variable, and credit spread and S&P 500 as financial varibles.

B Technical Appendix

This Appendix first provides the details about the prior distributions for the model parameters and elaborates some key sampling steps for estimating the SVMVAR model specified by equations (1)-(7). We also make some comments on the potential label switching issue for the order invariant SVMVAR and discuss a simple approach to overcome this issue.

B.1 Prior

We first introduce some notations for later reference. Let $\mathbf{x}$ as a vector and $\mathbf{X}$ as a matrix, we define diag($\mathbf{x}$) to be a diagonal matrix with its diagonal elements being $\mathbf{x}$, and vec($\mathbf{X}$) vectorizes the matrix $\mathbf{X}$ by stacking the columns of the matrix $\mathbf{X}$ on top of one another. We specify the following prior for the model parameters:

$$
\begin{aligned}
&\mathrm{vec}(\mathbf{B}_0) \sim \mathcal{N}\left(\mathrm{vec}(\mathbf{B}_0^0), \mathrm{diag}[\mathrm{vec}(\mathbf{V}_{b_0})]\right),\\
&\mathrm{vec}(\mathbf{B}_1, \ldots, \mathbf{B}_p) \sim \mathcal{N}\left(\mathrm{vec}(\mathbf{B}_1^0, \ldots, \mathbf{B}_p^0), \mathrm{diag}[\mathrm{vec}(\mathbf{V}_{b,1}, \ldots, \mathbf{V}_{b,p})]\right),\\
&\mathrm{vec}(\mathbf{A}_0, \ldots, \mathbf{A}_q) \sim \mathcal{N}\left(\mathrm{vec}(\mathbf{A}_1^0, \ldots, \mathbf{A}_p^0), \mathrm{diag}[\mathrm{vec}(\mathbf{V}_{a,1}, \ldots, \mathbf{V}_{a,q})]\right),\\
&\mathrm{vec}(\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_{p_h}) \sim \mathcal{N}\left(\mathrm{vec}(\mathbf{\Phi}_1^0, \ldots, \mathbf{\Phi}_{p_h}^0), \mathrm{diag}[\mathrm{vec}(\mathbf{V}_{\phi,1}, \ldots, \mathbf{V}_{\phi,p_h})]\right),\\
&\mathrm{vec}(\mathbf{\Psi}_1, \ldots, \mathbf{\Psi}_{p_y}) \sim \mathcal{N}\left(\mathrm{vec}(\mathbf{\Psi}_1^0, \ldots, \mathbf{\Psi}_{p_y}^0), \mathrm{diag}[\mathrm{vec}(\mathbf{V}_{\psi,1}, \ldots, \mathbf{V}_{\psi,p_y})]\right),\\
&\mathbf{\Sigma}_h \sim \mathcal{IW}(\nu_h, \mathbf{S}_h),\\
&\rho_{k,i} \sim \mathcal{N}(\rho_{k,i}^0, w_{k,i})\mathbf{1}(|\rho_{k,i}| < 1), \quad \sigma_{k,i}^2 \sim \mathcal{IG}(\nu_{k,i}, S_{k,i}), \quad i = 1, \ldots, n_k, \quad k \in \{m, f, u\},\\
&(p_{m,m}^i, p_{m,f}^i) \sim \mathcal{D}(\alpha_{m,m}^i, \alpha_{m,f}^i), \quad (p_{f,m}^i, p_{f,f}^i) \sim \mathcal{D}(\alpha_{f,m}^i, \alpha_{f,f}^i), \quad i = 1, \ldots, n_u.
\end{aligned}
$$

We consider the Minnesota-type adaptive hierarchical Horseshoe priors proposed by Chan (2021) for $(\mathbf{B}_1, \ldots, \mathbf{B}_p)$ and $(\mathbf{A}_0, \ldots, \mathbf{A}_q)$. To be specific, the prior means are set to $\mathbf{B}_j^0 = \mathbf{0}$, $j = 1 \ldots, p$ and $\mathbf{A}_j = \mathbf{0}, j = 1, \ldots, q$. For the variances of parameters in the coefficient matrices, we first denote $\mathbf{V}_{b,l}^{i,j}$ and $\mathbf{V}_{a,l}^{i,j}$ as the (i,j)th element of $\mathbf{V}_{b,l}$ and $\mathbf{V}_{b,l}$, and set

$$\mathbf{V}_{b,l}^{i,j} = \begin{cases} \frac{\kappa_1^b v_{i,j}^b}{l^2}, & \text{for } i = j \\ \frac{\kappa_2^b v_{i,j}^b}{l^2} \frac{d_i^2}{d_j^2}, & \text{for } i \neq j, \end{cases} \qquad \sqrt{v_{i,j}^b} \sim \mathcal{C}^+(0,1), \quad \sqrt{\kappa_1^b}, \sqrt{\kappa_2^b} \sim \mathcal{C}^+(0,1), \text{ for } l = 1, \ldots, p$$

$$\mathbf{V}_{a,l}^{i,j} = \begin{cases} \frac{\kappa_1^a v_{i,j}^a}{l+1^2}, & \text{for } i = j \\ \frac{\kappa_2^a v_{i,j}^a}{(l+1)^2} \frac{d_i^2}{o_j^2}, & \text{for } i \neq j, \end{cases} \qquad \sqrt{v_{i,j}^a} \sim \mathcal{C}^+(0,1), \quad \sqrt{\kappa_1^a}, \sqrt{\kappa_2^a} \sim \mathcal{C}^+(0,1), \text{ for } l = 0, \ldots, q$$

where $\mathcal{C}^+(0,1)$ is the standard half-Cauchy distribution. Following the standard Minnesota prior practice, we set the prior hyperparameters d_j^2 as the residual variances of AR(p) model for variable j and $o_j = \sum_{i=1}^{n_m} d_i^2$ if $j = 1$, and $o_j = \sum_{i=n_m+n_u+1}^{n} d_i^2$ if $j = 2$.

Similarly, we impose a Horseshoe prior on matrix $\mathbf{B}_0$. In particular, we set its mean $\mathbf{B}^0 = \mathbf{0}$ and let $\mathbf{V}_{b_0}^{i,j}$ be the (i,j)th element of $\mathbf{B}_0$:

$$\mathbf{V}_{b_0}^{i,j} = \kappa^{b_0} v_{i,j}^{b_0}, \quad \kappa^{b_0} \sim \mathcal{C}^+(0,1), \quad v_{i,j}^{b_0} \sim \mathcal{C}^+(0,1).$$

For VAR coefficients in the state equation (6), we set their prior means to $\mathbf{\Phi}_1^0 = 0.8\mathbf{I}_2$, $\mathbf{\Phi}_i^0 = \mathbf{0}, i = 2, \ldots, p_h$, and $\mathbf{\Psi}_j^0 = \mathbf{0}, j = 1, \ldots, p_y$. For the covariance matrices, we set $\mathbf{V}_{\phi,i} = 0.2^2 \mathbf{1}_{2\times 2}, i = 1, \ldots, p_h$ and $\mathbf{V}_{\psi,i} = 0.4^2 \mathbf{1}_{2\times n}, i = 1, \ldots, p_y$. For the the AR coefficients of the idiosyncratic log-volatilities specified in equations (7), we let $\rho_{k,i}^0 = 0.95$ and $w_{k,i} = 0.4^2$ for $k \in \{m, f\}$, $i = 1, \ldots, n_k$. For the inverse-gamma prior for the variance of innovation, we let degree of freedom and the scale parameter to $\nu_{k,i} = 10$ and $S_{k,i} = 0.03(\nu_{k,i} - 1)$ for $k \in \{m, f\}$, $i = 1, \ldots, n_k$, which implies that the each innovation has a prior mean 0.03.

For the inverse-Wishart prior for the covariance matrix of the innovation, we set its degree of freedom to $\nu_h = 10$. The scale matrix is set so as to center the prior distribution to have a prior mean $0.01\mathbf{I}_2$.

For the Markov transition probabilities governing the indicator variables $s_{i,t}$, we use concentration parameters of the Dirichlet priors to $(\alpha_{m,m}^i, \alpha_{m,f}^i) = (10, 1)$ and $(\alpha_{f,m}^i, \alpha_{f,f}^i) =$

$(1, 10)$, $i = 1, \ldots, n_u$.

B.2 MCMC Sampler

Given the prior distributions described in the previous Technical Appendix B, we provide the detailed steps for drawing $(\mathbf{h}_1, \ldots, \mathbf{h}_T)$, $(s_{i,t}, \ldots, s_{i,T})$, $(p^i_{m,m}, p^i_{m,f})$ and $(p^i_{f,m}, p^i_{f,f})$, $i = 1, \ldots, n_u$ in this section. The sampling procedures for other parameters of order dependent models can be found in Cross, Hou, Koop and Poon (2022) and Carriero, Clark and Marcellino (2018). For the order invariant models, we refer readers to Chan, Koop and Yu (2021) for more details for drawing the contemporaneous coefficient matrix $\mathbf{B}_0$. Lastly, the sampling steps for those hyperparamters in the Minnesota-type adaptive hierarchical Horseshoe prior can be found in Chan (2021).

B.2.1 Drawing $(\mathbf{h}_1, \ldots, \mathbf{h}_T)$

We modify the methods proposed by Cross, Hou, Koop and Poon (2021) (CHKP) to sample the log-volatility. To set the stage, we define $\eta^m_t = (\eta^m_{1,t}, \ldots, \eta^m_{n_m,t})'$, $\eta^f_t = (\eta^f_{1,t}, \ldots, \eta^f_{n_f,t})'$, $\eta^u_t = (\eta^u_{1,t}, \ldots, \eta^u_{n_u,t})'$ and let $\eta_t = (\eta^{m'}_t, \eta^{u'}_t, \eta^{f'}_t)'$. We first multiply $\mathbf{B}_0$ on both sides of equation (1) and then rescaled each equation by its idiosyncratic volatility, which gives

$$\widetilde{\mathbf{z}}_t = \sum_{i=0}^{q} \widetilde{\mathbf{C}}_{i,t}\mathbf{h}_{t-i} + \widetilde{\boldsymbol{\epsilon}}^y_t, \quad \widetilde{\boldsymbol{\epsilon}}^y_t \sim \mathcal{N}(\mathbf{0}, \widetilde{\mathbf{D}}_t), \tag{8}$$

where $\widetilde{\mathbf{z}}_t = \mathrm{diag}[\exp(-\frac{1}{2}\eta_t)]\,(\mathbf{B}_0\mathbf{y}_t - \sum_i^p \mathbf{B}_0\mathbf{B}_i\mathbf{y}_{t-i})$, $\widetilde{\mathbf{C}}_{i,t} = \mathrm{diag}[\exp(-\frac{1}{2}\eta_t)]\mathbf{B}_0\mathbf{A}_i$[5] and

$$\widetilde{\mathbf{D}}_t = \begin{pmatrix} h_{m,t}\mathbf{I}_{n_m} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & h_{s_{1,t},t} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & h_{s_{n_u,t},t} & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & h_{f,t}\mathbf{I}_{n_f} \end{pmatrix}.$$

Next, we construct a $n \times n$ selection matrix $\mathbf{Q}_t$ to reorder equation (8) so as to group all macro variables into the first block and financial variable into the second block. To be specific, let $M_t = \{i \in \{1, \ldots, n_u\} : s_{i,t} = m\}$ and $F_t = \{i \in \{1, \ldots, n_u\} : s_{i,t} = f\}$ be the sets collecting the indices of the unclassified variables that are belong to the macro block and financial block respectively, and we denote the number of elements in M_t by $|M_t|$ and the number of elements in F_t by $|F_t|$. We first define a $n_u \times n_u$ selection matrix $\widetilde{\mathbf{Q}}_t$ with its first $|M_t|$ rows being the rows of a $n_u \times n_u$ identify matrix indexed by M_t, and its last $|F_t|$ rows being the rows of a $n_u \times n_u$ identify matrix indexed by F_t. Then we define

$$\mathbf{Q}_t = \begin{pmatrix} \mathbf{I}_{n_m} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{Q}}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{n_f} \end{pmatrix}.$$

Premultiplying both sides of equation (8) by $\mathbf{Q}_t$, we can obtain the following expression:

$$\mathbf{z}_t = \sum_{i=0}^{q} \mathbf{C}_t\mathbf{h}_{t-i} + \boldsymbol{\epsilon}_t^z, \quad \boldsymbol{\epsilon}_t^z \sim \mathcal{N}(\mathbf{0}, \mathbf{D}_t), \tag{9}$$

where $\mathbf{z}_t = \mathbf{Q}_t\widetilde{\mathbf{z}}_t$, $\mathbf{C}_t = \mathbf{Q}_t\widetilde{\mathbf{C}}_t$ and $\mathbf{D}_t = \mathbf{Q}_t\widetilde{\mathbf{D}}_t\mathbf{Q}_t'$. In equation (9), it can be easily checked that the macro variables are ordered as the first $n_m + |M_t|$ variables in $\mathbf{z}_t$, and the financial

[5] For notational simplicity, we introduce a notation $\exp(x) = (e^{x_1}, \ldots, e^{x_m})'$ for a $m \times 1$ vector $x = (x_1, \ldots, x_m)'$.

variables are ordered as the last $n_f + |F_t|$ variables in $\mathbf{z}_t$. Thus equation (9) is now in a standard form of SVMVAR model, then the method proposed in CHKP can be used to efficiently draw $\mathbf{h}$. We refer readers to Cross, Hou, Koop and Poon (2022) for more details.

B.2.2 Drawing $(s_{i,1}, \ldots, s_{i,T}), i = 1, \ldots, n_u$

We first realize that the indicator variables across equations of the unclassified variables are conditionally independent, and the indicate variables enter the likelihood only via the conditional variance of the VAR system. To sample $(s_{i,1}, \ldots, s_{i,T})$, we first obtain the residuals

$$\mathbf{e}_t = \mathbf{B}_0 \left(\mathbf{y}_t - \sum_{i=1}^{p} \mathbf{B}_i \mathbf{y}_{t-i} - \sum_{j=0}^{q} \mathbf{A}_j \mathbf{h}_{t-j} \right),$$

then the residual for the ith unclassified variable, i.e., the $(n_m + i)$th element in $\mathbf{e}_t$, can be written as

$$\begin{aligned} e^u_{i,t} &\sim \mathcal{N}(0, e^{\omega^u_{i,t}}), \quad i = 1, \ldots, n_u, \\ \omega^u_{i,t} &= \eta^u_{i,t} + h_{s_{i,t},t}, \end{aligned}$$

then the conditional likelihood of $e^u_{i,t}$ is given by

$$p(e^u_{i,t}|s_{i,t}) \propto e^{-\frac{1}{2}h_{s_{i,t}}} \exp\left(-\frac{1}{2} e^{-(\eta^u_{i,t} + h_{s_{i,t},t})} \times (e^u_{i,t})^2 \right). \tag{10}$$

Here we have suppressed all the other conditional arguments except $s_{i,t}$ for notational convenience. Given the conditional likelihood in equation (10) and the Markov transition probability $p^i_{l,k}$, $k, l \in \{m, f\}$, the forward-backward algorithm of Chib (1996) can be applied for drawing the indicator variables $(s_{i,1}, \ldots, s_{i,T}), i = 1, \ldots, n_u$. To be specific, denote $e^u_{i,1:t} = (e^u_{i,1}, \ldots, e^u_{i,t})'$ and suppose we have $p(s_{i,t-1}|e^u_{i,1:t-1})$, the forward filtering process is conducted for $t = 1, \ldots, T$ as follows until we obtain $p(s_{i,T}|e^u_{1:T})$:

$$
\begin{aligned}
p(s_{i,t}|e^u_{i,1:t}) &= \frac{p(e^u_{i,t}|s_{i,t})p(s_{i,t}|e^u_{i,1:t-1})}{\sum_{s_{i,t}} p(e^u_{i,t}|s_{i,t})p(s_{i,t}|e^u_{i,1:t-1})} \\
&= \frac{\sum_{s_{i,t-1}} p(e^u_{i,t}|s_{i,t})p(s_{i,t}, s_{i,t-1}|e^u_{i,1:t-1})}{\sum_{s_{i,t}} \sum_{s_{i,t-1}} p(e^u_{i,t}|s_{i,t})p(s_{i,t}, s_{i,t-1}|e^u_{i,1:t-1})} \\
&= \frac{\sum_{s_{i,t-1}} p(e^u_{i,t}|s_{i,t})p(s_{i,t}|s_{i,t-1})p(s_{i,t-1}|e^u_{i,1:t-1})}{\sum_{s_{i,t}} \sum_{s_{i,t-1}} p(e^u_{i,t}|s_{i,t})p(s_{i,t}|s_{i,t-1})p(s_{i,t-1}|e^u_{i,1:t-1})},
\end{aligned}
$$

where we assume initial state $s_{i,1}$ to follow the stationary distribution of the Markov process given by $p^i_{l,k},\ k, l \in \{m, f\}$. The backward sampling step is implemented by first draw $s_{i,T}$ from $p(s_{i,T}|e^u_{1:T})$, and then sequentially drawing $s_{i,t}$ given $s_{i,t+1}$ from

$$
p(s_{i,t}|s_{i,t+1}, e^u_{i,1:T}) = \frac{p(s_{i,t}|e^u_{i,1:t})p(s_{i,t+1}|s_{i,t})}{\sum_{s_t} p(s_{i,t}|e^u_{i,1:t})p(s_{i,t+1}|s_{i,t})}.
$$

B.2.3 Drawing $(p^i_{m,m}, p^i_{m,f})$ and $(p^i_{f,m}, p^i_{f,f}), i = 1, \ldots, n_u$

Given the indicator variables $(s_{i,t}, \ldots, s_{i,T})$, $i = 1, \ldots, n_u$, and the Dirichlet distributed prior for $(p^i_{m,m}, p^i_{m,f})$ and $(p^i_{f,m}, p^i_{f,f})$, it follows that

$$
\begin{aligned}
(p^i_{m,m}, p^i_{m,f}|s_{i,t}, \ldots, s_{i,T}) &\sim \mathcal{D}(\alpha^i_{m,1} + N_{m.m}, \alpha^i_{m,2} + N_{m,f}), \\
(p^i_{f,m}, p^i_{f,f}|s_{i,t}, \ldots, s_{i,T}) &\sim \mathcal{D}(\alpha^i_{f,2} + N_{f,m}, \alpha^i_{f,1} + N_{f,f}),
\end{aligned}
$$

where $N_{k,l} = \sum_{j=1}^{T-1} \mathbb{1}(s_j = k, s_{j+1} = l)$, $k, l \in \{m, f\}$ and $\mathbb{1}(A)$ is a indicator function that is equal to one if A is true and zeros otherwise.

B.3 Label Switching Problem

This section discusses the label switching problem related to the estimation of the common stochastic volatilities $e^{h_{m,t}}$ and $e^{h_{f,t}}$. For notational simplicity, let $\widetilde{\mathbf{B}}_0 = \mathbf{B}_0^{-1}$. As each diagonal element of $\mathbf{U}_t$ specified in equation (2) is assumed to be changing over time, following the results in Bertsche and Braun (2020) and Chan, Koop and Yu (2021), it can be shown that $\widetilde{\mathbf{B}}_0$ is unique up to permutation of its columns and multiplication of its columns by -1. It is important to note that the signs of the columns of $\widetilde{\mathbf{B}}_0$ would not affect the identification of $e^{h_{m,t}}$ and $e^{h_{f,t}}$. However, the label switching problem resulting from permutation of columns of $\widetilde{\mathbf{B}}_0$ remains as an important issue. We will show that this label switching problem can be solved by setting different numbers of variables in the macro and financial blocks, i,e, setting $n_m \neq n_f$. In contrast, one should pay more attention on the case of $n_f = n_m$, as the label switching problem under this case may lead to invalid empirical inference on $e^{h_{m,t}}$ and $e^{h_{f,t}}$.

For expository purpose, we consider a model with two common SVs, $e^{h_{m,t}}$ and $e^{h_{f,t}}$, that does not include unclassified variables and idiosyncratic SVs, then the reduce-form variance-covariance matrix at time t is given by

$$\mathbf{\Sigma}_t = \widetilde{\mathbf{B}}_0 \widetilde{\mathbf{U}}_t \widetilde{\mathbf{B}}_0', \tag{11}$$

where $\widetilde{\mathbf{U}}_t = \mathrm{diag}(e^{h_{m,t}}\mathbf{I}_{n_m}, e^{h_{f,t}}\mathbf{I}_{n_f})$. It is easer for us to verify the potential label switching problem by rewriting equation (11) as

$$\mathbf{\Sigma}_t = e^{h_{m,t}} \left(\widetilde{\mathbf{b}}_1 \widetilde{\mathbf{b}}_1' + \ldots + \widetilde{\mathbf{b}}_{n_m} \widetilde{\mathbf{b}}_{n_m}' \right) + e^{h_{f,t}} \left(\widetilde{\mathbf{b}}_{n_m+1} \widetilde{\mathbf{b}}_{n_m+1}' + \ldots + \widetilde{\mathbf{b}}_{n_m+n_f} \widetilde{\mathbf{b}}_{n_m+n_f}' \right), \tag{12}$$

where $\widetilde{\mathbf{b}}_i$ is the ith column of $\widetilde{\mathbf{B}}_0$. Now we will show that there exists a label switching problem for the case of $n_f = n_m$. To see this, we first realize that $\mathbf{\Sigma}_t$ represented in equation (12) is a sum of two terms, and each of these terms is a sum of $n_m = n_f$ terms. Suppose we define $\bar{\mathbf{B}}_0 = (\widetilde{\mathbf{b}}_{n_m+1}, \ldots, \widetilde{\mathbf{b}}_{n_m+n_f}, \widetilde{\mathbf{b}}_1, \ldots, \widetilde{\mathbf{b}}_{n_m})$ then we can rewrite $\mathbf{\Sigma}_t$ as

$$\mathbf{\Sigma}_t = \bar{\mathbf{B}}_0 \bar{\mathbf{U}}_t \bar{\mathbf{B}}_0',$$

with $\bar{\mathbf{U}}_t = \mathrm{diag}(e^{h_{f,t}}\mathbf{I}_{n_f}, e^{h_{m,t}}\mathbf{I}_{m_n})$. It is worth highlighting that the label switching problem arises because $n_m = n_f$. As $e^{h_{m,t}}$ and $e^{h_{f,t}}$ are respectively associated with same number of columns of $\widetilde{\mathbf{B}}_0$, thus $(\bar{\mathbf{B}}_0, \bar{\mathbf{U}}_t)$ and $(\widetilde{\mathbf{B}}_0, \widetilde{\mathbf{U}}_t)$ are observationally equivalent.

Next, let's consider the case when $n_m \neq n_f$. We can represent the reduce-form variance-covariance matrix in the same way as in equation (12). However, in this case, the summations in the two parenthesis associated with $e^{h_{m,t}}$ and $e^{h_{f,t}}$ involve different numbers of terms. Since our model specification separates the variables into only two blocks, it is not possible to switch the ordering of the columns of $\widetilde{\mathbf{B}}_0$ for obtaining an observationally equivalent representation. To be specific, suppose that we follow the similar procedure as discussed above to switch the ordering of columns of $\widetilde{\mathbf{B}}_0$, then we will still have $\boldsymbol{\Sigma}_t = \bar{\mathbf{B}}_0\bar{\mathbf{U}}_t\bar{\mathbf{B}}_0' = \widetilde{\mathbf{B}}_0\widetilde{\mathbf{U}}_t\widetilde{\mathbf{B}}_0'$, but the parameterizations of $\bar{\mathbf{B}}_0\bar{\mathbf{U}}_t\bar{\mathbf{B}}_0'$ and $\widetilde{\mathbf{B}}_0\widetilde{\mathbf{U}}_t\widetilde{\mathbf{B}}_0'$ are different. This is because now $\widetilde{\mathbf{U}}_t$ groups its first n_m diagonal elements, while $\bar{\mathbf{U}}_t$ groups its first $n_f(\neq n_m)$ diagonal elements. Hence, when $n_m \neq n_f$, we do not have the label switching problem for $e^{h_{m,t}}$ and $e^{h_{f,t}}$ and we have $n_m = 17$ and $n_f = 10$ in our empirical study.

While the estimates for $e^{h_{m,t}}$ and $e^{h_{f,t}}$ are free from the label switching problem when $n_m \neq n_f$, the idiosyncratic SVs for those unclassified variables, i.e., $(\eta^u_{i,1}, \ldots, \eta^u_{i,T}), i = 1, \ldots, n_u$, still suffer from this problem. Our strategy to tackle this problem is by choosing appropriate MCMC initial values for $(\eta^u_{i,1}, \ldots, \eta^u_{i,T}), i = 1, \ldots, n_u$. More specifically, we set the initial values of $(\eta^u_{i,1}, \ldots, \eta^u_{i,T})$ to be the maximum values of the conditional posterior of a univariate SV model for the ith variable with the other parameters in the state equation fixed at their prior means. This can be done by implementing a similar Newton-Raphson method, with slight modifications, proposed in Cross, Hou, Koop and Poon (2022).[6] We have examined this simple strategy by running our MCMC sampler using different seeds of random number generators and we find that our empirical results are robust.

[6]In our empirical study, we also implement the Newton-Raphson method to initialize $(h_{m,1}, \ldots, h_{m,T})$, $(h_{m,1}, \ldots, h_{m,T})$, $(\eta^m_{i,1}, \ldots, \eta^m_{i,T}), i = 1, \ldots, n_m$ and $(\eta^f_{i,1}, \ldots, \eta^f_{i,T}), i = 1, \ldots, n_f$.

C. Supplementary Figures

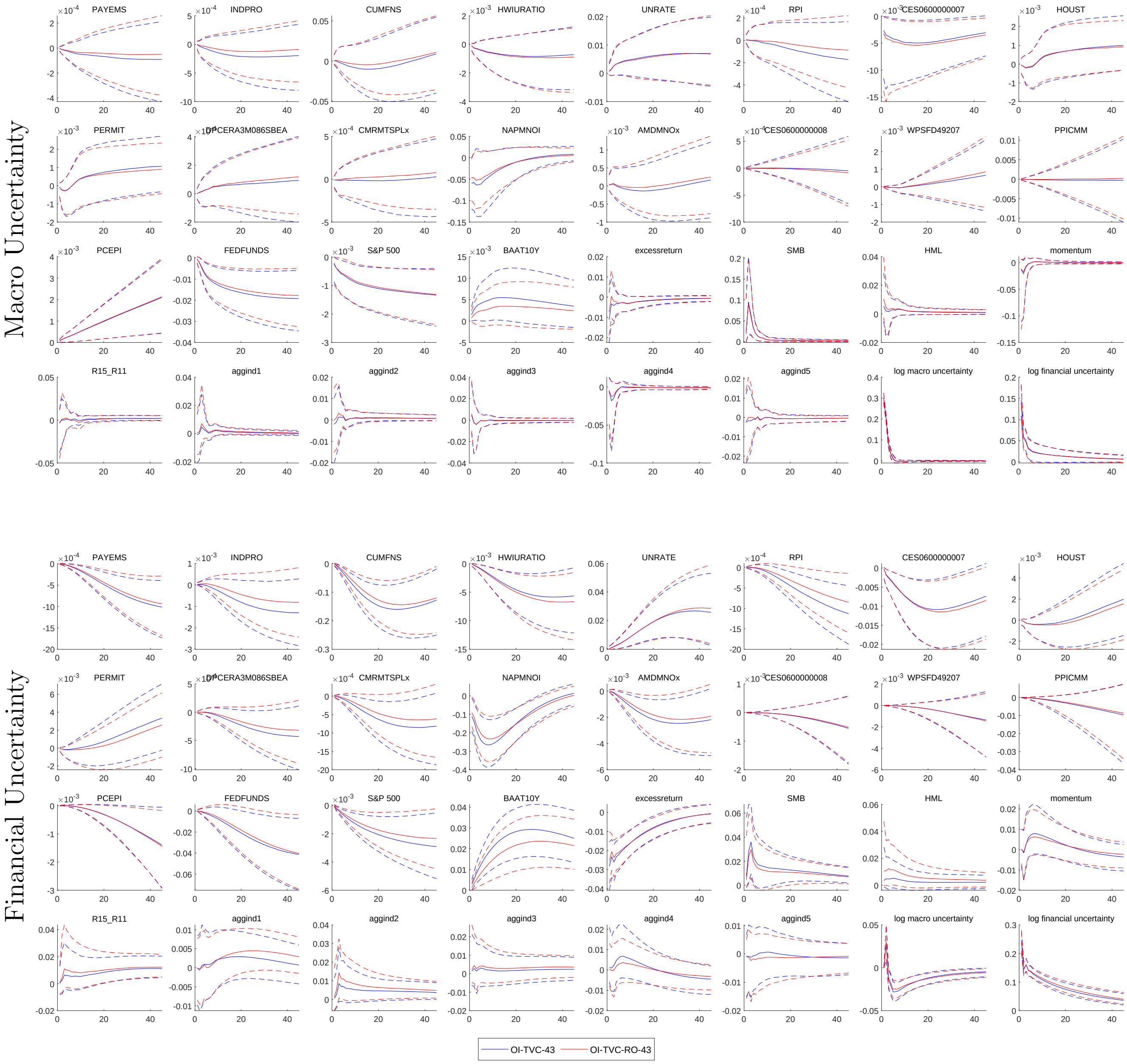

Figure 8: Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for OI-TVC-43 and OI-TVC-RO-43