Analysis of a Fractal Beamformer

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Overview

- Beamforming basics and analysis techniques
- Beam pattern
- Beamformer performance and complexity
- Sparse arrays
- Fractals and fractal arrays
- Full vs fractal array performance – beam efficiency
- Distributed fractal array – implementation challenges
- Conclusions and Future Work
Beamforming – Basics

- Due to direction of arrival and finite propagation speed the wavefront will arrive at different sensors with delay $\Delta \tau$
- Beamforming aligns sensors signals to create constructive interference at output
Beamforming – Basics

- Linear arrays
- Planar arrays
- 3D
- Element position defined in Cartesian coordinate system
- Elevation angle
- Azimuth angle
- Wave vector $\mathbf{k}$
Beamforming – Array Analysis

- Spatial Sampling:

\[
\min_{m,\mu} \| r_m - r_{\mu} \|_2 \leq \frac{\lambda_{\text{min}}}{2}
\]

\[
\lambda_{\text{min}} = \frac{2\pi c}{\omega_{\text{max}}}
\]

\[
k = \begin{bmatrix}
\sin \theta \cos \varphi \\
\sin \theta \sin \varphi \\
\cos \theta
\end{bmatrix}
\]

\[
x_m(t) = x(t - \Delta T_m) = x(t - \frac{k^T r_{\mu}}{c})
\]

- Temporal Sampling:

\[
x_m[n] = x_m(nT_s - \Delta T_m) = x_m((n - \frac{k^T r_{\mu}}{c T_s})T_s)
\]

\[
x_m[n - \tau_m]
\]

where:

\[
\tau_m = \frac{k^T r_{\mu}}{c T_s}
\]

\[
x_m[n] = e^{j\omega(n - \tau_m)T_s} = e^{j\Omega n}e^{-j\Omega \tau_m}
\]
Beamforming – Steering Vector

- Steering vector:

\[ x[n] = \begin{bmatrix} x_1[n] \\ x_2[n] \\ \vdots \\ x_M[n] \end{bmatrix} = e^{j\Omega n} \begin{bmatrix} e^{-j\Omega T_1} \\ e^{-j\Omega T_2} \\ \vdots \\ e^{-j\Omega T_M} \end{bmatrix} = \sqrt{M} e^{j\Omega n} s_{\varphi, \theta, \Omega} \]

- Complex multipliers \( w \)

\[ w^T s_{\Omega_0, \varphi_0, \theta_0} = 1 \]

\[ w = s_{\Omega_0, \varphi_0, \theta_0} \]
Beamforming – Gain Response

- Beam pattern:

\[ G(\Omega, \varphi, \theta) = w^T s_{\Omega, \varphi, \theta} \]
Beamforming – Gain Response

- Beam pattern:

\[ G(\Omega, \varphi, \theta) = w^T s_{\Omega, \varphi, \theta} \]
Beamforming – Thin/Sparse Array

• **Aim of array thinning:**
  – Reduce processing complexity
  – Maintain maximum spatial aperture of array
  – Adhere to spatial and temporal sampling requirements

• **Types of thinned arrays:**
  – Systematic and random thinning
  – Periodic and logarithmic spacing – log spacing
  – Fractal
    • Combines the benefits of periodic and randomly spaced arrays
Fractals
Beam Pattern – Fractal
Beamforming – Full vs Fractal
Beamforming – Power Concentration

Transmit power:

\[
\psi(\alpha, \Omega) = \int_0^{2\pi} \int_0^\alpha |G(\vartheta, \varphi, \Omega)|^2 \sin \vartheta \ \vartheta \ \varphi
\]

Power concentration:

\[
\rho(\alpha, \Omega) = \frac{\psi(\alpha, \Omega)}{\psi(\frac{\pi}{2}, \Omega)}
\]
Beamforming – Power Concentration

![Graph showing power concentration against cone opening angle](image)
Beamforming – Power Concentration

![Graph showing power concentration vs. cone opening angle](image)
Beamforming – Power Concentration

![Graph showing power concentration vs. normalised angular frequency](image)
Beamforming – Displacement Error
Beamforming – Element Failure
Conclusions & Future Work

• Purina fractal geometry offers:
  – Good performance especially at lower frequencies
  – Reduced complexity
  – Robustness to element failure and displacement

• Derived power concentration measures beam efficiency of various arrays

• Side lobe cancellation
  – Improve high frequency performance

• Distributed processing