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THE EFFECTS OF ENTRY IN THIN MARKETS

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The effects of entry in thin markets

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Abstract

We consider entry of additional firms into the market for a single commodity in which both sellers and buyers are permitted to interact strategically. We show that the market is quasi-competitive, in that the inclusion of an additional seller lowers the price and increases the volume of trade, as expected. However, whilst buyers benefit from this change under reasonable conditions on preferences, we cannot conclude that sellers are always made worse off in the face of more intense competition, contrary to the conventional wisdom. We characterize the conditions under which entry by new sellers may raise the equilibrium profit of existing sellers, which will depend in an intuitive way on the elasticity of a strategic analog of demand and the market share of existing sellers, and encompass completely standard economic environments.

Key words: bilateral oligopoly; entry; comparative statics.

JEL classification: C72; D21; D43; L13.

1 Introduction

The conventional wisdom in imperfectly competitive markets is that an increase in the number of firms is harmful to existing firms in the industry. In particular, there are several contributions to the literature on Cournot oligopoly that demonstrate that the profit of firms in an industry declines as the number of firms in that industry increases (see, for example, Seade (1980) and Amir and Lambson (2000)). The question we address in the current paper is this: if both buyers as well as sellers are permitted to behave strategically in a model of bilateral oligopoly, does this conventional wisdom apply? Our answer to this question is: "not always". To motivate our results we present an example of a completely standard economic environment in which there

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are few buyers and few sellers and increase the number of the latter by one. In the new equilibrium, not only are buyers made better off but existing sellers also receive a higher payoff in equilibrium. We then analyze a general model of bilateral oligopoly and, using methods that exploit the aggregative structure of the game played, derive conditions under which this perverse result survives, and when the conventional wisdom applies. These depend in an intuitive way on the market fundamentals.

The effect of entry by additional sellers to an oligopoly industry has seen much attention in the literature. Under standard restrictions on payoffs, it is generally found that markets organized à la Cournot are quasi-competitive; that is, industry output increases and price falls when additional firms enter (Frank (1965), Ruffin (1971), Okuguchi (1973)). A direct implication is that the (price-taking) buyers benefit from such a change. Study of existing firms' profit when additional firms enter the industry reveals that more intense competition lowers equilibrium profit (Seade (1980), Amir and Lambson (2000)). As such, under the threat of entry by additional firms, incumbent firms in an industry will take measures to deter such entry, and it is on this premise that the vast literature on entry deterrence has grown.

Our aim in this paper is to investigate whether these same conclusions that form the conventional wisdom in Cournot oligopoly apply equally in a model of bilateral oligopoly in which buyers as well as sellers are permitted to interact strategically. In bilateral oligopoly there is a single consumption good and money, and the set of agents is partitioned into a set of buyers and a set of sellers. Buyers have an initial endowment of money and sellers have the ability to produce the good. Trade takes place by buyers submitting a proportion of their endowment of money to a trading post to be exchanged for the good, and sellers deciding on a level of production to offer to the trading post. These bids and offers are aggregated and the rate of exchange of the good for money is determined as the ratio of bids to offers. Buyers then receive a proportional share of the total supply to the market, and sellers receive a proportional share of the total bids.

There have been some attempts to study the effects of increasing the number of players in bilateral oligopoly. Of note are the contributions of Bloch and Ghosal (1997) and Amir and Bloch (2009). Bloch and Ghosal (1997) demonstrate that, in a market in which all sellers and all buyers are identical, an increase in the number of traders on one side of the market increases the equilibrium payoff of traders on the opposite side of the market. They also comment on the possibility of perverse effects on traders' payoffs when the number of players on their own side of the market increases, but do not pursue this line of enquiry. Amir and Bloch (2009) study the effects of increasing the number of buyers in an environment where symmetry is imposed amongst sellers and buyers. When converted into the effects of increasing the number of sellers, their results imply that the quantity traded increases and price falls when the number of sellers increases, and that buyers' payoffs are non-decreasing in the number of sellers.

Working in bilateral oligopoly with a heterogeneous player set, we recall that the aggregative structure[1] of the game can be exploited to assist in identifying and analyzing equilibria in the game. (Dickson and Hartley 2008) In particular, strategic supply and demand functions can be constructed that represent the aggregate supply and demand of the sellers and buyers respectively that is consistent with a Nash equilibrium in which the price takes a particular value, and Nash equilibria are in one-to-one correspondence with intersections of these functions. The advantage of using this approach becomes apparent when a change in the economic environment is considered: whilst strategic demand is unaffected by additions to the set of sellers, strategic supply changes in a tractable way at prices at which the new sellers would be active in an equilibrium with that price. The effect on the intersection of strategic supply and demand is immediate, and we instantly gain knowledge of the change in the equilibrium price and volume of trade implied by the presence of additional sellers. This allows us to conclude that bilateral oligopoly is quasi-competitive (equilibrium volume of trade increases and price declines when the number of sellers increases), as Cournot oligopoly is. These aggregate comparative static results are complementary to those in the literature referred to above, in the sense that whilst our 'curvature restrictions' on payoffs are slightly stronger than those in Amir and Bloch (2009) (although we believe they are still very reasonable) we allow for a heterogeneous player set and don't make boundary restrictions that ensure all players are active in any equilibrium. We also show that, under reasonable conditions on preferences, buyers' equilibrium payoffs increase in the presence of additional sellers, again complementing the existing literature.

We then turn to address the effect on existing sellers of the presence of an additional seller that Bloch and Ghosal (1997) alluded to but, to the best of our knowledge has not been studied in the literature. Whilst the aggregate supply increases, individual sellers may either increase or reduce their supply at the new equilibrium. A necessary requirement for an existing seller's equilibrium payoff to increase is that she increases her supply, and our attention is drawn to studying the market conditions under which this is observed. By using the aggregative approach to analyze the game we characterize individual decisions according to the price and volume of trade in equilibrium. Having deduced the change in these in the presence of an additional seller it is then a simple task to deduce the change in individual supply consistent with the change in equilibrium. In doing this we exploit the strategic demand function to show that a seller in bilateral oligopoly will increase her supply in the presence of an additional active seller if and only if the elasticity of the strategic demand function exceeds some threshold, that depends inversely on the agent's market power as

[1] Roughly speaking, a game is aggregative when interactions between players occur only through the aggregation of strategies. Individual players are then only concerned with the aggregate value of strategies, not with exactly which players use which strategies. In games that possess this property different techniques to best response analysis can be used to analyze equilibria in a highly tractable way. See, for example, Novshek (1985) for an application to Cournot competition and Cornes and Hartley (2005) for an application to contests.

measured by their market share. Thus, in thin markets where sellers have significant market power incumbent sellers may increase their supply and therefore benefit from entry by additional sellers. Whilst the precise conditions characterizing when profit increases are convoluted, an example is provided of a perfectly standard economic environment where this is observed.

These results entertain the possibility that incumbent firms, far from undertaking strategic actions to deter entry, may actively encourage it, up to a point. For, active entrants will have a positive market share which reduces the market power of existing firms, meaning the necessary condition for profit to increase requires increasingly elastic demand in order to be satisfied. This means that after sufficient successive entry by new firms, the equilibrium profit of incumbents will always fall in the presence of additional firms, and the conventional wisdom will apply when markets are thick enough.

The rest of the paper is structured as follows. The next section outlines the economic environment and then in section 3 we present an example that raises the possibility of existing sellers being made better off in the presence of an additional seller, as motivation for our study. Section 4 outlines our characterization of equilibrium that utilizes strategic supply and demand curves, and in section 5 we use these to derive the effects on equilibrium aggregates of the introduction of an additional seller to the economy. Section 6 presents our analysis of individual agents, and in particular studies the effect on the payoffs of the aforementioned change, following which are our concluding remarks. All proofs are collected in an appendix.

2 The trading environment

We wish to model trade between sellers and buyers in the market for a single commodity where strategic behavior is permitted by *all* agents. To achieve this we turn to a model of bilateral oligopoly and impose the rules of a Shapley-Shubik strategic market game (Shapley and Shubik 1977). In this model, originally due to Gabszewicz and Michel (1997), there are two commodities. The first (denoted y_1) we take to be a consumption good, and the second (y_2) a commodity money. We partition the (index) set of agents I into $I^{\mathrm{S}} \cup I^{\mathrm{B}}$, $I^{\mathrm{S}} \cap I^{\mathrm{B}} = \emptyset$. An agent $i \in I^{\mathrm{B}}$ is endowed only with commodity money and so is a buyer; we denote by m_i the magnitude of buyer i's endowment. Conversely, agent $i \in I^{\mathrm{S}}$ has the ability to produce x units of the consumption good for a monetary cost of $C_i(x)$ (and no endowment of money). Such agents are called sellers. Our aim in the paper is to assess the effect on equilibrium when the set of sellers changes from I^{S} to $I^{\mathrm{S}\prime} = I^{\mathrm{S}} \cup k$.

The market operates as follows. Each seller $i \in I^{\mathrm{S}}$ decides on a quantity of the good to produce, denoted $x_i \geq 0$, which is then sent to a trading post to be exchanged for money. At the same time, each buyer decides on the proportion of her endowment of money $b_i \in [0, m_i]$ to be sent to the trading post to be exchanged for the good. Given a vector of such offers and bids detailing each trader's action, the market aggregates supply to $X = \sum_{j \in I^S} x_j$ and money bids to $B = \sum_{j \in I^B} b_j$, and determines the market clearing price as the ratio of the latter to the former: $p = B/X$. If $B \cdot X = 0$ the market is deemed closed and no trade takes place; buyers receive their initial endowment and sellers receive a zero payoff.

Sellers receive the revenue from their supply activity but incur costs of production, so a seller who supplied x receives a payoff of $xp - C_i(x)$. Buyers receive a proportional share b/B of the total quantity supplied to the market and evaluate their final allocation according to a real-valued utility function $u_i : \mathbb{R}^2_+ \to \mathbb{R}$; so the payoff to a buyer who made a bid of b is $u_i(b/p, e_i - b)$. We use $\partial_i(y_1, y_2)$ to denote the marginal rate of substitution at the allocation (y_1, y_2).

These trading rules constitute a well-defined game, and the equilibrium concept we use is that of Nash equilibrium in pure strategies. The game always has an equilibrium in which every agent uses their null strategy of bidding/supplying zero to the market: it is easily checked that when the bids/supply of all other players are zero the null strategy is a best response for each player. (Note well, however, that there are no (autarkic) Nash equilibria in which $B > 0$ and $X = 0$ or $X > 0$ and $B = 0$.) In the sequel we confine our attention to non-autarkic Nash equilibria in which at least some trade takes place.

We employ the following assumptions on traders' payoff functions. Throughout, we suppose that the utility function of each buyer $i \in I^B$ is binormal; that is, if $y_1 \leq y_1'$ and $y_2 \geq y_2'$ then $\partial_i(y) \geq \partial_i(y')$, where the final inequality is strict if $y_1 < y_1'$ and $y_2 > 0$. This assumption is fairly innocuous, and means marginal rates of substitution increase under moves to the north-west, implying that (competitive) income expansion paths are upward-sloping. For the sellers, we assume that costs are increasing and convex, and that there are no fixed costs, that is, for each $i \in I^S$ $C_i(0) = 0$, $C_i' > 0$ and $C_i'' \geq 0$. These assumptions will give rise to a very desirable property; namely that if a non-autarkic Nash equilibrium exists then it will be unique. Whilst they are not necessary for the study of bilateral oligopoly using the methods that follow we believe the assumptions are not too restrictive in the current environment, and give rise to a highly tractable analysis of the comparative static properties of equilibrium.

The economic environment we have adopted, whilst easily extended to general equilibrium environments by adding more commodities, has a partial equilibrium feel. In particular, Cournot competition can be modeled as a quantity-setting game amongst the sellers assuming the buyers are price-takers and are represented by an inverse demand curve. When considering entry by additional firms into the market it has been shown in the literature that Cournot's model is quasi-competitive, in that entry by additional firms increases industry output and the equilibrium price reduces (see Frank (1965), Ruffin (1971) and Okuguchi (1973)). Moreover, it is well-known that more competition is bad for existing firms, i.e. their profit reduces when additional firms enter the market (Seade (1980, Result R4), Amir and Lambson (2000, theorem 2.2)).[2] It also follows that price-taking buyers, facing lower prices, are better off in the presence of more firms. The observation that incumbent firms' profit reduces in the presence of new entrants is at the heart of the extensive literature on entry deterrence strategies and anti-competitive behavior in industries in the face of the threat of new entrants to an industry.

The purpose of the current study is to investigate whether this conventional wisdom also applies in bilateral oligopoly where all agents behave strategically and trade takes place according to the rules prescribed above. We begin with an example that motivates our study.

3 Example

In this section we present an example of a bilateral oligopoly environment with few sellers and few buyers, and consider the effect of an additional seller on the equilibrium. We show that, whilst aggregate supply increases and price reduces, which accords with the conventional wisdom, existing sellers benefit from the presence of an additional seller, for they receive higher profits in the new equilibrium. Since buyers also receive higher utility we conclude that the addition of a seller to the market is a Pareto improvement.

Suppose there are three buyers each with a unit endowment of money and quadratic preferences given by $u(y_1, y_2) = 3y_1 - {}^1\!/_4 y_1^2 + y_2$, and 2 sellers each with quadratic costs given by $C(x) = {}^3\!/_4 x^2 + {}^1\!/_2 x$. It is easily verified that there is a symmetric non-autarkic equilibrium[3] in this market, and the first-order conditions reveal that the equilibrium supply of each seller is $\hat{x} = 9/29$ and the equilibrium bid of each buyer is $\hat{b} = 336/841$. The resulting equilibrium price is $\hat{p} = 56/29$. Equilibrium payoffs are $\hat{u} \approx 1.210$ for the buyers and $\hat{\pi} \approx 0.372$ for the sellers.

If a further seller enters the market then the new equilibrium supply from each seller is $\hat{x}' = 15/31$ and the equilibrium bid of each buyer is $\hat{b}' = 855/961$. The equilibrium price is thus $\hat{p}' = 57/31$. The payoff to each buyer at the new equilibrium is $\hat{u}' \approx 1.503$ and the payoff to each seller is $\hat{\pi}' \approx 0.472$.

In accord with expectations, the addition of a seller reduces the equilibrium price and increases total supply to the market. Buyers are also made better off. Contrary to expectations, and despite the fact that there is nothing out of the ordinary about this particular economic environment, existing sellers' profit also increases. As such, the change in the market structure has had a Pareto improving effect.

[2] In fact all these results have been proved for symmetric markets in which all firms are homogeneous.

[3] Our arguments in the sequel ensure this is the unique non-autarkic equilibrium.

In this example the addition of a seller causes incumbent sellers to increase their supply to the market. Yet, accompanied by this increase in supply is only a mild reduction in the price. This results in each seller's revenue increasing to the extent that it dominates their increased costs. Whether existing sellers benefit from this change in the market environment depends on the responsiveness of the equilibrium price to changes in supply, which in turn depends on the reactions of the buyers. In the sequel, we implement novel methods that give rise to a strategic version of demand from the buyers. After defining the elasticity of this strategic demand function we then go on to characterize the conditions under which incumbent sellers are made better off in the presence of new sellers, and when the conventional wisdom applies, which will depend in an intuitive way on both the elasticity of strategic demand and on a measure of the extent of market power of existing sellers.

4 Characterization of equilibria

We begin by outlining a novel method of analyzing the game played that exploits its aggregative properties. This culminates in strategic supply and demand functions that represent the aggregate behavior of sellers and buyers respectively that is consistent with equilibrium, and intersections of these functions are in one-to-one correspondence with non-autarkic Nash equilibria in the game. This provides a highly tractable alternative to the direct analysis of best responses, even in heterogeneous environments. The details of the analysis are published in Dickson and Hartley (2008), to which we refer the reader for full details. Here we recall the main arguments under our currently imposed conditions on preferences with the aim of providing a self-contained treatment, since we use several of the constructs of the method for our analysis in our subsequent arguments.

Whilst the approach is effective in identifying equilibria in bilateral oligopoly, its usefulness is magnified when considering comparative static properties of equilibria, as we do in the sequel. With consideration of the effects of entry in mind, it is noteworthy that, for a given set of buyers, strategic demand is invariant to the composition of the set of sellers. Moreover, the addition of a seller to the economy manifests itself in a tractable way as a change in strategic supply. As such, we will immediately gain insight into the effect on the equilibrium price and volume of trade without initially being concerned with changes in individual equilibrium strategies. However, since we characterize individual behavior according to precisely these variables the effects on individuals are easily deduced. Thus, rather than needing to aggregate changes in individual strategies to determine the effects on equilibrium aggregates the method allows us to look at aggregates directly. In this way, we are able to derive interesting comparative static properties in rather general environments that would otherwise have been intractable.

With this motivation in mind we turn to describe the method of analysis, beginning with the supply side of the market. (For a full description see Dickson and Hartley (2008).) Each seller $i \in I^{\mathrm{S}}$ may be seen as choosing her supply to maximize her profit given the choices of all other agents in the economy, thereby solving the problem

$$\max_{x \in \mathbb{R}_+} xp - C_i(x),$$

where $p = B/X$, $X = x + X_{-i}$ and $X_{-i} = \sum_{j \neq i \in I^{\mathrm{S}}} x_j$. The first-order condition which, under the assumption that costs are increasing and convex, is both necessary and sufficient for identifying best responses, reveals that the best response of seller $i \in I^{\mathrm{S}}$ to a supply of other sellers totalling X_{-i} and bids totalling B takes the form

$$\left\{ x : \left(1 - \frac{x}{x + X_{-i}}\right) \frac{B}{x + X_{-i}} \leq C_i'(x) \right\},$$

with equality if $x > 0$.

Rather than working with best responses directly, we instead derive from these the behavior of agents that is consistent with a Nash equilibrium in which the aggregates take particular values. Accordingly, the supply of seller $i \in I^{\mathrm{S}}$ consistent with a non-autarkic Nash equilibrium in which the aggregate supply is $X > 0$ and the price is p is given by $X s_i^{\mathrm{S}}(p; X)$, where s_i^{S} is the *share function* of seller i, which is given by the unique value of $\sigma \in [0, 1]$ that satisfies

$$(1 - \sigma)p \leq C_i'(\sigma X), \tag{1}$$

with equality if $\sigma > 0$. Define $p_i^* = C_i'(0)$ as the price below which seller i's supply would be zero regardless of the actions of her opponents, then we find that for $p \leq p_i^*$ $s_i^{\mathrm{S}}(p; X) = 0$ for all $X > 0$, whilst for $p > p_i^*$ the share function will take positive values, and slight modifications of the arguments in Dickson and Hartley (2008) reveal that it varies continuously with X and p, is strictly decreasing in $X > 0$ with the property that $\lim_{X \to 0} s_i^{\mathrm{S}}(p; X) = 1 - \frac{p_i^*}{p}$ (Dickson and Hartley 2008, Lemma 3.1), and is non-decreasing in p (Dickson and Hartley 2008, Lemma 5.2).

Share functions give consistent behavior at the individual level. Equilibrium requires consistency of aggregate behavior. Consistency of aggregate supply with a non-autarkic Nash equilibrium in which the price takes a particular value requires the sum of the individual supplies of sellers to be equal to the aggregate supply, or for the aggregate share function $S^{\mathrm{S}}(p; X) = \sum_{j \in I^{\mathrm{S}}} s_j^{\mathrm{S}}(p; X)$ to be equal to one. Defining $\mathcal{X}(p)$ by

$$S^{\mathrm{S}}(p; \mathcal{X}(p)) = 1, \tag{2}$$

the values of X belonging to $\mathcal{X}(p)$ give the levels of aggregate supply that are consistent with a Nash equilibrium in which the price is p. It turns out that there is at most one value of X satisfying $S^{\mathrm{S}}(p; X) = 1$ for each p, and we call $\mathcal{X}(p)$ the *strategic supply function*.

Strategic supply is defined only above some cutoff price, P^{S}, which is such that

$$\sum_{j \in I^{\mathrm{S}}} \max \left\{ 0, 1 - \frac{p_j^*}{P^{\mathrm{S}}} \right\} = 1,$$

and when costs are convex it follows that strategic supply not only varies continuously with $p > P^{\mathrm{S}}$ but also has the desirable property that it is non-decreasing in p (Dickson and Hartley 2008, Lemma 5.2).

Similar considerations apply to the demand side, which will culminate in a strategic demand function. Each buyer $i \in I^{\mathrm{B}}$ can be seen as maximizing her utility from consumption over her choice of bid, i.e. solving the program

$$\max_{b \in [0, m_i]} u_i(b/p, m_i - b),$$

where again $p = B/X$, $B = b + B_{-i}$ and $B_{-i} = \sum_{j \neq i \in I^{\mathrm{B}}} b_j$. Under the assumption of binormality of preferences the first order condition is both necessary and sufficient in identifying best responses, which take the form

$$\left\{ b : \partial_i \left(\frac{b}{b + B_{-i}} X, e_i - b \right) \leq \left(1 - \frac{b_i}{b + B_{-i}} \right)^{-1} \frac{B}{X} \right\},$$

with equality if $b > 0$. Looking for buyer i's bids that are consistent with a Nash equilibrium in which the aggregate bid is $B > 0$ and the price is p, we define the share function of buyer $i \in I^{\mathrm{B}}$ by $s_i^{\mathrm{B}}(p; B) = \min\{\sigma, m_i/B\}$, where $\sigma \in [0, 1]$ is the unique solution to

$$\partial_i(\sigma B/p, e_i - \sigma B) \leq (1 - \sigma)^{-1} p, \tag{3}$$

with equality if $\sigma > 0$. When multiplied by B, the share function gives the bid of buyer $i \in I^{\mathrm{B}}$ consistent with a non-autarkic Nash equilibrium in which the aggregate bid is $B > 0$ and the price is p.

For each $i \in I^{\mathrm{B}}$ define $p_i^* = \partial_i(0, m_i)$, which is the price above which buyer i would never make a bid for the consumption good regardless of the actions of other buyers. Then buyer i's share function is defined for all $0 < p < p_i^*$ where it is continuous, decreasing in both B and p and has the property that $\lim_{B \to 0} s_i^{\mathrm{B}}(p; B) = 1 - \frac{p}{p_i^*}$ (Dickson and Hartley 2008, Lemmas 3.3 and 5.1).

We again look for consistent behavior at the aggregate level, which requires the sum of individual bids to be equal to the aggregate bid, or for the aggregate share function of the buyers $S^{\mathrm{B}}(p; B) = \sum_{j \in I^{\mathrm{B}}} s_j^{\mathrm{B}}(p; B)$ to be equal to one. Instead of looking for the aggregate bid consistent with a particular price it is more convenient to consider the level of demand, given by the ratio of aggregate bid to price, that is consistent with a non-autarkic equilibrium in which the price takes a particular value. Thus, let $\mathcal{D}(p)$ be defined by

$$S^{\mathrm{B}}(p; p\mathcal{D}(p)) = 1, \tag{4}$$

then $\mathcal{D}(p)$ is the *strategic demand function*, that gives the level of demand consistent with a non-autarkic Nash equilibrium in which the price is p. Strategic demand is defined for all $0 < p < P^{\mathrm{B}}$ where P^{B} is a cutoff price above which demand is zero in any Nash equilibrium with such a price, and is defined by

$$\sum_{j \in I^{\mathrm{B}}} \max\left\{0, 1 - \frac{P^{\mathrm{B}}}{p_j^*}\right\} = 1.$$

When preferences are binormal it follows from the fact that individual share functions (and therefore the aggregate) are strictly decreasing in both B and p where positive that the strategic demand function is strictly decreasing in $0 < p < P^{\mathrm{B}}$. (Dickson and Hartley 2008, Lemma 5.1.)

The purpose of constructing strategic supply and demand functions lies in the fact that non-autarkic Nash equilibria in bilateral oligopoly are in one-to-one correspondence with intersections of strategic supply and demand (Dickson and Hartley 2008, Proposition 3.5). In particular there is a non-autarkic Nash equilibrium with price $\hat{p}$ if and only if

$$\mathcal{X}(\hat{p}) = \mathcal{D}(\hat{p}). \tag{5}$$

This fact allows us to deduce that a non-autarkic equilibrium exists if and only if $P^{\mathrm{S}} < P^{\mathrm{B}}$ (i.e. strategic supply and demand cross) and that under our current assumptions such an equilibrium is unique (since the monotonicity properties imply they cross only once). The condition $P^{\mathrm{S}} < P^{\mathrm{B}}$ requires that there are 'sufficient' gains from trade in the economy, and for the remainder of the paper we suppose that preferences and cost functions are such that this holds to rule out the case that autarky is the only equilibrium. At the non-autarkic equilibrium, the volume of trade is $\hat{X} = \mathcal{X}(\hat{p})$and the aggregate bid from the buyers is $\hat{B} = \hat{p}\hat{X}$. The equilibrium supply of seller $i \in I^{\mathrm{S}}$ is $\hat{x}_i = \hat{X}s_i^{\mathrm{S}}(\hat{p}; \hat{X})$ and the equilibrium bid of buyer $i \in I^{\mathrm{B}}$ is $\hat{b}_i = \hat{p}\hat{X}s_i^{\mathrm{B}}(\hat{p}; \hat{p}\hat{X})$.

The aim of the next section is to investigate how a change in the economic environment in the form of the addition of a seller affects equilibrium outcomes, that we now have the tools to identify. Studies that undertake this task usually restrict all sellers to be identical and differentiate variables with respect to the number of sellers. The way we analyze equilibria here, however, means there is no need to appeal to homogeneity assumptions. For this reason we can introduce a further seller that need not be the same as any existing player in the game and consider how her presence changes strategic supply and demand, and therefore the effect on the equilibrium price and volume of trade. Armed with this knowledge we will then turn in the proceeding section to investigate whether anything can be said of the change in individual behavior and payoffs.

5 Aggregate comparative statics

In this section we investigate the change in the equilibrium price, volume of trade and aggregate bid when an additional seller enters the economy. Thus, suppose that the set of sellers becomes $I^{S\prime} = I^S \cup \{k\}$ (post change quantities will be denoted with primes). Our analysis of equilibrium in the previous section allowed the game to be separated into two 'partial games', consistency between which determines equilibrium. Of note is that the addition of a seller to the economy makes no difference to the partial game played by the buyers, and so strategic demand remains unchanged. Conversely, strategic supply will be affected. p_k^* is the price below which seller k will be inactive in a Nash equilibrium with this price. Whilst at prices less than p_k^* strategic supply remains unchanged, at prices above p_k^* seller k will want to supply a positive amount and strategic supply will adjust to reflect this. Note that this will not simply be an addition of seller k's supply to the function, as there will be a change in optimal strategies of other sellers induced by her presence. The following lemma summarizes the effect of the introduction of an additional seller on the strategic supply function.

Lemma 1. *Suppose all sellers' cost functions are increasing and convex. Then when an additional seller k enters the economy, strategic supply increases for all prices exceeding p_k^* and is otherwise unchanged: $\mathcal{X}'(p) > \mathcal{X}(p)$ for all $p > p_k^*$.*

As figure 1 illustrates this result implies that if seller k's cutoff price p_k^* is strictly below the price at the original non-autarkic equilibrium, i.e. $p_k^* < \hat{p}$, then there will also be a non-autarkic equilibrium in the enlarged economy in which seller k is active and the price is lower and the aggregate volume of trade higher than previously.

Proposition 2. *Suppose all buyers' preferences are binormal, the cost functions of all sellers are increasing and convex, and $P^S < P^B$. Then there is a non-autarkic Nash equilibrium with price $\hat{p}$ such that $\mathcal{X}(\hat{p}) = \mathcal{D}(\hat{p})$. Suppose an additional seller k joins the economy whose costs satisfy the conditions above. Then if $p_k^* \geq \hat{p}$ the Nash equilibrium is unchanged. Conversely, if $p_k^* < \hat{p}$ the equilibrium price falls and the quantity of the good traded increases: $\hat{p}' < \hat{p}$ and $\hat{X}' > \hat{X}$.*

This confirms that when the preferences of all buyers are binormal and sellers' cost functions are increasing and convex bilateral oligopoly is quasi-competitive, exhibiting the features of the price reducing and the total volume of trade increasing when new sellers enter the economy.

Amir and Bloch (2009) also study the effects on equilibrium aggregates of entry in bilateral oligopoly (but where the number if buyers is increased). Our results are complementary to theirs due to our differing restrictions on preferences. Whilst they impose weaker functional restrictions than we do (requiring only the consumption commodity (money) to be normal for buyers (sellers)), they require boundary conditions on preferences (that ensure an interior solution) and impose symmetry amongst

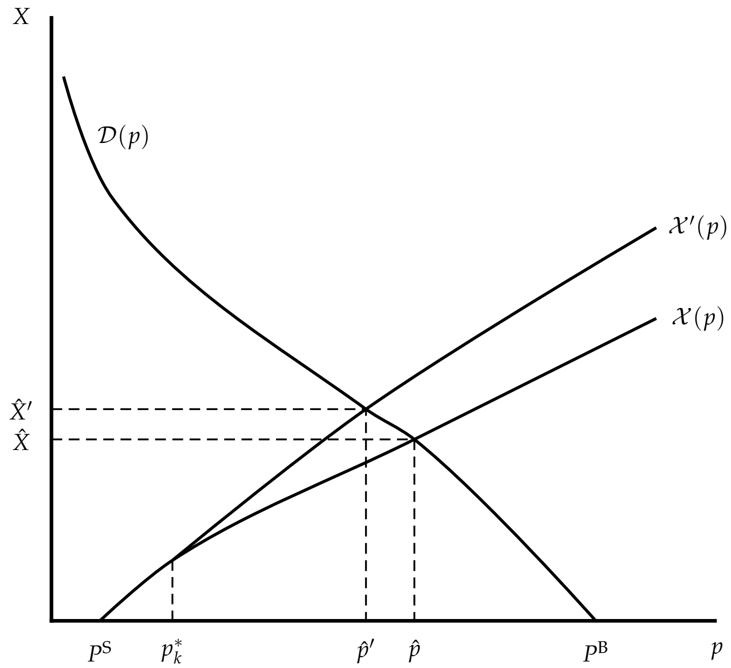

Figure 1: The effect on equilibrium aggregates of an additional active seller.

buyers and sellers. Under these conditions multiple equilibria are possible and they derive analogous results to our Proposition 2 (Amir and Bloch 2009, Propositions 1 and 2) applied to extremal equilibria.

Whilst we can derive definitive results concerning the effects on the quantity traded and the price of the addition of a seller to the economy under our current assumptions, the same is not true of aggregate bids when buyers' preferences are merely binormal. Additional conditions can, however, be imposed on buyers' payoff functions that imply that the aggregate bid is inversely related to the price. Writing $\mathcal{B}(p) = p\mathcal{D}(p)$ for the aggregate bid consistent with a non-autarkic Nash equilibrium in which the price is p we have the following lemma.

Lemma 3. *Suppose all buyers' preferences are binormal. Then if, in addition, all buyers preferences are such that $y_1\partial_i(y_1, y_2)$ increases (decreases) in $y_1 > 0$ for fixed $y_2 > 0$ then $\mathcal{B}(p') \leq (\geq)\mathcal{B}(p)$ for any $P^{\mathrm{B}} > p' > p > 0$.*

Binormality of preferences implies that for fixed y_2, the marginal rate of substitution $\partial_i(y_1, y_2)$ decreases in y_1. Requiring that the product $y_1\partial_i(y_1, y_2)$ increases in y_1 bounds this decrease, which ensures that (competitive) income expansion paths are not too steep; in other words, as more consumption opportunities open up to the buyer she always wants to 'sufficiently' increase her consumption of the good. If this is true for all buyers then as the price reduces buyers want to increase their consumption sufficiently, and their optimal decisions cause the aggregate bid to increase. When the product $y_1\partial_i(y_1, y_2)$ decreases in y_1 for all buyers, the converse is true.

Since we know the effect on the equilibrium price when an additional seller enters the economy (from Proposition 2) it is a simple consequence of Lemma 3 to deduce the effect on the equilibrium aggregate bid.

Corollary 4. *Suppose the preferences of all buyers are binormal and suppose an additional active seller enters the economy. Then if, in addition, all buyers' preferences are such that $y_1\partial_i(y_1, y_2)$ increases (decreases) in $y_1 > 0$ for fixed y_2 then $\hat{B}' \geq (\leq)\hat{B}$.*

At this juncture it is convenient to digress to consider the aggregate bid in a little more detail. The aggregate bid made by the buyers is the revenue that is shared amongst the sellers in proportion to their supply. Recall that if the demand side of a market can be represented by competitive demand then the total revenue the sellers receive is increasing (decreasing) in total supply if and only if that demand function is elastic (inelastic). Using our strategic demand function, we proceed now to derive an analogous result for fully strategic bilateral oligopoly. Since strategic demand is continuous and strictly decreasing it is invertible to a function, that we denote $\mathcal{P}(X)$, which is itself continuous and decreasing in its argument. $\mathcal{P}(X)$, which we call strategic inverse demand, gives us the price that will emerge in a Nash equilibrium in which the aggregate supply is $X > 0$, taking into account the buyers (strategic) behavior. As such, the total revenue when supply is $X > 0$ is $X\mathcal{P}(X)$, and in order for $X > 0$ to be a Nash equilibrium the aggregate bid from the buyers when supply is $X > 0$ must exactly equal this value. Writing $\tilde{\mathcal{B}}(X)$ for the aggregate bid consistent with a Nash equilibrium in the partial game played by the buyers in which the aggregate supply $X > 0$, the condition for X to be an equilibrium level of supply is

$$\tilde{\mathcal{B}}(X) = X\mathcal{P}(X). \tag{6}$$

This is essentially a re-statement of condition (5) in supply, rather than price, space.

Now define

$$\eta(X, \Delta X) = -\frac{\mathcal{P}(X)}{X(\Delta\mathcal{P}(X)/\Delta X)} \tag{7}$$

as the elasticity of strategic demand over the change ΔX, measured along the strategic demand curve. Then for $X' > X$, $\tilde{\mathcal{B}}(X') \geq (\leq)\tilde{\mathcal{B}}(X)$ if and only if $\eta(X, \Delta X) \geq (\leq)1$, where $\Delta X = X' - X$.

As we have noted, for a given set of buyers the strategic demand function is fixed and since it is strictly decreasing in p any change to the economic environment will bring about opposing effects on the equilibrium volume of trade and the equilibrium price. Since we know from Lemma 3 how the aggregate bid changes according to the price, our deductions above allow us to draw the conclusion in the following proposition.

Proposition 5. *Suppose the set of buyers is fixed and all buyers' preferences are binormal. Consider a change to the economy that increases the equilibrium volume of trade from $\hat{X}$ to $\hat{X}' > \hat{X}$. Then $\hat{B}' \geq (\leq)\hat{B}$ if and only if $\eta(\hat{X}, \Delta\hat{X}) \geq (\leq)1$ where $\Delta\hat{X} = \hat{X}' - \hat{X}$. As such, if the preferences of every buyer $i \in I^{\mathrm{B}}$ are such that $y_1\partial_i(y_1, y_2)$ increases (decreases) in y_1 for fixed $y_2 > 0$ then $\eta(\hat{X}, \Delta\hat{X}) \geq (\leq)1$ (since $\hat{B}' \geq (\leq)\hat{B}$).*

By exploiting the aggregative properties of the game to construct strategic supply and demand functions we have been able to deduce, in a highly tractable fashion, the effects on equilibrium aggregates without having to first deal with the complexities of individual behavior and then perform an aggregation process. Armed with knowledge of these effects, we turn in the next section to our representations of individual behavior and attempt to characterize individual responses and the effect on payoffs.

6 Individual comparative statics

We begin by looking at the response of individual buyers and the effect on their payoff, following which we turn to analyze sellers. In order to make progress we assume changes are small enough that we can differentiate so that we can use the first-order conditions to simplify expressions.

In the previous section we concluded that the effect of an additional seller on the aggregate bid can only be signed when we place additional restrictions on all buyers' preferences. When all buyers are homogeneous these same restrictions allow us to conclude similar effects for individual bids (which are equal to $\hat{B}/|I^{\mathrm{B}}|$). However, with a heterogeneous buyer set nothing we have said yet allows us to conclude the effect on individual bids, since the bid of buyer i is given by $\hat{b}_i = \hat{B}s_i^{\mathrm{B}}(\hat{p}; \hat{B})$ and we cannot sign the change in the equilibrium value of the share function. The following lemma resolves this ambiguity.

Lemma 6. *Suppose all buyers' preferences are both binormal and such that $y_1\partial_i(y_1, y_2)$ increases (decreases) in y_1 for fixed $y_2 > 0$. Then when an additional seller enters the economy no buyer's bid is lower (higher) than previously: $\hat{b}_i' \geq (\leq)\hat{b}_i$ for all $i \in I^{\mathrm{B}}$.*

We turn next to look at the change in the equilibrium payoffs of the buyers. The equilibrium payoff of buyer $i \in I^{\mathrm{B}}$ can be written as $\hat{u}_i = u_i(\hat{b}_i/\hat{p}, e_i - \hat{b}_i)$. The change in her payoff is thus

$$\begin{aligned} \mathrm{d}\hat{u}_i &= ((1/\hat{p})u_i^1 - u_i^2)\mathrm{d}\hat{b}_i - (\hat{b}_i/\hat{p}^2)u_i^1\mathrm{d}\hat{p} \\ &= u_i^1/\hat{p}[(\hat{b}_i/\hat{B})\mathrm{d}\hat{b}_i - (\hat{b}_i/\hat{p})\mathrm{d}\hat{p}], \end{aligned}$$

where the second line utilizes the first-order condition. From this expression it is transparent that so long as the buyer doesn't reduce her bid in the presence of a new seller she will receive a higher level of utility in equilibrium than with fewer sellers (since the price strictly declines), and the previous lemma allowed us to conclude that bids are non-decreasing under additional restrictions on preferences, or indeed when buyers are homogeneous.

Proposition 7. *Suppose the preferences of all buyers are binormal and either a) all buyers' preferences are also such that $y_1 \partial_i(y_1, y_2)$ increases in y_1 or b) all buyers are homogeneous. Suppose further that all sellers' cost functions are increasing and convex. Then when an additional active seller enters the economy all buyers are made strictly better off.*

If a buyer's bid in the new Nash equilibrium is lower than previously this raises the possibility that she will receive a lower level of utility in the new equilibrium. Note well, however, that it cannot be the case that *all* buyers are made worse off, even if they all reduce their bids (which will be the case when $y_1 \partial_i(y_1, y_2)$ is decreasing in y_1 for all buyers). For, if a buyer reduces her bid she retains more money. In order for her payoff to reduce, therefore, it is necessary that her allocation of the good, given by $\hat{\sigma}_i \hat{X}$ (where $\hat{\sigma}_i$ is buyer i's equilibrium share) reduces. Whilst this can be true for individual buyers it cannot be true for all (since $\hat{X}$ increases and equilibrium requires $\sum_{j \in I^{\mathrm{B}}} \hat{\sigma}_j = 1$); at least one buyer's payoff must increase. This allows us to conclude the following.

Proposition 8. *Suppose all buyers' preferences are binormal and all sellers' cost functions are increasing and convex. Then the Nash equilibrium in the presence of an additional active seller cannot be Pareto inferior to the Nash equilibrium in the original economy.*

This concludes our study of the buyers. Next we turn to the sellers.

The equilibrium payoff of seller $i \in I^{\mathrm{S}}$ is given by $\hat{\pi}_i = \hat{x}_i \hat{p} - C_i(\hat{x}_i)$. If an additional seller enters the economy the change in her payoff is

$$\begin{aligned} \mathrm{d}\hat{\pi}_i &= \hat{x}_i \mathrm{d}\hat{p} + p \mathrm{d}\hat{x}_i - C_i'(\hat{x}_i) \mathrm{d}\hat{x}_i \\ &= \hat{x}_i \mathrm{d}\hat{p} + \frac{\hat{x}_i}{\hat{X}} \hat{p} \mathrm{d}\hat{x}_i, \end{aligned}$$

where the second line uses the first-order condition. From Proposition 2 we know that in the presence of an additional active seller $\mathrm{d}\hat{p} < 0$. Thus, if seller i's supply in the new Nash equilibrium is lower than before she will receive a lower equilibrium payoff. However, if she increases her supply in equilibrium this raises the possibility that she could be made better off.

To investigate whether this will ever be the case, we note that the supply of seller $i \in I^{\mathrm{S}}$ consistent with a Nash equilibrium in which the aggregate supply is $X > 0$ and the price is p can be written $\tilde{x}_i(p; X) = X s_i^{\mathrm{S}}(p; X)$. Explicitly,

$$\tilde{x}_i(p; X) = \{x : (1 - x/X)p \leq C_i'(x)\},$$

with equality if $x > 0$. If seller i is active in the original Nash equilibrium then $(1 - \hat{x}_i/\hat{X})\hat{p} = C_i'(\hat{x})$. Since costs are convex C_i' is increasing in its argument, and since the left-hand side is decreasing in x it follows that with a change in the economic environment seller i's supply increases (decreases) if and only if $\Delta(1-\hat{x}_i/\hat{X})\hat{p} > (<) 0$. This change is given by

$$\begin{aligned}\Delta(1-\hat{x}/\hat{X})\hat{p} &= \frac{\hat{x}}{\hat{X}^2}\hat{p}\Delta\hat{X} + (1-\hat{x}/\hat{X})\Delta\hat{p} \\ &= \Delta\hat{p}[1-\hat{\sigma}_i(1+\eta(\hat{X},\Delta\hat{X}))]\end{aligned}$$

since, in equilibrium, $\hat{p} = \mathcal{P}(\hat{X})$. In the above expression, $\Delta\hat{p} = \hat{p}' - \hat{p}$, $\Delta\hat{X} = \hat{X}' - \hat{X}$ and $\eta(\cdot,\cdot)$ is as defined in expression (7). In the presence of an additional seller we know from Proposition 2 that $\Delta\hat{p} < 0$. As such, seller i increases (decreases) her supply to the market if and only if $1-\hat{\sigma}_i(1+\eta(\hat{X},\Delta\hat{X})) < (>) 0$, i.e. if

$$\eta(\hat{X},\Delta\hat{X}) > (<) \frac{1}{\hat{\sigma}_i} - 1. \tag{8}$$

From this it becomes transparent that the equilibrium reaction of an existing seller to the presence of a new seller depends on her share of the market and on the elasticity of the strategic demand function, which captures the buyers' optimal responses. If the elasticity of strategic demand exceeds a critical value that is inversely related to the seller's market share then that seller will increase her supply at the new equilibrium, otherwise it will decline. Thus, if the elasticity of demand is sufficiently low and/or the seller's share in the market is small then her optimal supply will reduce[4] and it can be deduced from the expression for the change in the equilibrium payoff that under these circumstances the profit of the existing seller will decline. If sellers are sufficiently numerous, no seller has a market share that is too great and demand is not too elastic then the conventional wisdom that entry by an additional seller reduces existing sellers' profit will apply.

Notice that if there are no dominant firms (i.e. when $\hat{\sigma}_i < 1/2$ then strategic demand being inelastic is a sufficient condition for all existing firms to reduce their equilibrium supply when an additional active seller enters. Recall from Proposition 5 that inelastic strategic demand is equivalent to the aggregate bid of buyers decreasing, and that this occurs when the preferences of all buyers satisfy the restriction that $y_1\partial_i(y_1,y_2)$ decreases in y_1. This allows us to draw the following conclusion concerning when the conventional comparative statics apply.

Proposition 9. *Suppose the preferences of all buyers are both binormal and such that $y_1\partial_i(y_1,y_2)$ is decreasing in y_1, the costs of all sellers are increasing and convex, and there are no dominant firms. Then when an additional active seller enters the economy existing sellers reduce their supply and their equilibrium profit decreases.*

[4] Indeed, the presence of a new active seller may cause some existing sellers to be inactive in the new equilibrium.

If strategic demand is elastic and there are firms whose share of the market is not too small such sellers may increase their supply when an additional firm enters, and this raises the possibility that their equilibrium payoff may increase. Seller $i \in I^S$ will increase her supply when $\eta(\hat{X}, \Delta\hat{X}) > 1/\hat{\sigma}_i - 1$, and whilst the full decomposition of the change in profit is convoluted and doesn't reveal informative threshold conditions, the intuition is simple: when an additional seller enters this lowers the price, but if an existing seller increases her supply and the fall in price is small enough the revenue she receives will increase (this is more likely the more elastic demand is). If the increase in revenue exceeds the additional cost incurred from increasing supply then her profit increases.

As is evidenced by the example at the start of the paper, there are standard economic environments in which this is the case. If the set of buyers have preferences such that demand is elastic and there are sellers whose share of the market is not too small the presence of an additional active seller might make existing sellers better off. Contrary to the vast literature on entry deterrence, in thin bilateral oligopoly environments in which strategic demand is sufficiently elastic there may be firms that actively encourage the entry of additional firms in the market.

The conditions required for this, according to Proposition 5, necessitate an increase in the aggregate bid, which will be the case when all buyers preferences satisfy the restriction that $y_1 \partial_i(y_1, y_2)$ is increasing in y_1. Under these conditions on preferences it was shown in Proposition 7 that buyers are made better off in the presence of an additional seller. Thus, if the condition in (8) is satisfied with a $>$ inequality for all sellers, then entry by an additional seller will be Pareto improving.

Note well, however, that even if market conditions are such that entry is encouraged by some existing sellers, successive entry by new active sellers will reduce the market share of existing sellers and the critical level that the elasticity of strategic demand must exceed will increase. As such, there will be a critical density of firms beyond which further entry will cause existing firms to reduce their supply implying their profit will fall, and the conventional wisdom will apply when markets are thick enough.[5]

7 Conclusions

By exploiting the aggregative structure of the game we have been able to undertake a comprehensive study of the change in equilibrium when a new seller enters a bilateral oligopoly, without making any assumptions that restrict the heterogeneity of agents or imposing overly restrictive assumptions on players' payoff functions. We found that bilateral oligopoly is quasi-competitive (the volume of trade increases and price reduces when additional firms enter the market), and that under reasonable restrictions

[5] If there are $|I^S|$ homogeneous sellers the condition is $\eta(\hat{X}, \Delta\hat{X}) > |I^S| - 1$.

on buyers' preferences their payoffs increase in the presence of additional sellers. The sellers themselves may also be made better off in the face of more intense competition. In particular, if the elasticity of demand exceeds a threshold which is inversely related to a firm's market share that firm will enjoy a higher level of profit at the new equilibrium. If this is true for all sellers the presence of an additional seller is Pareto improving. Whilst successive entry by additional sellers eventually annihilates this result, it raises the possibility that in thin markets incumbent firms may actively seek new entrants to the industry, in contrast to the entry deterrence literature.

A Appendix - Proofs

Proof of Lemma 1. If $p \leq p_k^*$ seller k's share function is identically zero for all $X > 0$ so the aggregate share function remains the same so for such prices the value of X where the aggregate share function equals one, which is precisely strategic supply, ramains unchanged. For $p > p_k^*$, $s_k^{\mathrm{S}}(p; X) > 0$ for all $X > 0$ and so $S^{\mathrm{S}'}(p; X) > S^{\mathrm{S}}(p; X)$ for all $X > 0$. In particular $S^{\mathrm{S}'}(p; \mathcal{X}(p)) > S^{\mathrm{S}}(p; \mathcal{X}(p)) = 1$ for all $p > p_k^*$. Since both S^{S} and $S^{\mathrm{S}'}$ are strictly decreasing in $X > 0$ when sellers' costs are convex this implies the value of X where $S^{\mathrm{S}'}(p; X) = 1$ exceeds $\mathcal{X}(p)$; $\mathcal{X}'(p) > \mathcal{X}(p)$ for all $p > p_k^*$. $\square$

Proof of Lemma 3. For $0 < p < P^{\mathrm{B}}$ the consistent aggregate bid $\mathcal{B}(p)$ is given by $S^{\mathrm{B}}(p; \mathcal{B}(p)) = 1$. We have already noted that individual share functions are strictly decreasing in $B > 0$, and the aggregate share function inherits this property. As we will see, it will therefore suffice to show that the aggregate share function is non-increasing in p when $y_1 \partial_i(y_1, y_2)$ is increasing in y_1 for all buyers and non-decreasing if $y_1 \partial_i(y_1, y_2)$ decreases in y_1, and we proceed by demonstrating these properties for individual share functions.

Suppose that for buyer $i \in I^{\mathrm{B}}$ $y_1 \partial_i(y_1, y_2)$ is increasing in y_1. Then suppose, contrary to our claim, that for $p' > p$ we have $\sigma' = s_i^{\mathrm{B}}(p'; B) > s_i^{\mathrm{B}}(p; B) = \sigma$. Then we have $e_i - \sigma' B < e_i - \sigma B$, and we split the proof into two cases. In case (i) $\sigma' B / p' \geq \sigma B / p$. Then binormality implies $\partial_i(\sigma' B / p', e_i - \sigma' B) \leq \partial_i(\sigma B / p, e_i - \sigma B)$. But, using the first order condition this implies $(1 - \sigma')^{-1} p' \leq (1 - \sigma)^{-1} p$, a contradiction. In case (ii) $\sigma' B / p' < \sigma B / p$. Then we use the fact that $y_1 \partial_i(y_1, y_2)$ is increasing in y_1 to deduce that

$$\frac{\sigma' B}{p'} \partial_i \left(\frac{\sigma' B}{p'}, e_i - \sigma' B \right) < \frac{\sigma B}{p} \partial_i \left(\frac{\sigma B}{p}, e_i - \sigma' B \right).$$

But since $e_i - \sigma' B < e_i - \sigma B$ binormality implies that

$$\frac{\sigma B}{p} \partial_i \left(\frac{\sigma B}{p}, e_i - \sigma' B \right) < \frac{\sigma B}{p} \partial_i \left(\frac{\sigma B}{p}, e_i - \sigma B \right).$$

Combining these inequalities and using the first-order condition gives

$$\frac{\sigma' B}{p'}(1-\sigma')^{-1}p' < \frac{\sigma B}{p}(1-\sigma)^{-1}p$$

which, after canceling terms leads to a contradiction.

Thus, for $p' > p$ we have that for each $i \in I^{\mathrm{B}}$, $s_i^{\mathrm{B}}(p';B) \leq s_i^{\mathrm{B}}(p;B)$ for all $B > 0$. As such, $S^{\mathrm{B}}(p';B) \leq S^{\mathrm{B}}(p;B)$ for all $B > 0$. In particular we have that for $0 < p < P^{\mathrm{B}}$, $S^{\mathrm{B}}(p';\mathcal{B}(p)) \leq S^{\mathrm{B}}(p;\mathcal{B}(p)) = 1$. Since S^{B} is strictly decreasing in $B > 0$ this implies that $\mathcal{B}(p') \leq \mathcal{B}(p)$.

Next we turn to the case when $y_1\partial_i(y_1,y_2)$ decreases in y_1. Suppose contrary to our claim that for $p' > p$, $\sigma' = s_i^{\mathrm{B}}(p';B) < s_i^{\mathrm{B}}(p;B) = \sigma$. Then we have $e_i - \sigma' B > e_i - \sigma B$ and $\sigma' B/p' < \sigma B/p$. Then the fact that $y_1\partial_i(y_1,y_2)$ decreases in y_1 can be used to deduce that

$$\frac{\sigma' B}{p'}\partial_i\left(\frac{\sigma' B}{p'}, e - \sigma' B\right) > \frac{\sigma B}{p}\partial_i\left(\frac{\sigma B}{p}, e_i - \sigma' B\right) > \frac{\sigma B}{p}\partial_i\left(\frac{\sigma B}{p}, e_i - \sigma B\right)$$

where the last inequality is due to binormality. But using the first-order condition, this implies

$$\frac{\sigma' B}{p'}(1-\sigma')^{-1}p' > \frac{\sigma B}{p}(1-\sigma)^{-1}p$$

which, after canceling terms, yields a contradiction. Thus, $s_i^{\mathrm{B}}(p';B) \geq s_i^{\mathrm{B}}(p;B)$ and if the additional restriction on preferences holds for all buyers this implies $S^{\mathrm{B}}(p',\mathcal{B}(p)) \geq S^{\mathrm{B}}(p,\mathcal{B}(p)) = 1$ with the implication that $\mathcal{B}(p') > \mathcal{B}(p)$. □

Proof of Lemma 6. The bid of buyer $i \in I^{\mathrm{B}}$ consistent with a Nash equilibrium in which the aggregate bid is $B > 0$ and the price is p is given by $\tilde{b}_i(p;B) = Bs_i^{\mathrm{B}}(p;B)$. We show that this function is non-decreasing in $B > 0$ and, when $y_1\partial_i(y_1,y_2)$ is increasing (decreasing) in y_1 that $\tilde{b}_i$ is non-increasing (non-decreasing) in p. Since, in the presence of an additional seller, $\hat{p}' < \hat{p}$ and $\hat{B}' \geq (\leq)\hat{B}$ when $y_1\partial_i(y_1,y_2)$ is increasing (decreasing) in y_1 for all $i \in I^{\mathrm{B}}$, this achieves the desired result.

Note that

$$\tilde{b}_i(p;B) = \{b : \partial_i(b/p, e_i - b) = (1 - b/B)^{-1}p\}.$$

To show that $\tilde{b}_i$ is non-decreasing in $B > 0$ suppose, to the contrary, that for $B' > B$ we have $b' = \tilde{b}_i(p';B) < \tilde{b}_i(p;B) = b$. Then $b'/p < b/p$ and $e_i - b' > e_i - b$ and binormality implies $\partial_i(b'/p, e_i - b') > \partial_i(b/p, e_i - b)$. But then the first-order condition implies $(1 - b'/B')^{-1}p > (1 - b/B)^{-1}p$, a contradiction, which establishes our claim.

Turning to the monotonicity properties with respect to p, suppose that $y_1\partial_i(y_1,y_2)$ is increasing in y_1, $p' > p$ and, contrary to our claim, that $b' = \tilde{b}_i(p';B) > \tilde{b}_i(p;B) = b$. Then $e_i - b' < e_i - b$, and we split the proof into two cases. In case (i) $b'/p' > b/p$, and so binormality implies $\partial_i(b'/p', e_i - b') < \partial_i(b/p, e_i - b)$. Utilizing the first-order condition, this implies $(1 - b'/B)^{-1}p' < (1 - b/B)^{-1}p$, a contradiction. In case (ii) $b'/p' < b/p$ and the fact that $y_1\partial_i(y_1, y_2)$ increases in y_1 is used to deduce that

$$\frac{b'}{p'}\partial_i\left(\frac{b'}{p'}, e_i - b'\right) < \frac{b}{p}\partial_i\left(\frac{b}{p}, e_i - b'\right) < \frac{b}{p}\partial_i\left(\frac{b}{p}, e_i - b\right),$$

where the last inequality follows from binormality. Using the first-order condition, this implies

$$\frac{b'}{p'}\left(1 - \frac{b'}{B}\right)^{-1} p' < \frac{b}{p}\left(1 - \frac{b}{B}\right)^{-1} p,$$

which, after canceling terms, yields a contradiction. Thus, $\tilde{b}_i(p; B)$ decreases in p when $y_1\partial_i(y_1, y_2)$ increases in y_1.

Next suppose that $y_1\partial_i(y_1, y_2)$ is decreasing in y_1. Also suppose that for $p' > p$ we have $b' = \tilde{b}_i(p'; B) < \tilde{b}_i(p; B) = b$, contrary to our claim. Then $e_i - b' > e_i - b$ and $b'/p' < b/p$. The latter inequality means we can use the fact that $y_1\partial_i(y_1, y_2)$ is decreasing in y_1 to deduce that

$$\frac{b'}{p'}\partial_i\left(\frac{b'}{p'}, e_i - b'\right) > \frac{b}{p}\partial_i\left(\frac{b}{p}, e_i - b'\right) > \frac{b}{p}\partial_i\left(\frac{b}{p}, e_i - b\right),$$

the last inequality being due to binormality. But then the first-order condition implies

$$\frac{b'}{p'}\left(1 - \frac{b'}{B}\right)^{-1} p' > \frac{b}{p}\left(1 - \frac{b}{B}\right)^{-1} p,$$

which after canceling terms gives rise to a contradiction. Thus, when $y_1\partial_i(y_1, y_2)$ is decreasing in y_1, $\tilde{b}_i(p; B)$ increases in p. This concludes the proof. □

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