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MODEL OF DELEVERAGING

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The Carbon Bubble:
Climate Policy in a Fire-Sale Model of Deleveraging

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Abstract

Credible implementation of climate change policy, consistent with the 2°C limit, requires a large proportion of current fossil fuel reserves to remain unused. This issue, named the Carbon Bubble, is usually presented as a required asset write-off, with implications for investors. For the first time, we discuss its implications for macroeconomic policy and for climate policy itself. We embed the Carbon Bubble in a macroeconomic model exhibiting a financial accelerator: if investors are leveraged, the Carbon Bubble may precipitate a fire-sale of assets across the economy, and generate a large and persistent fall in output and investment. We find a role for policy in mitigating the Carbon Bubble.

Keywords: Carbon Bubble, fire-sale, deleveraging, resource substitution, 2°C target.

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1 Introduction

In 1996, EU Governments set a global temperature target of two degree Celsius (°C) above pre-industrial level which was made international policy at the 2009 United Nations Climate Change Conference in Copenhagen.[1] A global mean temperature increase of 2°C is considered as a threshold separating safety from extreme events: significant extinctions of species, reductions in water availability and food production, catastrophic ice sheet disintegration, and sea level rise (EU Climate Change Expert Group, 2008). The Potsdam Climate Institute has calculated that if we want to reduce the probability of exceeding 2°C warming to 20%, then only one-fifth of the Earth's proven fossil fuel reserves can be burned unabated (Carbon Tracker Initiative, 2011). As a consequence, there is a global "carbon budget" of allowable emissions, whilst the rest is "unburnable carbon". Broadly, this translates into a near term cessation of all coal use, though proven oil and gas reserves can be partly exploited to fuel the global economy[2] as it implements a transition to zero carbon infrastructure.[3]

The imposition of a climate policy consistent with the Potsdam Climate Institute's calculations would mean that the fundamental value of many fossil fuel assets must be written-off. The Carbon Tracker Initiative's (2011) report terms this issue "Carbon Bubble", as the current market value of these "stranded assets" must be made up of a zero fundamental value, and a "bubble" component.[4] According to Robins et al. (2012), carbon constraints could impact the valuations of coal assets by as much as 44%, with average impact to the stock market valuation of mining companies between 3% and 7%. Moreover, "given that the mining sector comprises around 12% of the FSTE100 index, the risk is potentially also relevant to the broader market" (Robins et al., 2012, page 6). As well as direct financial losses on fossil fuel assets, the use value (and hence the financial value) of assets that are strongly complementary with fossil fuel use is also at risk, e.g. power stations, vehicles, factories, and the whole broad network of infrastructure that determines the location of, and enables the flows of, energy, people, goods and economic activity.

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[1] See Jaeger and Jaeger (2011) for a summary of how the target emerged and evolved.

[2] See McGlade and Ekins (2015) who suggest that "globally, a third of oil reserves, half of gas reserves and over 80 per cent of current coal reserves should remain unused ... in order to meet the target of 2°C".

[3] We should view the replacement zero carbon infrastructure as a combination of zero carbon energy generation (e.g. renewables), the energy storage and associated technologies required to make this as convenient as current fossil fuels (e.g. electric cars), and energy efficiency improvements (e.g. mass transit and densification).

[4] What has been termed the Carbon Bubble is a real asset which has positive fundamental value in one state of the world (no regulation) but not in another (with regulation). It is not an economic bubble of the form described in the economics literature that follows from Tirole (1985). However, see Jovanovic (2013), who suggests that the price of oil does contain a bubble. This term was coined by the Carbon Tracker Initiative (2011) and popularised by Rolling Stone (2012): according to Google Trends, web search on the term "Carbon Bubble" reached a high around May 2015.

However, the financial importance of some of these write-offs can be questioned. Most of the fossil reserves that will have to stay in the ground are coal, which is quite costly to extract (relative to the price obtained for the energy that can be so generated). Further, unconventional oil and gas fracking is currently a key marginal oil and gas resource: to the extent that its extraction seems to start as soon as the price is only marginally over extraction costs, this suggests that the owners of these deposits do not expect prices to rise a lot in the future. Therefore, the economic rent associated with, and hence the value of, these reserves is likely low. Conversely, the fossil fuel assets with significant value (e.g. conventional oil and gas with low extraction costs) are the very assets that will still be allowed to be used within a global carbon budget. Furthermore, not all the non-fossil fuel assets that are strongly complementary with fossil fuels will be exposed to the Carbon Bubble. Assets that depreciate fairly quickly, e.g. cars, will be used almost fully even under a Carbon Bubble scenario. Some assets which depreciate slowly, e.g. the road network, may be valuable in the post-fossil fuel era, though some proportion of its value is likely at risk (e.g. it may be in a relatively sub-optimal location, or it may be utilised at a very low capacity in a Carbon Bubble scenario). A Carbon Bubble scenario, compared to laissez-faire, will strongly constrain the future working life of many slowly depreciating assets, such as newly built coal power stations, whose financial value is therefore at great risk.

If policymakers enforce compliance with the 2°C target, certain assets will have their use restricted, and thus will be revalued by the markets. While the extent of the loss to the capital stock under a Carbon Bubble scenario is a matter of debate, it is clearly non-zero and it carries potentially profound implications. Indeed, the severity of the financial crisis has proven that a financial market disruption, induced by a problem in a small portion of the economy, can cause a deep recession, through the financial accelerator mechanism of feedback between the financial and non-financial sectors. Credible climate policy implementation will lead to asset price falls. If investors are using their holdings of such exposed financial assets as collateral, then these write-downs could lead to a breakdown of credit relationships and a general decline in the amount of total credit supplied to the economy. Forward looking markets could turn an announced carbon budget into a "sudden stop" akin to, or worse than, the 2008 Financial Crisis (Mendoza, 2010): the Carbon Bubble could burst.

A recessionary response is particularly damaging with respect to the implementation of the climate policy itself, as one of its aims is to provide the incentives for investments in alternative energy capital, in order to replace the current fossil fuel based energy infrastructure. A substantial stock of zero carbon productive capacity will need to be in place at the point at which the carbon budget is exhausted, but the bursting of the Carbon Bubble could throw the economy into a deep recession, thus depriving green technology of investment funds when they are most needed. Even if the fossil fuels assets really should

be written-off to avoid disastrous global warming, the implementation of such a policy must pay cognisance to the impact that it will have upon investment.

The main contribution of this article is to link, for the first time, the issue of the Carbon Bubble with the financial accelerator mechanism, and to analyse the interaction between climate policy and macroeconomic stabilisation.[5] We model the consequences of a write-off of energy capital following the implementation of a climate policy in the spirit of the 2°C target. We assume a binding cumulative emissions allowance (a carbon budget), and incorporate a financial accelerator effect by using the credit amplification mechanism of Kiyotaki and Moore (1997), where entrepreneurs borrow from savers using their current asset holdings as collateral. This framework allows us to go beyond the discussions of the Carbon Bubble that have appeared to date, which focus upon the risks from climate policy to individual investors,[6] and to start considering the link from the impact of climate policy to financial markets and the macroeconomy, back to the appropriate climate policy itself. Alongside the credible implementation of climate policy consistent with the 2°C limit, we consider policies that transfer investors' debts to the government, subsidise investment, and provide government guarantees on investors' borrowings. We show that macroeconomic policies that mitigate the impact of the Carbon Bubble upon the balance sheets of investors can be welfare enhancing (though not necessarily Pareto improving), even if such policies are welfare destroying under normal circumstances. We chose a tractable model to create this first meeting between macro-financial and climate-economy models, and with which to form initial conclusions: our aim is to start the conversation and to provoke further research.

The remainder of the article is organised as follows. Section 2 reviews the literature, and Section 3 presents the theoretical model we employ. Section 4 describes the process we used to calibrate the model, so that it broadly replicates the outcomes seen over the 2008-09 Financial Crisis. Section 5 sets out the Carbon Bubble scenario in which the planner bans investment in new high carbon energy capital, and implements restrictions on the usage of existing carbon assets. This section also examines policies which the planner can additionally implement, in order to minimise the business cycle response, and to boost alternative energy capital investment and welfare. Finally, Section 6 concludes.

2   Related Literature

Gerke et al. (2013) show that most models of the financial accelerator share qualitatively similar features. We choose to work with the model of Kiyotaki and Moore (1997) because it has several characteristics that are attractive given our exercise. Firstly, it is a tractable

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[5]Though the Bank of England has also signalled that it is investigating this issue: see Carney (2014).
[6]See Carbon Tracker Initiative (2011, 2013).

model with fire-sale dynamics which we can modify to examine the interaction between climate policy induced energy capital write-offs and the macroeconomy. Secondly, their model does not return to steady state following a very large negative shock. This allows us to make the rhetorical point that such a shock can wipe out the entrepreneurial sector, and that other mechanisms or interventions are necessary to restart leveraged investment.[7,8] Thirdly, in the Kiyotaki and Moore's (1997) model, entrepreneurs use a Leontief combination of fixed capital and reproducible idiosyncratic capital. We introduce two flavours of idiosyncratic capital: a more productive high carbon variety and a less productive zero carbon variety. The Leontief specification is a convenient modelling device to capture the very low elasticity of substitution between energy and other inputs to production, at least in the short-run, measured by Hassler et al. (2015).

This framework allows us to model the Carbon Bubble, which has hitherto not been considered as part of the literature on the economics of climate change. The standard approach to the economics of climate change, Nordhaus's (2008) Integrated Assessment Model (IAM), considers an optimal economic growth framework which includes damages from climate change. Typically, IAMs balance the economic benefits of fossil fuel emissions for production against the economic damages from climate change, to produce some optimal timepath for emissions reduction which is implemented with a timepath of carbon taxes.

The scientific literature, on the other hand, suggests that the first-order impact of emissions in any given period is related to their contribution to the overall cumulative emissions, which is the main driver behind climate change (Allen et al., 2009; IPCC, 2014). This is also consistent with the headlines from the Carbon Tracker Initiative's (2011) report which talked of a "carbon budget". In this article, we will use this idea of a cumulative emissions constraint, or carbon budget, which makes the modelling exercise easier: we model a cumulative emissions limit separating non-catastrophic damages, which are broadly undetectable in the social welfare function, from catastrophic damages which

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[7]See Footnote 26 for a discussion of the other steady states of the Kiyotaki and Moore's (1997) model.

[8]We introduce a putative debt renegotiation process that ensures the model can always return to the interior steady state, similarly to Cordoba and Ripoll (2004). However, relying too much on debt renegotiation ruins the story that models of this sort tell: we thus calibrate the model in such a way that this debt renegotiation process is not actually needed in any of the scenarios we present in Section 5. See Appendix A.6 for the particulars of the calibration, and Appendix A.9 for more information on the renegotiation process.

cause infinitely negative social welfare and so must be avoided at all costs.[9]

One way to think about imposing such a cumulative emissions constraint that embeds it within the standard approach is to say that we are arguing probabilistically, and invoke Weitzman (2009). Perhaps the damages associated with climate change have an uncertainty that grows with their median size. With low emissions, within our allowed carbon budget, we have low median damages and further, the uncertainty on these damages has a thin-tailed distribution: the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is zero. The expected impact of such emissions is close to the medium impact and it is almost undetectable in terms of overall social welfare. Our carbon budget represents some threshold between a thin-tailed and a fat-tailed distribution for damages from emissions. With a fat-tailed distribution of damages, the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is infinitely negative. Therefore, for emissions greater than the carbon budget, although the median impact is smoothly increasing in emission levels, the expected value tends to infinity across this threshold. Therefore, treating climate damages as approximately zero within the carbon budget and infinite beyond the carbon budget can be rationalised, and it simplifies the modelling substantially.

3   The Model

We develop a two-agent closed economy model which extends the full model of Kiyotaki and Moore (1997, Chapter III) by allowing entrepreneurs to choose between two types of investment good with different productivity. We also introduce a simple government or policymaker.

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. There are two types of infinitely lived agents: a continuum of entrepreneurs of mass one, and a continuum of savers of mass m.[10] Entrepreneurs and savers maximize the expected discounted utilities from

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[9]A cumulative emissions constraint may be subject to the so-called "Green Paradox" issue raised by Sinn (2012) in which future restrictions on fossil fuels, or future incentives to green technologies, lead to greater contemporary use of fossil fuels, as current owners of these assets compete to exploit them before the conditions for their use deteriorate. In the context of a cumulative emissions constraint that will not be breached, this means that fossil fuel asset owners fight for their asset not to be one of those assets which is "stranded". A more fundamental problem raised by the Green Paradox is the ability of a policy maker to implement a cumulative emissions constraint and a fossil fuel investment ban which has the effect of allowing some asset holders to exploit their assets whilst stranding other assets. In this first-order exercise, we assume a certain omnipotence for the policy maker, and leave complications such as the ability to implement and enforce this policy for future research. In support of this model simplifying assumption is the contention of van der Ploeg (2013) and Tietenberg and Lewis (2014) that the Green Paradox does not seem to be a serious obstacle to climate policy.

[10]Variables regarding the savers are identified by the prime. Aggregate variables will be capitalized. Steady state variables will be starred. For a list of variables and parameters, and their definitions, see Appendix A.1.

consumption,

$$\max_{\{x_s\}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} x_s \right] \qquad \text{and} \qquad \max_{\{x'_s\}} E_t \left[\sum_{s=t}^{\infty} (\beta')^{s-t} x'_s \right] \qquad (1)$$

where x_t and x'_t represent consumption at date t of the entrepreneur and the saver respectively; $\beta, \beta' \in (0,1)$ indicate the discount factors; and E_t indicates expectations formed at date t. Both types of agents are risk neutral but they differ in their rates of time preference: entrepreneurs are more impatient,[11]

Assumption A *(Impatient entrepreneurs)* $\quad \beta < \beta'$.

There are three types of goods: fixed capital (K), energy capital (Z), and non-durable commodity. The energy capital has two flavours: high carbon and zero carbon, indexed by H and L respectively. The non-durable commodity cannot be stored but can be consumed or invested in energy capital.[12] The fixed capital does not depreciate and is available in a fixed aggregate amount, given by $\bar{K}$, while both types of energy capital depreciate at rate $1 - \lambda$ per period. The government can levy a tax on the output of an entrepreneur who uses high carbon energy capital, i.e. a carbon tax, and provide a green subsidy to entrepreneurs using zero carbon energy capital. The net position of the government is either financed through a lump-sum tax or distributed through a lump-sum transfer on a per capita basis i.e. the government runs a balanced budget.

At the end of each time period $t-1$, there is a competitive asset market and a competitive one-period credit market. In the former, one unit of the fixed capital is exchanged for q_{t-1} units of the commodity; in the second, one unit of the commodity at date $t-1$ is exchanged for R_{t-1} units of the commodity at date t. The commodity is assumed to be the numeraire, so that its price is normalised to unity. Then q_t represents the price per unit of fixed capital, and R_t is the gross interest rate. At the start of a new period t, markets are closed (although there is a window of opportunity for debt renegotiation): stocks of fixed capital, energy capital, and debt holdings are state variables. Production then takes place over period t.

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[11]Exogenous ex-ante heterogeneity on the subjective discount factors not only allows us to keep the model tractable but also ensures the model simultaneously has borrowers and lenders. This is in line with many dynamic (stochastic) general equilibrium models of financial friction e.g. Kiyotaki and Moore (1997), Iacoviello (2005), Iacoviello and Neri (2010), Devereux and Yetman (2010), Pariès et al. (2011), and Liu et al. (2013).

[12]For example, the energy capital includes coal mines, power stations, wind farms, and a balanced electricity grid, while the fixed capital includes the factory which uses the electricity, but does not directly care how this electricity has been produced (though such a factory does care indirectly about the production techniques used in the creation of its electricity inputs, since the productivity of this production will affect electricity prices and thus affect the factory's cost base).

3.1 Entrepreneurs

An entrepreneur produces a quantity of the commodity, y, with a one-period Leontief production function: fixed capital, k, is combined with energy capital, z, in $1:1$ proportion.

This period's decisions affect next period's production. The entrepreneur can choose between two technologies. Choosing the first, k_{t-1} units of fixed capital are combined with z_{t-1}^H units of the high carbon energy capital, producing y_t units of the commodity. However, this choice implies that the after tax output available to the entrepreneur will be reduced by any proportional carbon tax implemented, $\tilde{\tau}_t$:

$$
\begin{aligned}
y_t = F_H(k_{t-1}, z_{t-1}^H) &= \left(a^H + c\right) \times \min\left(k_{t-1}, z_{t-1}^H\right) \\
(1 - \tilde{\tau}_t)\, y_t &= \left(a^H - \tau_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^H\right).
\end{aligned}
\tag{2}
$$

Choosing the second, the entrepreneur combines k_{t-1} units of fixed capital with z_{t-1}^L units of the zero carbon energy capital and benefits from a proportional subsidy, $\tilde{\varsigma}_t$. The output available to the entrepreneur, however, will be increased only by a fraction $\delta \in [0,1]$ of the subsidy implemented, where δ is a structural parameter representing the effectiveness of the subsidy:[13],[14]

$$
\begin{aligned}
y_t = F_L\left(k_{t-1}, z_{t-1}^L\right) &= \left(a^L - (1-\delta)\varsigma_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^L\right) \\
(1 + \tilde{\varsigma}_t)\, y_t &= \left(a^L + \delta\varsigma_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^L\right).
\end{aligned}
\tag{3}
$$

No matter the technology used, ck_{t-1} units of the y_t units of output produced at date t are not tradable and must be consumed by the entrepreneurs (who therefore must pay any carbon tax levied out of tradable output).[15]

In line with Acemoglu et al. (2012), we assume that zero carbon energy capital is

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[13]Being credit constrained, the entrepreneurial sector will use a sub-optimally low quantity of capital in equilibrium: a subsidy would therefore move the economy towards first best by mitigating against the credit frictions. We want to look at policies that mitigate the problem of the Carbon Bubble, but we do not want to eliminate credit constraints in steady state. Thus we introduce this productivity destroying distortion, which will be calibrated so that the policymaker does not want to use the subsidy in steady state. If there are any benefits (measured using the policymaker's objective function) in applying a subsidy, these must therefore be due to the Carbon Bubble issue. More details on the subsidy can be found in Appendix A.3.

[14]In the remainder of the article, we discuss only $\tau_t = \tilde{\tau}_t(a^H + c)$ and $\varsigma_t = \tilde{\varsigma}_t\left(a^L - (1-\delta)\varsigma_t + c\right)$, positive bijective transformations of the proportional tax rate and subsidy rate into units that can be compared to the productivities of the two alternative technologies.

[15]The ratio $a^i/(a^i + c)$, $i = \{H, L\}$, represents an upper bound on the entrepreneur's savings rate. Alternatively, c could represent a sort of minimum dividend that needs to be distributed to the shareholders. This non-tradable quantity of output is introduced in Kiyotaki and Moore (1997) to avoid the possibility that the entrepreneur keeps postponing consumption. Indeed, since preferences are linear, entrepreneurs would like to not consume and increase investment. While this assumption and the presence of linear preferences but different discount factors can be considered as unorthodox modelling choices, Kiyotaki and Moore (1997, Appendix) show that the same qualitative results can be obtained using an overlapping generations model with standard concave preferences and conventional saving/consumption decisions.

intrinsically less productive than high carbon energy capital,

Assumption B *(Productivity advantage of the carbon sector)* $\quad a^H > a^L$.

The commodity can be consumed or invested. For that portion of their output which is invested, the entrepreneur converts ϕ units of the commodity into one unit of energy capital: ϕ is the output cost of investing in one unit of energy capital.[16]

As in Kiyotaki and Moore (1997), we impose an upper limit on the obligations of the entrepreneurs. Suppose that, if the entrepreneur repudiates their contract, the lender can only repossess the fixed asset.[17] Entrepreneurs are therefore subject to the following borrowing constraint:[18]

$$b_t \leq \frac{q_{t+1}k_t}{R_{t+1}}. \tag{4}$$

Consider an entrepreneur who holds k_{t-1} units of fixed capital, $z_{t-1} = k_{t-1}$ units of energy capital, and has gross debt b_{t-1} at the end of period $t-1$. At date t they receive net income from production of $a_t^i k_{t-1}$ units of tradable output (depending on the technology used), they incur a new loan b_t and acquire more fixed capital, $k_t - k_{t-1}$. Having experienced depreciation and having increased their fixed capital holdings, the entrepreneur will have to convert part of the tradable output to energy capital. In general, they will have to invest $\phi(k_t - \lambda k_{t-1})$ in order to have enough energy capital to cover depreciation and new fixed capital acquisition. They then repay the accumulated debt, $R_t b_{t-1}$, and choose how much to consume in excess of the amount of non-tradable output, $(x_t - ck_{t-1})$. In addition, they receive a per capita transfer from the government or pay the per capita tax, g_t, depending on the net position of the government. Thus, the entrepreneur's flow-of-funds constraint,[19] as at the end of period t, is given by

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) + \tau_t k_{t-1} = a^H k_{t-1} + b_t + g_t \tag{5a}$$

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) = a^L k_{t-1} + \delta\varsigma k_{t-1} + b_t + g_t. \tag{5b}$$

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[16]Note that, instead of writing the model in terms of differing productivities of high and zero carbon technologies and a single cost of investment, $\{a^H, a^L, \phi\}$, we reach qualitatively the same results by writing the model in terms of a single productivity and differing output costs of investing in energy capital, $\{a, \phi^H, \phi^L\}$, as in van der Zwaan et al. (2002). Since results are qualitatively similar, we do not present this alternative model here.

[17]Two critical assumptions in Kiyotaki and Moore (1997) are imposed here. Firstly, the entrepreneur cannot pre-commit to work and can freely decide to withdraw their labour: Hart and Moore (1994) refer to this option as "inalienability of human capital". Secondly, the entrepreneur's technology and energy capital are idiosyncratic. Thus, if they decide to withdraw their labour between dates t and $t+1$, there would be only the fixed capital k_t and no output at $t+1$.

[18]Rather than the amount of collateral depending upon the relative bargaining power of the agents, Hart and Moore (1994) suggest that the lender may be able to require the full value of their counterpart's assets as collateral.

[19]This can be interpreted as imposing that profits of the entrepreneurs must be zero. Positive profits would be invested.

The first equation refers to an entrepreneur who uses the high carbon energy capital, while the second relates to the use of the zero carbon energy capital.

Each period only a fraction $\pi \in (0, 1)$ of entrepreneurs have an investment opportunity.[20] Thus, with probability $1 - \pi$, the entrepreneur cannot invest and must downsize its scale of operation, since the depreciation of their energy capital implies $z_t^i = \lambda z_{t-1}^i$. This probabilistic investment assumption,[21] when combined with Leontief production, means that with probability $1 - \pi$ the entrepreneur also faces the constraint

$$k_t \leq \lambda k_{t-1}. \tag{6}$$

3.2 Representative Saver

Savers are willing to lend commodities to entrepreneurs in return for debt contracts, and they also produce commodities by means of a decreasing return to scale technology which uses the fixed capital as an input and takes one period, according to

$$y_t' = \Psi\left(k_{t-1}'\right) \qquad \text{with } \Psi' > 0, \quad \Psi'' < 0. \tag{7}$$

Savers are never credit constrained because they can trade all their output and no particular skill is required in their production process. Savers solve the relevant maximization problem in (1), subject to their budget constraint,

$$q_t\left(k_t' - k_{t-1}'\right) + R_t b_{t-1}' + x_t' = \Psi\left(k_{t-1}'\right) + b_t' + g_t. \tag{8}$$

3.3 Competitive Equilibrium

An equilibrium consists of a sequence of prices $\{q_t, R_t, \tau_t, \varsigma_t\}$, allocations for the entrepreneur $\{x_t, k_t, z_t, b_t\}$ and the saver $\{x_t', k_t', b_t'\}$ such that, taking the prices as given, each entrepreneur solves the relevant maximization problem in (1) subject to the technological constraints in either (2) or (3) and, if appropriate, (6), the borrowing constraint in (4) and the flow-of-funds constraint in (5a) or (5b); each saver maximizes the relevant part of (1) subject to the technological constraint in (7) and the budget constraint in (8); the government always runs a balanced budget; and the goods, asset, and credit markets clear.

Using $\gamma_t \in [0, 1]$ to indicate the share of aggregate entrepreneurs' fixed capital holdings which are combined with high carbon energy capital at time t, let I_t^H, I_t^L, B_t, $mb_t' \equiv B_t'$, K_t, $mk_t' \equiv K_t'$, X_t, $mx_t' \equiv X_t'$, Y_t, $my_t' \equiv Y_t'$, $\tau_t \gamma_t K_t \equiv T_t$, $\varsigma_t(1 - \gamma_t)K_t \equiv P_t$, $(1+m)g_t \equiv G_t$

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[20]The arrival rate of the investment opportunity is independent through time and across agents.

[21]This assumption is introduced by Kiyotaki and Moore (1997, page 229 - 230) to capture the idea that "investment in fixed assets is typically occasional and lumpy".

be aggregate investment flows, borrowing, fixed capital holdings, consumption, output, carbon tax, green subsidy, and aggregate lump-sum transfer $(+ve)$ or tax $(-ve)$. Then the government budget constraint and the market clearing conditions for assets, credit, and goods are, respectively,

$$T_t - P_t = G_t \tag{9a}$$

$$K_t + K'_t = \bar{K} \tag{9b}$$

$$B_t + B'_t = 0 \tag{9c}$$

$$I_t^H + I_t^L + X_t + X'_t + G_t - T_t + P_t = Y_t + Y'_t. \tag{9d}$$

Note that, given assumption A, the impatient entrepreneurs borrow from the patient savers in equilibrium. Moreover, given that savers are risk neutral and there is no uncertainty, the rate of interest, R_t, is constant and determined by the patient saver's rate of time preference i.e. $R_t = 1/\beta' \equiv R$.

To characterize equilibrium, we start with the savers. Since they are not credit constrained, their fixed capital holdings are such that they are indifferent between buying and selling this capital. This is the case if the rate of return from buying fixed capital is equal to the rate of return of selling,

$$\frac{\Psi'(k'_t)}{u_t} = R, \tag{10}$$

where the "user cost of capital",

$$u_t \equiv q_t - \frac{q_{t+1}}{R}, \tag{11}$$

is both the down payment required to purchase one unit of the fixed capital, and the opportunity cost of holding fixed capital for savers.

Using (9b) together with (10), the following asset market equilibrium condition is obtained:

$$u_t = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K_t}{m}\right) \equiv u(K_t). \tag{12}$$

The ratio $(\bar{K} - K_t)/m$ is the representative saver's fixed capital holdings. An increase in the saver' demand for fixed capital causes the middle term of Equation (12) to decrease, given the assumption of decreasing marginal productivity in (7). Equivalently, an increase in entrepreneurs' demand for fixed capital needs a decrease in savers' demand for the market to clear: this is achieved by a rise in the user cost, u_t. Thus, $u' > 0$.

Entrepreneurs who can invest at date t will prefer borrowing up to the limit and investing, rather than saving or consuming, hence limiting their consumption to the current non-tradable output $(x_t = ck_{t-1})$: for them, the credit constraint in (4) is binding. Conversely, an entrepreneur who cannot invest at t, given that they will not want to waste

their remaining stock of energy capital, will adjust their levels of debt and fixed capital such that Equation (6) will hold with equality.[22] For ease of exposition, let a_t represents the net (i.e. after tax and/or subsidy) productivity of the entrepreneurial technology. At the aggregate level, the behaviour of the entrepreneurs implies that entrepreneurs' aggregate fixed capital holdings and borrowing evolve according to[23]

$$K_t = \frac{\pi}{q_t + \phi - \frac{q_{t+1}}{R}} \left[(q_t + \phi\lambda + a_t) K_{t-1} - RB_{t-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t-1} \right] + \\ + (1-\pi)\lambda K_{t-1} \tag{13}$$

$$B_t = q_t (K_t - K_{t-1}) + \phi (K_t - \lambda K_{t-1}) + RB_{t-1} - a_t K_{t-1} - \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t-1}. \tag{14}$$

One interesting implication of Equation (13) is that demand for fixed capital from the entrepreneurial sector increases given an increase, in equal proportion, of both today's and tomorrow's fixed capital prices. A rise in the current price increases entrepreneur's net worth and a rise in the future prices strengthens the value of the collateral (thus allowing the entrepreneurs to borrow more) and this more than compensates for the price-increase induced reduction in demand.

We are now able to characterize, for given K_{t-1} and B_{t-1}, the perfect foresight competitive equilibrium from date t onward as the paths of aggregate entrepreneurs' fixed capital holdings and debts, and fixed capital prices, $\{K_{t+s}, B_{t+s}, q_{t+s}\}_{s=0}^{\infty}$, such that Equations (12), (13), and (14) are satisfied for all t.[24]

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[22]We refer the interested reader to Kiyotaki and Moore (1997, footnote 22) for the full proof of the claims on the behaviour of investing and non-investing entrepreneurs. They show that by Assumption A investment strictly dominates saving while Assumption G in Appendix A.2 ensures that an entrepreneur prefers to invest (if they can) or save (if they cannot invest) rather than consuming the marginal unit of tradable output.

[23]Note that $\gamma \in (0,1)$ only if the entrepreneur is indifferent between the two technologies.

[24]Note that Equations (13) and (14) are very similar to the equations of motion derived in Kiyotaki and Moore (1997). Our putative addition of a debt renegotiation mechanism, based on Cordoba and Ripoll (2004), does not affect the equations of motion under perfect foresight, since adverse shocks and hence debt renegotiation do not occur under perfect foresight. We return to the debt renegotiation mechanism in Appendix A.6 and A.7 where its incorporation will allow the economy to recover from very large exogenous shocks imposed at time t, through an instantaneous adjustment at time t^+, with the economy thereafter following the perfect foresight, risk-free path.

3.4   Steady State

Given constant a,[25] there exists a continuum of steady state equilibria, $(q^\star, K^\star, B^\star)$, with associated $u^\star$, indexed by $\gamma \in [0, 1]$, where[26]

$$\left(\frac{B}{K}\right)^\star = \frac{\phi\lambda - \phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}}{R - 1} \tag{15a}$$

$$u^\star = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K^\star}{m}\right) = \frac{R - 1}{R}q^\star \tag{15b}$$

$$u^\star = \frac{\pi\left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right] - \phi(1 - \lambda)(1 - R + R\pi)}{\pi\lambda + (1 - \lambda)(1 - R + R\pi)}. \tag{15c}$$

Once the government has effectively set a private "productivity target" for the entrepreneurial sector, a, through τ and ς, Equation (15a) says that in steady state the entrepreneurs use the amount of tradable output, $aK^\star$, together with (net of) the transfer (tax) from the government, $K^\star[\gamma\tau - (1 - \gamma)\varsigma]/(1 + m)$, to repay the interest on the debt, $(R - 1)B^\star$, and to replace the amount of energy capital that has depreciated in the period, $\phi(1 - \lambda)K^\star$. As a result, the scale of operation of the entrepreneurial sector neither increases nor decreases.

Figure 1 provides a visual representation. The horizontal axis shows demand for fixed capital from the entrepreneurs from left to right and from the savers from right to left. Since the market for fixed capital clears, the sum of the two demands is equal to $\bar{K}$. The vertical axis consists of the marginal product of fixed capital, which is constant at $a^i + c$, $i = \{H, L\}$, for entrepreneurs but decreasing with fixed capital usage for savers.

Were the debt enforcement problem absent, and absent any government policy, the economy would be able to reach the first best allocation, E_{FB}, in which the entirety of the aggregate entrepreneurs' fixed capital holdings are used with high carbon energy capital. In this scenario, entrepreneurs are not constrained in the amount they can borrow. Thus, the marginal products of the two sectors are identical. In contrast, in the constrained economy too much of fixed capital is left in the hands of the savers and entrepreneurs have a higher marginal product than savers.

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[25]If $a \equiv a^H - \tau = a^L + \delta\varsigma$, entrepreneurs are indifferent over which technology they use. If this is not the case, entrepreneurs would use either 100% low ($\gamma = 0$), or high ($\gamma = 1$) carbon energy capital.

[26]See Appendix A.8, for the analysis of the stability of the system. See Appendix A.2 for further assumptions used in obtaining the model's equilibrium. Given Assumptions E and F, $(B/K)^\star$ and $u^\star$ are positive. Note that, for any (γ, a), this interior steady state is unique. However, as in Kiyotaki and Moore (1997), there are two other steady states: (1) fixed capital price below savers' marginal product when using $\bar{K}$, so that all fixed capital lies in the hands of the savers; (2) fixed capital price and debt holdings both tending to infinity, all the fixed capital is in the hands of the entrepreneurs, and the price growth is such that next period collateral value is always sufficient to take on the required debt levels this period. We use debt renegotiation and our calibration strategy to ensure that the economy can converge back to the interior steady state.
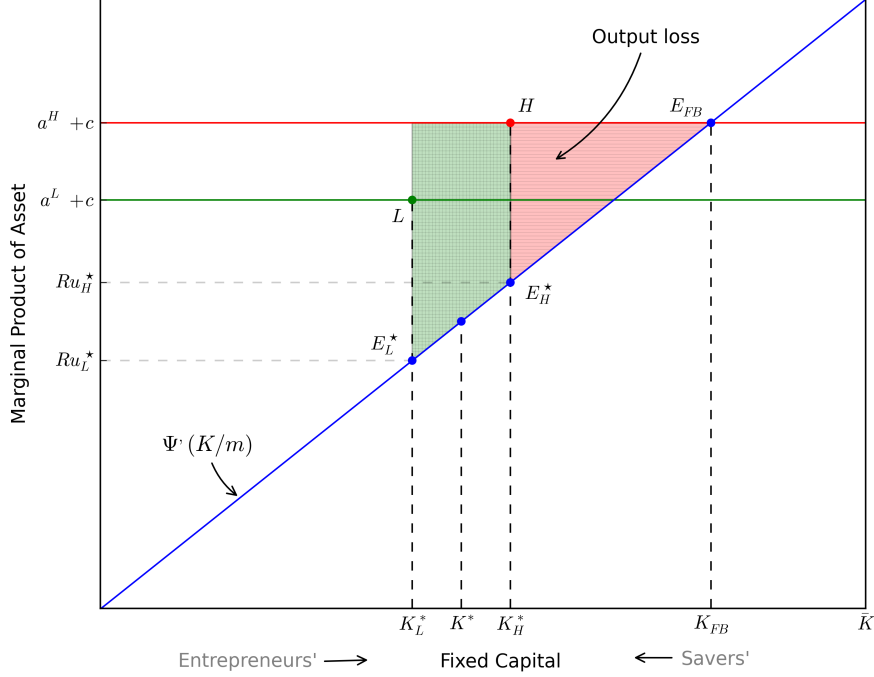
Figure 1: Comparison of steady states

Consider two particular equilibria. In a world which only uses the high carbon energy capital (i.e. $\gamma = 1$), and with $a = a^H$ (i.e. no carbon tax), the equilibrium is given by $E_H^\star$, where the aggregate entrepreneurs' fixed capital holding is $K_H^\star$. Conversely, the fully decarbonised equilibrium (i.e. $\gamma = 0$) with $a = a^L$ (i.e. no green subsidy) is $E_L^\star$, with corresponding $K_L^\star$. It easy to show that the former equilibrium provides a larger share of fixed capital to the entrepreneurs compared to the latter, $K_H^\star > K_L^\star$. As a consequence, output, investment, borrowing and consumption are higher. Intuitively, having the government set a lower private productivity target for the entrepreneurial sector, this not only earns less revenue with respect to a higher private productivity target, but also has lower net worth. Thus, in general, entrepreneurs can borrow, invest, and produce less. To clear the market, the demand for fixed capital by the savers must be higher in the decarbonised world, which requires a lower user cost. But a lower user cost is associated with a lower fixed capital price and thus with a lower net worth of the constrained sector, which translates into less collateral. Less collateral means lower investment and production, and so on in a vicious circle.

The amount of fixed capital used by the entrepreneurs, $K^\star(a, \gamma)$, for any $a \in [a^L, a^H]$, $\gamma \in [0, 1]$, is within the interval $[K_L^\star, K_H^\star]$ and is a monotonically increasing function of the share of fixed capital used in conjunction with high carbon energy capital, $\gamma \in [0, 1]$.[27]

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[27]The same argument does not need to hold for the private productivity of the entrepreneurs' technology, $a \in [a^L, a^H]$, because of the distortionary impact of the subsidy.

Increasing γ results in a higher net worth of the entrepreneurial sector, through an increase in the carbon tax revenues and the per capita transfer made by the government. As a consequence, the representative entrepreneur can afford higher fixed capital holdings with positive repercussions on investment and output.[28]

4 Calibration Strategy

In this section, we develop our calibration strategy.[29] We match energy data, and broadly match the experience of the 2008-09 Financial Crisis. We chose the calibration delivering the shortest cycle length, not requiring debt renegotiation, and with strictly positive debt at all dates.[30]

4.1 Savers Production Function, Definition of Welfare, and Time

We start by following Kiyotaki and Moore (1997) and impose the following linear structure for the user cost,

Assumption C *(Linear user cost structure)* $\quad u(K) = \frac{1}{R}\Psi'\left(\frac{\bar{K}-K}{m}\right) \equiv K - \nu$.

Integrating the savers' production function up, means that we have some constant production flow, $const/m$, independent of the level of fixed capital used by the savers, that must be calibrated in order to look at aggregate production. For definitional convenience, we assume that this constant is such that (decarbonised) steady state consumption flow is the same for both individual savers and entrepreneurs. By using Assumption C, we

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[28]Appendix A.5 presents an interesting result: given the presence of credit frictions, we may see higher absolute investment levels in zero carbon energy if there is also some investment in high carbon energy capital. This is because the higher *gamma*, the higher the net worth of the entrepreneurs: those investing in zero carbon can borrow, invest, and produce more. However, this result does not hold given the calibration we use (see Section 4), under which the highest steady state level of zero carbon investment is achieved with $\gamma = 0$.

[29]Kiyotaki and Moore's (1997, Chapter III) parametrisation is not suitable for our exercise because, in the absence of debt renegotiation, the economy can only return to steady state for extremely small negative shocks. Moreover, debt renegotiation makes the world increasingly "classical" in that it eliminates the debt overhang and the persistent negative effects from the shock; all the adjustment happens at $t = 0$, and after this the economy is able to ramp up investment and return to steady state, similarly to a neoclassical growth model exhibiting conditional convergence. See Appendix A.6.

[30]Qualitatively there is very little dependence upon the calibration: our general conclusions about policy effectiveness are robust to the particulars of this calibration. The calibration satisfies all the assumptions made on parameter restrictions from Appendix A.2. More details can be found in Appendix A.6.

obtain aggregate output in the economy as

$$Y_t = \left[\gamma_t a^H + (1 - \gamma_t)\left(a^L - (1 - \delta)\varsigma_t\right) + c\right] K_{t-1} +$$

$$+ R\left(\bar{K} - \nu\right)\left(\bar{K} - K_{t-1}\right) - \frac{R\left(\bar{K} - K_{t-1}\right)^2}{2} - const$$

and average per period consumption for entrepreneurs and per period consumption for savers as, respectively,[31]

$$E[x_t] = cK_{t-1}$$

$$x_t' = \frac{R\left[(\bar{K} - \nu)(\bar{K} - K_{t-1}) - \frac{(\bar{K} - K_{t-1})^2}{2}\right]}{m} - \frac{const}{m} + g_t + \frac{RB_{t-1} - B_t}{m} + \frac{q_t\left(K_t - K_{t-1}\right)}{m}.$$

The utilitarian social welfare function maximised by the policymaker at $t = 0$ is assumed to be the discounted present value of future consumption. We assume that savers are infinitesimally more patient than the entrepreneurs and thus, for any practical calculation, their discount factors are the same. Therefore, policy is chosen by the policymaker to maximise the present discounted value of all future "Net National Income" flows in the model,

$$W_t = \sum_{s=t+1}^{\infty} \left[\beta^{s-t} E\left[x_s\right] + (\beta')^{s-t} m x_s'\right] \approx \sum_{s=t+1}^{\infty} R^{t-s} \left[E[x_s] + m x_s'\right].$$

We follow Kiyotaki and Moore (1997), and set the depreciation rate of energy capital, $\lambda = 0.975$, and the interest rate, $R = 1.01$, so that time periods can be interpreted as quarter years. These correspond to a depreciation rate of 10% per annum for energy capital, and to an annual interest rate on debt of 4%. Finally, we normalise productivity, $a^L = 1$.

4.2   The Financial Crisis

The financial crisis began with the realisation that the fundamental value of subprime mortgages (and the CDOs into which they were bundled) was much lower than had previously been recognised. Hellwig (2009) estimated that the total value of subprime mortgages outstanding was \$1.1tn in the second quarter of 2008. Dietz et al. (2016) use data from the Financial Stability Board to claim that the total value of global non-bank financial assets in 2013 was \$143.3tn. Since both figures are approximation, we use 1/150

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[31]Entrepreneurs are not all identical as their personal holdings of energy and fixed capital will depend upon their idiosyncratic histories, but in expectation (given a unit mass of entrepreneurs) their individual fixed capital holdings are represented by the aggregate entrepreneurial fixed capital holding. Conversely, if we imagine that all savers save through some financial intermediary institution, then they are all homogeneous.

as a rounded figure for the loss in entrepreneurial wealth. We imagine a scenario in which there is a one-time initial unexpected shock to the wealth of the entrepreneurs such that their wealth at the end of the first period is reduced by $\Delta K^\star$, where Δ is given by

$$\frac{\Delta K^\star}{\phi K^\star + q^\star \bar{K}} = \frac{1}{150}.$$

We calibrate the parameters m, ϕ, π, and ν to the impact of the financial crisis on output and upon asset values. With regards to the former, data from FRED[32] suggests that annual percentage changes in "Constant GDP per capita for the World" were consistently just below 3% prior to the financial crisis, but fell to less than -3% when the crisis struck. This data suggests that the financial crisis scenario should involve a fall in output of around 6% in our steady state model, with no growth in per capita incomes. With regards to the latter, a back of the envelope calculation[33] suggests that a well diversified investor experienced a fall in asset values of around 20-25%.

4.3 The Energy Sector

According to Newell et al. (2016), and to energy mix figures from EIA (2016, Table 1.2), fossil fuels represent around 80% of energy generation. This gives us a calibrated value of $\gamma = 0.8$. The EIA (2015, Table 1) provides figures on the "total system levelized costs of electricity", which we apply to their energy mix figures to estimate that fossil fuel generation costs around 10% less per unit of energy supplied. This allows us to set $a^H = 1.10$.

Both fossil fuels and alternative energy generating capacity exist in the data, and we can only replicate this in the model if their net private productivities, after taxes and subsidies, have been equalised. We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy rate from the planner's perspective in the initial steady

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[32]Accessible at https://research.stlouisfed.org/fred2/series/NYGDPPCAPKDWLD.

[33]The loss of $1/150$ should represent a relatively mild adverse event to a well diversified investor. Instead we saw the S&P500 decline by 40% (percent change from 1 year ago, viewed in quarterly timesteps, from https://research.stlouisfed.org/fred2/series/SP500), and corporate bond spreads rise. For example, "Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity" rose from just over 1.5% prior to the financial crisis, to more than 5.5% at the height of the crisis (see https://research.stlouisfed.org/fred2/series/BAA10YM). On the other hand, the effective value of public assets, inferred from government bond prices, rose as interest rates fell (at least in non-Eurozone periphery countries). Assume public assets are 20% of total assets, and private assets are funded 50:50 debt and equity. Public assets grow in value by around 10% (calculated as 4% income, plus an interest rate fall of 2% on a bond portfolio with a discounted mean term of 5 years). Equity falls by 40% (with no income, since the S&P500 is a total return index), and corporate debt falls by 10% (calculated as income of 4%, plus an interest rate rise of 2% on a bond portfolio with a discounted mean term of around 5 years).

state is $\varsigma = 0$.[34] This means that net private productivity is $a = a^L$, and carbon tax $\tau = a^H - a^L$.

EIA (2016, Table 1.2) suggests that energy expenditure as a percentage of total GDP is around 7.5%. In order to interpret this within the model, we imagine that in steady state the expenditure by the entrepreneurial final goods production sector on energy intermediate goods is equal to the flow value of the energy capital value i.e. $(R-1)\phi K^\star$ and that this represents total expenditures on energy. Therefore, we have the following object as a calibration target:

$$\frac{(R-1)\phi K^\star}{(a+c+\gamma\tau)K^\star + R(\bar{K}-\nu)(\bar{K}-K^\star) - 0.5R(\bar{K}-K^\star)^2 - const} = 7.5\%.$$

5 Dynamic Simulations

Now that we have developed the analytic framework, and calibrated the model, in this section we turn to the issue of the Carbon Bubble.

Here we imagine a scenario loosely modelled upon the current state of the global economy's capital stock: efforts have been made to provide incentives to develop and deploy zero carbon energy capital, but at the global level, the stock of high carbon energy capital is not falling; global reserves of fossil fuels are more than sufficient to exceed some carbon budget; and energy capital investments that lock the economy into high carbon patterns of use are still being made. Therefore, as described in Section 4.3, in the periods prior to the start of our dynamic simulations, we consider the global economy to be in a steady state in which the private returns from investment in both high and zero carbon energy capital are equalised via the imposition of a carbon tax, but that we are in a steady state characterised by $\gamma = 0.8$. The values for the fixed capital used by, and the debt holdings of, entrepreneurs in this steady state are $K_F^\star$ and $B_F^\star$ respectively. Therefore, the steady state high carbon energy capital stock is $Z^H = \gamma K_F^\star$.

At the start of our simulation, the planner makes an announcement: future investment in high carbon energy capital is banned, and the total future use of high carbon energy capital is limited. The carbon production in period t is linear in the amount of high carbon investment energy capital, Z_t^H, used in production at t. Since we can choose units, let this amount of carbon production also equal Z_t^H for simplicity. Effectively, the

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[34]This means that the optimal subsidy from the planner's perspective in the decarbonised steady state is actually negative if we allow negative distortions because the distortion is so large. This is because this optimal $\varsigma = 0$ calibration target requires a distortion large enough to offset the benefits of a higher private productivity applied to the 80% of output that is produced using the undistorted high carbon energy capital. However, we do not allow negative distortions and so the optimal subsidy from the planner's perspective in the decarbonised steady state is again zero. See Appendix A.3 for more information on the subsidy.

policymaker announces a carbon budget, $\bar{S}$, which satisfies

$$\bar{S} = (1 - \kappa) \times \sum_{t=0}^{\infty} \lambda^t Z^H = (1 - \kappa) \times \frac{\gamma K_F^{\star}}{1 - \lambda},$$

where κ is the share of high carbon good that must be left in the ground.

As discussed in Section 1, whereas a substantial amount of companies' current reserves cannot be burnt if we are to remain within the 2°C limit for climate change (with some probability),[35] the financial importance of some of these write-offs can be questioned. Instead of guessing a financial value that will have to be written-off, we assume this to be of the same magnitude as the shock that precipitated the financial crisis.[36] This means that our Carbon Bubble scenario is precipitated by writing-off a fraction κ of the current stock of carbon emitting energy capital,[37] such that

$$\kappa \left(\frac{\gamma \phi K_F^{\star}}{\phi K_F^{\star} + q_F^{\star} \bar{K}} \right) = \frac{1}{150}.$$

This gives $\kappa \approx 12\%$.

Entrepreneurs must decide the share of the remaining high carbon energy capital they will use in this period, and the resulting rate at which they will retire these goods. This choice is a function of prices, q_1, and determines the values of the state variables for the next period: K_1 and B_1. Entrepreneurs make this choice optimally, and it turns out that they always choose to produce at the maximum rate that they are able to, and delay the retirement of the remaining high carbon asset to the last period available.[38]

Figure 2 gives an overview of the responses of the economy to implementing $\bar{S}$ at

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[35]The Carbon Tracker Initiative (2013, page 15) says "an estimated 65-80% of listed companies' current reserves cannot be burnt unmitigated", while IEA (2012, page 3) says "No more than one-third of proven reserves of fossil fuels can be consumed".

[36]In previous versions of this article, we presented the simulations with write-off ranging from 20% to 80% of the initial steady state amount of high carbon goods, reflecting various combinations of the recommendations from IEA (2012) and Carbon Tracker Initiative (2013), among others. Obviously, the quantitative results depend on the size of the shock, but the qualitative results are unchanged: the policies that we consider in the next subsections are always associated with a welfare increase (the bigger the shock, the bigger the potential welfare increase). These alternative simulations are available on request.

[37]Note that this is a conservative interpretation of the impact of the Carbon Bubble scenario in that we are reducing the stock of high carbon energy capital that can be used, but this capital is not collateralisable. Entrepreneurial net worth is affected, but their stock of collateralisable assets is not affected. The impact of the Carbon Bubble in this scenario comes because the reduction in entrepreneurial net worth reduces entrepreneurial demand for credit which feeds through into reduced demand for fixed capital, lower prices for fixed capital, and hence fire-sale dynamics.

[38]An interested reader can find the technical details in Appendix A.7.

$t = 0.$[39] It shows movement in $K/K^\star$, $Y/Y^\star$, $B/B^\star$, $q/q^\star$, and $I/I^\star$ i.e. the ratios of entrepreneurs' fixed capital, total output, investors' debt, price of fixed capital, and aggregate investment flow, to their respective decarbonised steady state values.
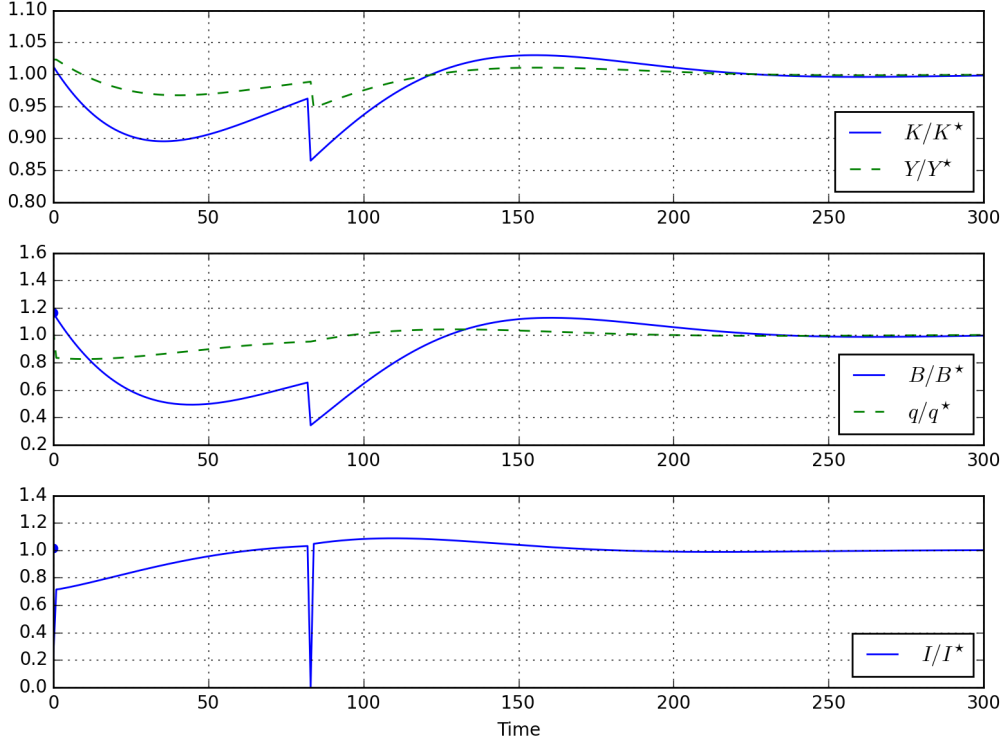

Figure 2: The burst of the bubble

Notes. In the lower panel, the line represents investment in zero carbon energy capital weighted over its decarbonised steady state level. However, the dot represents the ratio of the steady state level of investment at $t = 0$, which consists of both low and high carbon energy capital, to the decarbonised steady state level, which by definition is composed by investment in zero carbon energy capital alone.

As soon as the carbon budget is announced, the price of fixed capital collapses by approximately 20%. The price fall has damaged the entrepreneurs' balance sheets such that they choose to sell fixed capital back to the savers and repay debt. The fall in asset values precipitates forced sales to ensure borrowing and collateral requirements are aligned, but this forced sale causes prices to fall again which causes further forced sales, and further price falls, and so on i.e. we see fire-sale dynamics. The process stops when fixed capital becomes so unproductive in the hands of the savers that the entrepreneurs can once again afford the lowered price, and the economy recovers towards the new steady

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[39]The dynamics of the model are solved for using numerical simulations of the forward shooting method. Details of the algorithm are given in Appendix A.9 but the rough approach is to guess the discontinuous change in the fixed capital price following the shock and iterate the economy forward through time to see if it converges back to steady state. If the price eventually explodes (tends to zero), the initial guess is revised downward (upward). This "guess and check" procedure is repeated until the fixed capital price is within some tolerance level of its steady state value at the end of the projection.

state.

In the dynamics associated with the announcement of the Carbon Bubble, the entrepreneurial sector deleverages, reducing both assets and debt, until around period 40 (10 years after the announcement if we interpret periods as quarters). At this point debt levels and the holdings of fixed capital in the hands of the entrepreneurs are around 49% and 88% of steady state levels. Since a large share of fixed capital is employed in the low productivity sector, output collapses. Output bottoms out at almost 6% below the previous steady state value, and at more than 3% below the new steady state value. Investment levels fall markedly, by around 30%, even though the economy is in short supply of energy capital.

After approximately 80 periods, the economy reaches its carbon budget: the remaining high carbon assets must be retired, and thus entrepreneurs return fixed capital and debt to the savers, but prices do not fall because this time the shock is "expected". The new decarbonised economy then takes approximately 40 periods to fully recover and start stabilising around the new decarbonised steady state.

In the next subsections, we consider four possible additional actions for the planner that mitigate some of the welfare loss associated with writing-off the high carbon energy capital.

5.1 Tax Funded Transfer of Investors' Debt

The entrepreneurial sector is credit constrained, and following the imposition of climate policy, it is burdened with excessive debt relative to its assets. Perhaps the planner can achieve a better outcome if the burden of this debt is shifted to an economic actor who is not credit constrained.

We suppose that the planner first announces the carbon budget $\bar{S}$, and than takes over some share $\omega \in [0, 1]$ of the entrepreneurs' debt, funding the debt repayments by raising a constant per capita tax, τ^G, over $T = 100$ periods.[40] The social planner chooses the value of ω to maximise our measure of social welfare. Figure 3 gives an overview of the responses of the economy to implementing the optimal policy, $\omega = 35\%$, at $t = 0$.

Following the policymaker's actions, the price of fixed capital increases by around 12%. As a consequence of the debt transfer, entrepreneurs' borrowing starts at 65% of the steady state value, and slightly increases over the first period as the entrepreneurs use their cash flow to balance their fixed capital holdings with their energy capital stocks. This involves slightly increasing their fixed capital holdings and increasing investment levels

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[40]The choice of $T = 100$ periods is relatively arbitrary. A less arbitrary choice would have been the issue of perpetuities, but this would have changed the steady state, which is problematic since we are running a numerical rather than analytic analysis. This length was chosen because 25 years is a common term for new issues of government debt.
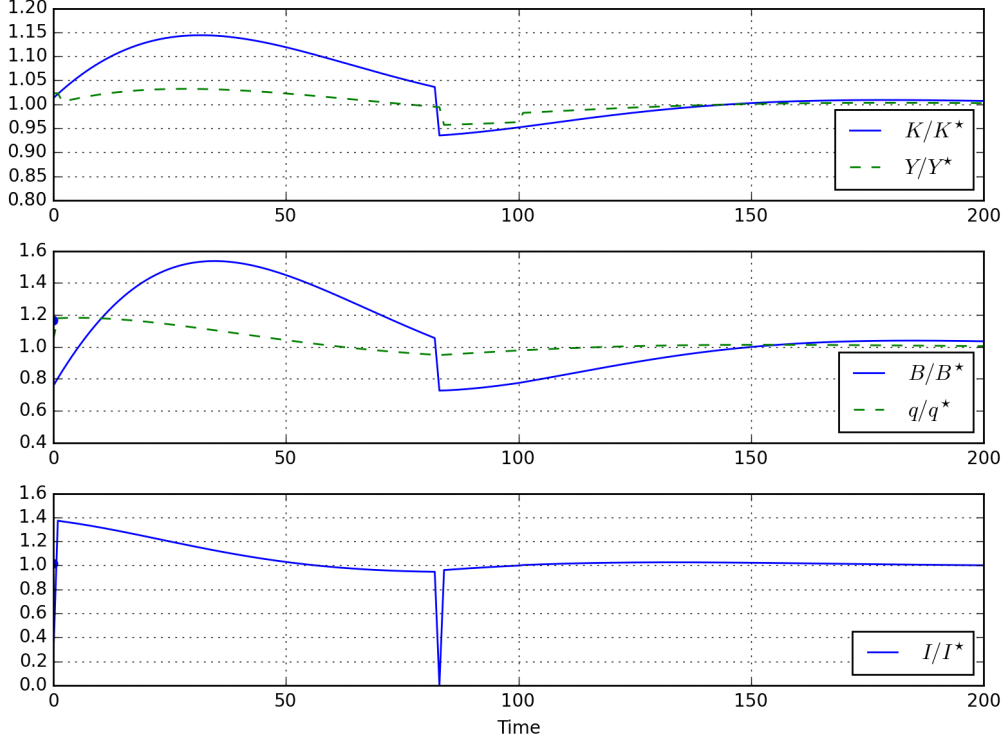
Figure 3: Transferring entrepreneurs' debt

Notes. In the middle panel, the dot represents the ratio of the steady state value of debt before the write-off of high carbon energy capital to the decarbonised steady state value, while the line starts at the post-transfer value.

to 1.35 times the steady state level. The entrepreneurs start to take on debt, to operate with more of the fixed asset than in steady state, and to build up stocks of zero carbon energy capital to a level above their steady state value. As before, around period 80, entrepreneurs retire the remaining high carbon asset. After that, the economy is heading towards a "steady state" with taxes, which is characterised by lower output, entrepreneur fixed asset holdings, debt, asset values, and investment levels, than in the true steady state. Once the taxes and government intervention in the debt market cease at $t = 100$, the economy converges to the model's true steady state. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 7% higher than in the no-policy scenario.

The welfare gain over 200 periods induced by implementing this optimal $\omega = 35\%$ policy is $+2.5\%$.[41] Implementing this debt transfer policy does not however represent

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[41]We want to underline that any benefits from this policy is a consequence of the Carbon Bubble issue, but debt redistribution would be welfare increasing also if applied in the steady state. Thus we have introduced a deadweight loss associated with τ^G in the production function of the entrepreneurs, $y_t = (a + c - \mu(\tau^G)^2)k_{t-1}$. The deadweight loss parameter μ is calibrated in such way that it is optimal for the social planner to use zero debt redistribution in steady state: any benefits from this policy must be due to the Carbon Bubble issue.

a Pareto improvement over the Carbon Bubble with no-policy scenario: the welfare improvement is composed of $+12.3\%$ for entrepreneurs, and -2.2% for savers. Savers have limited upside from this policy, but they still pay taxes to fund it.

5.2 Subsidy

After banning new high carbon investment and announcing the carbon budget that constrains the use of existing high carbon energy capital, in this policy scenario the social planner also announces an increased level of subsidy paid to entrepreneurs to boost the private productivity of their production.[42,43] This subsidy, for simplicity, will linearly decrease back to its optimal level over 100 periods.[44] We find that with the Carbon Bubble, there is a clear optimal subsidy which boosts private productivity by, initially, around 40%. Figure 4 shows the dynamics following the carbon budget announcement, when the planner implements this optimal subsidy program.

The dynamics are very similar to the debt reallocation scenario. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 7% higher, and welfare is almost 2% higher, than in the no-policy scenario.[45] Again, this is not a Pareto improvement with respect to the Carbon Bubble with no-policy scenario. Savers are worse off (-2.4%) because of the increased tax they have to pay to fund the subsidy, whereas the entrepreneurs benefit from the subsidy $(+10.8\%)$.

5.3 Government Guarantee

In this policy scenario, we model a government guarantee which reassures lenders and relaxes credit constraints. Specifically, we imagine a guarantee that effectively multiplies an entrepreneur's collateral: for a given quantity of collateral, the entrepreneurs can borrow more. Analytically, Equation (4) is modified to $b_t \leq R^{-1}q_{t+1}k_t\left(1 + gtee_t\right)$, where $gtee_t$ is the government guarantee in t.

Immediately after the same carbon budget announcement as in the no-policy scenario, the planner announces a linearly reducing guarantee which reaches zero after 100

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[42]Alternatively the extra subsidy could be targeted to output produced with only zero carbon energy capital. Since new investment in high carbon energy capital is banned, there is no incentive problem with simply paying a subsidy, independent of the technology used, that increases the private productivity of all production in the entrepreneurial sector. The only difference between these policies is that, for a given level of subsidy, the targeted subsidy provides a lower boost to entrepreneurs' incomes (we also checked that entrepreneurs would not want to use high carbon energy capital given their lower private productivity under the targeted subsidy).

[43]A renewables subsidy raises the spectre of what Sinn (2012) calls "the Green Paradox". See footnote 9 for a discussion of why it is not a problem here.

[44]This length was chosen for comparability with the debt transfer policy.

[45]The productivity destroying distortion is such that the optimal subsidy is zero in steady state. Any benefits from this policy is thus due to the Carbon Bubble issue. See Appendix A.3.
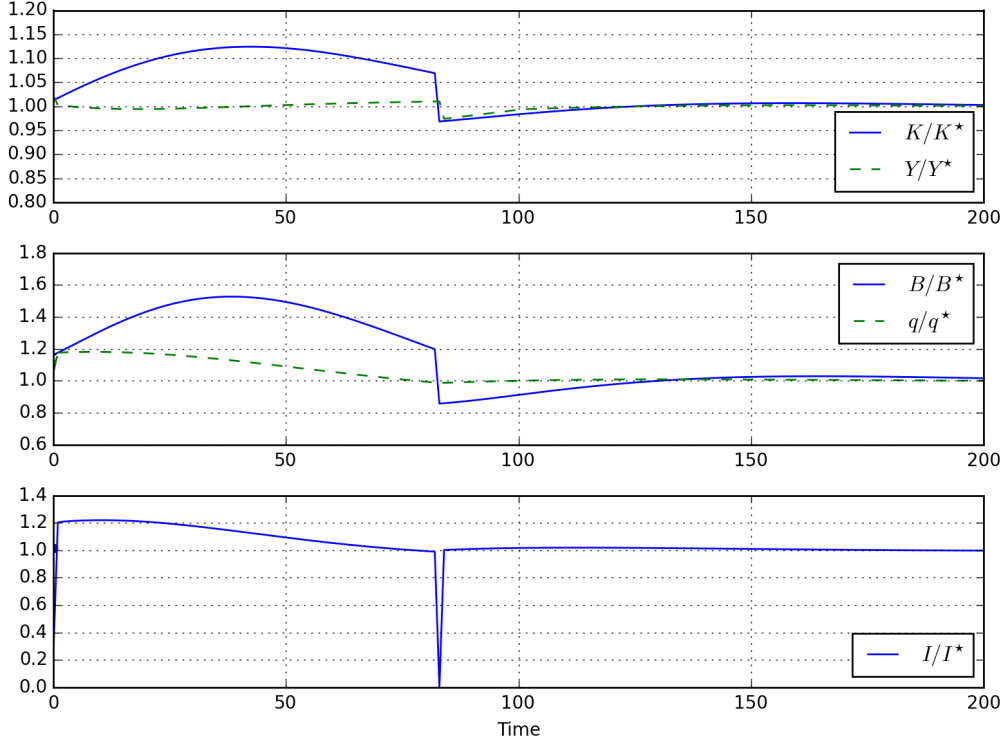
Figure 4:   Subsidising entrepreneurs

periods.[46] The steady state to which the economy is converging is therefore unchanged. Figure 5 shows the dynamics following the carbon budget announcement, when the planner implements a guarantee starting at $gtee_0 = 5\%$, which is approximately optimal given our parameters.

The price jump is slightly smaller than in the no-policy scenario, at approximately -19%. However, the government guarantee allows entrepreneurs to have access to more debt, and the entrepreneurs use the proceeds of this borrowing to maintain higher fixed capital holdings and higher investments levels than in the no-policy scenario. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 3% higher, and welfare is almost 3% higher, than in the no-policy scenario.

Contrary to the first two policies, this policy does produce a Pareto improvement with respect to the no-policy scenario: both savers $(+0.4\%)$ and entrepreneurs $(+8.2\%)$ are better off. This is because this policy relaxes credit constraints at no cost, which may of course be unrealistic.[47]

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[46]Again, 100 periods is chosen for consistency with the previous two policies. This implies $gtee_t = \max \{0, (100 - t)/100\} \times gtee_0$.

[47]There is, however, an implicit cost related to the government guarantee: an higher guarantee makes entrepreneurs rely more on external borrowings than internal funds for investment. Whereas this leads to an increase of entrepreneurs' capital holdings in the short-run, in the long-run they accumulate less net worth. For more details, see Luangaram (2003).
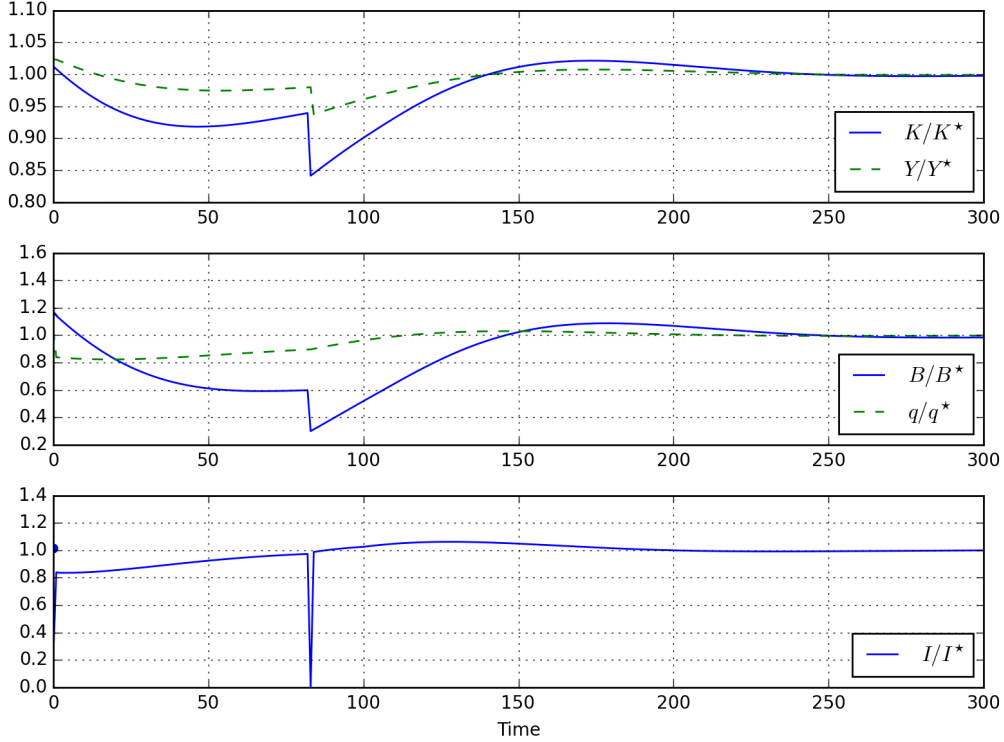
Figure 5: Providing a government guarantee

5.4   Deception

Here we consider a different possible action for the planner: she can vary the amount of current high carbon energy capital that it tells the market is allowed to be used, $\hat{S}$. The planner announces $\hat{S} \geq \bar{S}$. For $\hat{S} > \bar{S}$, the economy's actual carbon budget, $\bar{S}$, is used at some time T. When this happens, $\bar{S}$ is revealed to all agents and the entrepreneurs are compelled to leave unused their remaining high carbon energy capital: high carbon production is abruptly banned in a desperate attempt to avoid catastrophic climate change and consequential societal collapse.

In a canonical growth or business cycle model, the social planner does not have any incentive to lie: stating an $\hat{S} > \bar{S}$ would cause a welfare destroying discontinuity in consumption across the period in which $\bar{S}$ is revealed. In this model, conversely, overstating the actual carbon budget limits the fall in the price of fixed capital and thus the decrease in the value of the collateral.[48] This allows higher investment in zero carbon technology, and potentially generates enough productive capacity between period 0 and T, when $\bar{S}$ is revealed, to mean that the present value of consumption flows is higher under deception.

Figure 6 presents the simulation for the welfare-maximizing value of $\hat{S}$, consistent with

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[48]In addition to risk neutrality meaning that consumption discontinuities are not welfare destroying here.

approximately 105% of the size of the actual carbon budget being announced at $t = 0$.[49]
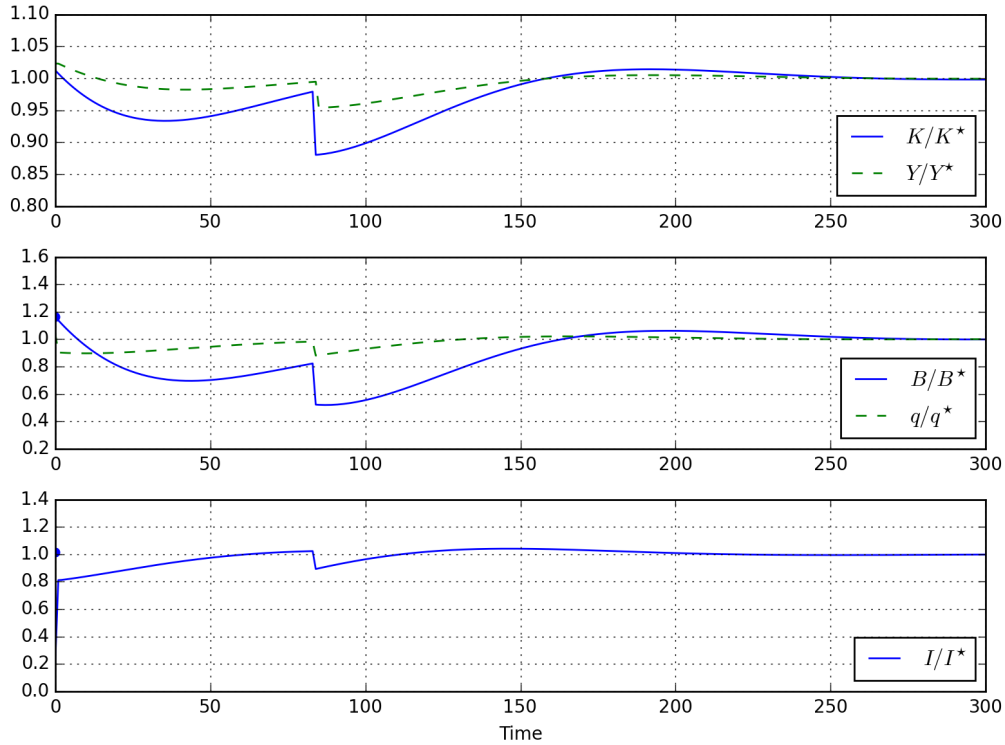

Figure 6: Dishonest social planner

The dynamics are similar to the ones following the truthful announcement, but the initial shock is smaller. The price of fixed capital falls immediately by approximately 13%. Investment does not fall as much as in the no-policy scenario, and when the actual carbon budget runs out, despite a large negative shock which induces further deleveraging, investment can soon continue. At T, around 90% of aggregate entrepreneurs' asset holdings are already dedicated to zero carbon energy capital so that, when the remaining high carbon resources must be left unused, an alternative productive capacity already exists. This limits the magnitude of the recession which results.

Over 200 periods, the cumulative investment flow in zero carbon energy capital is almost 3% higher than in the no-policy scenario; and welfare is around 3% higher (+8.6% for entrepreneurs and +0.6% for savers). This policy is a Pareto improvement on the no-policy scenario.

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[49]This implies that the announced write-off is approximately 60% of the truthful write-off.

6   Conclusions

This article analyses the effects of the credible implementation of climate change targets, which imply that a substantial proportion of fossil fuel assets become "stranded", in an economy characterized by collateral constraints. To do this, we extend the Kiyotaki and Moore's (1997) model to allow for two investment goods representing high carbon and zero carbon energy capital. This framework allows us to model, for the first time in the economics of climate change, the so-called "Carbon Bubble". This was introduced by the Carbon Tracker Initiative's (2011) report as a warning to investors: climate change mandates a policy response, and you, as an investor, should protect your portfolio from this policy response. By incorporating the Carbon Bubble issue within a macro-financial model, we can start the conversation around appropriate macroeconomic policies that should accompany the Carbon Bubble.

We take as given that climate science mandates a severe climate policy response, such that society has a limited "carbon budget" relative to its ability to emit carbon pollution. Imposing this carbon budget severely damages the balance sheets of investors, and in the presence of financial frictions, this has major macroeconomic implications. We consider the social planner's problem in facilitating the transition from a high carbon economy to the carbon-free era, by choosing policies to maximise social welfare.

We allow the public sector to take over the debt obligations of the credit constrained entrepreneurs, or to provide investment subsidies. In both cases, the improved net asset position of entrepreneurs allows them to invest more than in the no-policy case, driving the economy out of the recession faster. However, despite the improved macroeconomic performance, it is entrepreneurs that capture the major part of the benefits, and savers lose out overall due to the need to pay the taxes to fund these policies.

We then consider a government guarantee that provides a collateral multiplier to the entrepreneurs. By allowing more lending after the Carbon Bubble shock, this policy allows entrepreneurs to maintain higher fixed capital holdings and higher investment levels than is the case without policy, with positive macroeconomic consequences. Finally, we allow the planner to dishonestly announce a larger carbon budget than is really the case. Between the announcement of the policy, and the point at which the true carbon budget is revealed, economic activity is higher than it would have been given an honest announcement, and so investment in replacement zero carbon energy capital has also been greater. While these policies represent a Pareto improvement, we caution that these results may be unrealistic.

This article represents a first-order exercise, the purpose of which is to start the technical analysis of the interaction between climate policy and macroeconomic stabilisation, and to provoke further research. We showed that policies which mitigate the impact of the Carbon Bubble upon investors' balance sheets can be beneficial. The "global balance

sheet" will be used to fund the zero carbon infrastructure which must be built to replace our fossil fuel based economy, and the bursting of the Carbon Bubble could throw the economy into a deep recession, depriving green technology of investment funds when they are most needed. Thus, even if the fossil fuels assets really should be written-off to avoid disastrous global warming, it is likely to be sub-optimal to do this naïvely.

There are many areas of the model which could be made more sophisticated in order to fully quantify the impact of the Carbon Bubble and to design optimal policy. Whereas the approximate amount of companies' current reserves that cannot be burned if we are to remain within the 2°limit has been calculated before, an approximate financial value of these assets, taking into account their economic rent and use under a Carbon Bubble scenario, is still unknown. The financial accelerator mechanism could be embedded in a standard climate-economy Integrated Assessment Model rather than the reduced form that we have used where you are either within or beyond the carbon budget.[50] The supply side of the model could be made more realistic, with depreciation of, and investment in, the non-energy capital. Endogenous growth is likely an important aspect that should be considered: if learning-by-doing is important then any under-utilisation of capital induced by the Carbon Bubble could be more damaging than in the model presented here.[51] The "black box" distortion associated with the subsidy could also be micro-founded in an endogenous growth framework.

On the macro-financial side, perhaps the next step should be the addition of a banking sector.[52] By micro-founding the financial intermediation process, both the economic response to the balance sheet impact of the Carbon Bubble, and the design of optimal policy to mitigate the shock, can be altered.[53] One could also properly model the bankruptcy process, incorporating a fuller description of the capital structure of investors' balance sheets with different priority creditors, and costs of financial distress. Heterogeneous agents may be an important element to add to the model, as in Punzi and Rabitsch (2015). The costless government guarantee suggests that aggregate uncertainty or stochastic noise should be added, so that such a guarantee had an expected cost along the equilibrium path.

Other elements might be improved as well. For example, the Pareto sub-optimality of some of the macroeconomically beneficial policies suggests that a political process

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[50]van der Ploeg and Rezai (2017) have taken a step in this direction.

[51]Ghisetti et al. (2015) analyse the role of financial barriers behind the adoption of environmental innovations.

[52]See for example Gersbach and Rochet (2012), Gertler et al. (2012), Caballero and Simsek (2013), and Gertler and Kiyotaki (2015).

[53]The key mechanism in this article is the fall in the collateral value of the entrepreneurs following a tighter climate policy. But, in reality, it is not only entrepreneurs that owns fossil capital or fossil reserves. In a model with a financial intermediation sector, we would observe costlier bank credit and bank runs if the tighter climate policy influences, directly or indirectly, the balance sheets of the intermediaries. See, for example, Gertler and Kiyotaki (2015).

is important. Perhaps this Pareto sub-optimality problem would be reduced though if savers also supplied labour to the entrepreneurial sector, and there was the possibility of unemployment in recessions. The issue of international spillovers is also important: we have modelled the Carbon Bubble issue as if there is a single global policymaker; but what would be the incentives for a national policymaker enforcing a Carbon Bubble restriction with or without the cooperation of other national policymakers? Another political economy issue that may be desirable to add to the model is some measure of the costs of acting through the planner.[54]

There remains much to do in fully specifying a model which will allow a macroeconomic forecast of the impact of the Carbon Bubble, and which will allow an optimal policy response to be designed. This article has started this modelling, and shown that there is a role for policy in mitigating its impact. Policy which protects investors' balance sheets mitigates the macroeconomic downturn, and leads to higher investment in the replacement zero carbon productive capacity over the period in which we still use carbon emitting productive capacity.

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[54]In the model as it stands, an obvious optimal policy would be to nationalise investment in energy so that the government, which is not credit constrained, maintains investment in the face of the Carbon Bubble.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous. 2012. "The environment and directed technical change." *American Economic Review*, 102(1): 131 – 66.

Allen, M. R., D. J. Frame, C. Huntingford, C. D. Jones, J. A. Lowe, M. Meinshausen, and N. Meinshausen. 2009. "Warming caused by cumulative carbon emissions towards the trillionth tonne." *Nature*, 458: 1163 – 1166.

Caballero, R. J., and A. Simsek. 2013. "Fire sales in a model of complexity." *The Journal of Finance*, 68(6): 2549 – 2587.

Carbon Tracker Initiative. 2011. "Unburnable carbon: Are the world's financial markets carrying a carbon bubble?"Technical report, Investor Watch.

Carbon Tracker Initiative. 2013. "Unburnable carbon 2013: Wasted capital and stranded assets."Technical report, Investor Watch.

Carney, M. 2014. "Letter to Joan Walley MP, Chair of House of Commons Environmental Audit Committee." http://www.parliament.uk/documents/commons-committees/environmental-audit/Letter-from-Mark-Carney-on-Stranded-Assets.pdf.

Cordoba, J. C., and M. Ripoll. 2004. "Collateral constraints in a monetary economy." *Journal of the European Economic Association*, 2(6): 1172 – 1205.

Devereux, M. B., and J. Yetman. 2010. "Financial deleveraging and the international transmission of shocks." In *The international financial crisis and policy challenges in Asia and the Pacific*. Ed. by Bank for International Settlements, 52 of BIS Papers chapters: Bank for International Settlements, 274 – 298.

Dietz, S., A. Bowen, C. Dixon, and P. Gradwell. 2016. "'Climate value at risk' of global financial assets." *Nature Climate Change*.

EIA. 2015. "Levelized cost and levelized avoided cost of new generation resources in the Annual Energy Outlook 2015."Technical report, US Energy Information Administration.

EIA. 2016. "Monthly energy review - April 2016."Technical report, US Energy Information Administration.

EU Climate Change Expert Group. 2008. "The 2°C target: Background on impacts, emission pathways, mitigation options and costs."

Gerke, R., M. Jonsson, M. Kliema, M. Kolasa, P. Lafourcade, A. Locarno, K. Makarski, and P. McAdam. 2013. "Assessing macro-financial linkages: A model comparison exercise." *Economic Modelling*, 31: 253 – 264.

Gersbach, H., and J. C. Rochet. 2012. "Aggregate investment externalities and macroprudential regulation." *Journal of Money, Credit and Banking*, 44(2): 73 – 109.

Gertler, M., and N. Kiyotaki. 2015. "Banking, liquidity, and bank runs in an infinite horizon economy." *American Economic Review*, 105(7): 2011 – 2043.

Gertler, M., N. Kiyotaki, and A. Queralto. 2012. "Financial crises, bank risk exposure and government financial policy." *Journal of Monetary Economics*, 15(Supplement): S17 – S34.

Ghisetti, C., M. Mazzanti, S. Mancinelli, and M. Zoli. 2015. "Do financial constraints make the environment worse off? Understanding the effects of financial barriers on environmental innovations." SEEDS Working Papers 0115, SEEDS, Sustainability Environmental Economics and Dynamics Studies.

Hart, O., and J. H. Moore. 1994. "A theory of debt based on the inalienability of human capital." *Quaterly Journal of Economics*, 109(4): 841 – 879.

Hassler, J., P. Krusell, and C. Olovsson. 2015. "Energy-saving technical change." *Centre for Macroeconomics Discussion Paper No. 1529.*

Hellwig, M. F. 2009. "Systemic risk in the financial sector: An analysis of the subprime-mortgage financial crisis." *De Economist*, 157(2): 129 – 207.

Hunter, J. D. 2007. "Matplotlib: A 2D graphics environment." *Computing in Science & Engineering*, 9(90): 90 – 95.

Iacoviello, M. 2005. "House prices, borrowing constraints, and monetary policy in the business cycle." *American Economic Review*, 95(3): 739 – 764.

Iacoviello, M., and S. Neri. 2010. "Housing market spillovers: Evidence from an estimated DSGE model." *American Economic Journal: Macroeconomics*, 2(2): 125 – 164.

IEA. 2012. "World energy outlook 2012."Technical report, International Energy Agency.

IPCC. 2014. "Climate change 2014."Technical report, Intergovernmental Panel on Climate Change.

Jaeger, C. C., and J. Jaeger. 2011. "Three views of two degrees." *Regional Environmental Change*, 11(1S): 15 – 26.

Jovanovic, B. 2013. "Bubbles in prices of exhaustible resources." *International Economic Review*, 54(1): 1 – 34.

Kiyotaki, N., and J. H. Moore. 1995. "Credit cycles." Discussion Paper no. TE/95/285. London: London School Econ., Suntory-Toyota Internat. Centre Econ. and Related Disciplines, 1995; Working Paper no. 5083. Cambridge, Mass.

Kiyotaki, N., and J. H. Moore. 1997. "Credit cycles." *Journal of Political Economy*, 105(2): 211 – 248.

Liu, Z., P. Wang, and T. Zha. 2013. "Land price dynamics and macroeconomic fluctuations." *Econometrica*, 81(3): 1147 – 1184.

Luangaram, P. 2003. "Asset prices, leverage and financial crisis: The case of Thailand." Ph.D. dissertation, The University of Warvick, Department of Economics.

McGlade, C., and P. Ekins. 2015. "The geographical distribution of fossil fuels unused when limiting global warming to 2°C." *Nature*, 517: 187 – 190.

Mendoza, E. G. 2010. "Sudden stops, financial crises, and leverage." *American Economic Review*, 100(5): 1941 – 1966.

Newell, R. G., Y. Qian, and D. Raimi. 2016. "Global energy outlook 2015." *NBER Working Paper No. 22075.*

Nordhaus, W. D. 2008. *A question of balance.* London: Yale University Press.

Pariès, M. D., C. K. Sørensen, and D. Rodriguez-Palenzuela. 2011. "Macroeconomic propagation under different regulatory regimes: Evidence from an estimated DSGE model for the euro area." *International Journal of Central Banking*, 7(4): 49 – 113.

van der Ploeg, F. 2013. "Cumulative carbon emissions and the green paradox." *Annual Review of Resource Economics*, 5: 281 – 300.

van der Ploeg, F., and A. Rezai. 2017. "Cumulative emissions, unburnable fossil fuel, and the optimal carbon tax." *Technological Forecasting and Social Change*, 116(C): 216–222.

Punzi, M. T., and K. Rabitsch. 2015. "Investor borrowing heterogeneity in a Kiyotaki-Moore style macro model." *Economics Letters*, 130(C): 75 – 79.

Robins, N., A. Keen, and Z. Knight. 2012. "Coal and carbon: Stranded assets: Assessing the risk."Technical report, HSBC Global Research.

Rolling Stone. 2012. "Global Warming's Terrifying New Math." Available online at http://www.rollingstone.com/politics/news/global-warmings-terrifying-new-math-20120719, written by Bill McKibben.

Sinn, H. W. 2012. *The Green Paradox*. Cambridge: MIT Press.

Tietenberg, T. H., and L. Lewis. 2014. *Environmental and Natural Resource Economics*. Abingdon: Routledge.

Tirole, J. 1985. "Asset bubbles and overlapping generations." *Econometrica*, 53(6): 1499 – 1528.

van der Zwaan, B. C. C., R. Gerlagh, G. Klaassen, and L. Schrattenholzer. 2002. "Endogenous technological change in climate change modelling." *Energy Economics*, 24(1): 1 – 19.

van der Walt, S., S. C. Colbert, and G. Varoquaux. 2011. "The NumPy array: A structure for efficient numerical computation." *Computing in Science & Engineering*, 13(22): 22 – 30.

Weitzman, M. L. 2009. "On modeling and interpreting the economics of catastrophic climate change." *The Review of Economics and Statistics*, 91(1): 1 – 19.

A Appendix

A.1 Glossary of variables and parameters

In the text, lower case letters indicate variables for a representative entrepreneur or for a representative saver if followed by a prime symbol. Upper case letters are aggregate variables. Starred letters represent steady state equilibrium variables.

Table 1: Variables and parameters definition

$t = 0, 1, ..., \infty$	time
m	relative savers' population size
x	consumption
β	discount factor
k	entrepreneur's fixed capital holdings
$\bar{K}$	total supply of fixed capital
z^H	high carbon energy capital
z^L	zero carbon energy capital
$1 - \lambda$	energy capital depreciation rate
q	relative price of fixed capital
R	gross interest rate
y	output
a^i	tradable proportion of output
c	non-tradable proportion of output
τ	carbon tax rate
ς	green subsidy rate
δ	effectiveness of the green subsidy
b	debt
g	per capita government tax or transfer
π	proportion of entrepreneurs with investment opportunity
ϕ	output cost of investing
I	aggregate investment flow
u	user's cost of asset
γ	proportion of entrepreneurs using high carbon technology
a	net private productivity
$1 - \lambda_H$	high carbon energy capital retirement rate
$1 - \rho$	share of entrepreneurs' high carbon good written off at $t = 1$
S	cumulative emissions
$\bar{S}$	actual carbon budget
$\hat{S}$	carbon budget announced by social planner
τ^G	tax funding the transfer of investors' debt
B^G	social planner's debt
ω	share of entrepreneurs' debt taken over by social planner
$gtee$	government guarantee

A.2   Model assumptions not given in main text

We specify below some further model assumptions that are omitted from the main text and which are relevant restrictions in the derivation of the steady state.

We avoid a corner solution i.e. we ensure that, in the neighbourhood of the steady state, both types of agent produce,

Assumption D $\Psi'\left(\frac{\bar{K}}{m}\right) < \frac{\pi\left[a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right]-\phi(1-\lambda)(1+R\pi-R)}{\pi\lambda+(1-\lambda)(1-R+R\pi)} < \Psi'(0);$

we assume that the tradable output is at least enough to substitute the depreciated energy capital,

Assumption E $a^L > (1-\lambda)\phi,$

and that the probability of investment is not too small,

Assumption F $\pi > \frac{R-1}{R}.$

Assumptions E and F are also used to ensure that the steady state values $(q^\star, K^\star, B^\star)$ and the associated $u^\star$ are positive.

To guarantee that the entrepreneur will not want to consume more than the non-tradable output, we assume

Assumption G $c > \frac{1-\beta R\lambda(1-\pi)}{\beta R[\pi\lambda+(1-\lambda)(1-R+R\pi)]}\left(\frac{1}{\beta}-1\right)(a^L+\lambda\phi).$

Note that, since β and R are close to one, both Assumptions F and G are weak. Finally, we avoid the explosion in asset prices with the following transversality condition,

Assumption H $\lim\limits_{s\to\infty} E_t(R^{-s}q_{t+s}) = 0.$

A.3   Subsidy induced distortion

In a decarbonised world ($\gamma = 0$), since $K^\star$ is a monotonically increasing function of the productivity target set by the government, the amount of fixed capital used by the entrepreneurs increases with the subsidy. Indeed, an entrepreneur benefits fully from the presence of the subsidy while they only partly contribute to the per capita tax (since this is paid by savers too). However, when there is a cost associated with the subsidy, an increase in the target productivity set by the government has an ambiguous effect on social welfare. Indeed, while entrepreneurs' utility always increases in a (as it is a constant multiple, c, of $K^\star$), the increase in savers' income from increased debt interest as lending increases may not compensate the decrease due to increasing taxes. We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy rate from the social planner's perspective is $\varsigma = 0$.

Panel (a) of Figure A.1 shows that any positive subsidy is welfare destroying in the neighbourhood of the decarbonised steady state. Conversely, Panel (b) shows that there is a clear optimal subsidy following the announcement of the carbon budget, because a positive subsidy ameliorates the balance sheet position of the entrepreneurs, and thus allows more investment in alternative productive capacity.

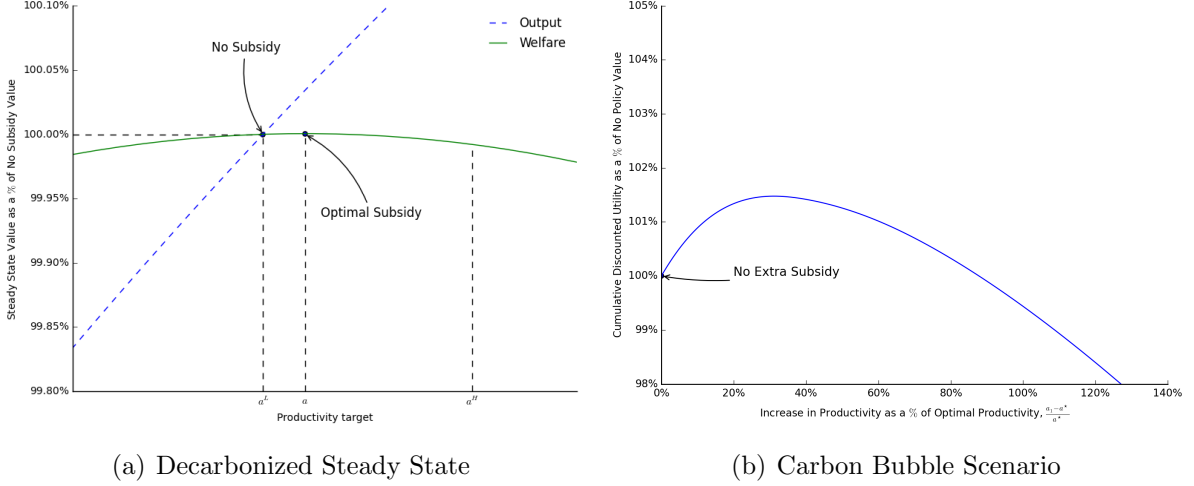(a) Decarbonized Steady State

(b) Carbon Bubble Scenario

Figure A.1: Welfare effects of the subsidy

A.4 Tax Induced Deadweight Loss

Similarly to the above, we add a deadweight loss parameter associated with the transfer of entrepreneurs' debt, such that the optimal transfer in the decarbonised steady state is zero.

Consider a social planner taking over a share $\omega \in [0, 1]$ of the entrepreneurs' debt, $B_G^\star$, in the decarbonised steady state. The debt repayments are funded through lump-sum taxes, τ^G, over T years. This implies

$$\tau^G = \frac{(1 - \beta) \omega B_G^\star}{(1 + m) \beta (1 - \beta^T)}.$$

The deadweight loss associated with the tax, $\mu \left(\tau^G \right)^2$, modifies the production function of the entrepreneurs to

$$y_t = \left[a^L - (1 - \delta) \varsigma_t + c - \mu \left(\tau^G \right)^2 \right] \times \min \left(k_{t-1}, z_{t-1}^L \right)$$

and thus modifies for T periods the equations of motion of K_t and B_t. We numerically find the value of μ that makes $\omega = 0$ (subject to $\omega \geq 0$) optimal in the decarbonised steady state.

A.5 Steady state zero carbon energy investment

One interesting result in the initial steady state is that, in an economy with credit frictions, unlike its frictionless equivalent, the high carbon proportion, γ, that maximises steady state zero carbon investment may be greater than zero. Under certain conditions, the relationship between the proportion of high carbon production, γ, and the absolute value of zero carbon investment is not monotonic: indeed, the higher the share, γ, of entrepreneurs using high carbon production and investing in high carbon energy capital, the higher is the net productivity of the fixed capital, and the higher are tax revenues and so the per capita transfer. This means that entrepreneurs have higher net worth and so can hold more of the fixed capital. Since the fixed capital is more productive in the hands of the entrepreneurs, its value increases. This potentially allows the entrepreneurs who are

35

using zero carbon production and investing in zero carbon energy capital to borrow more, invest more and produce more. Crucially we show that this non-monotonic relationship is due to the presence of credit frictions.

The steady state value of aggregate entrepreneurs' fixed capital holdings is[55]

$$K^\star = \frac{\pi \left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} \right] - \phi(1-\lambda)(1 - R + R\pi)}{\pi\lambda + (1-\lambda)(1 - R + R\pi)} + \nu.$$

Since zero carbon investment is given by $I_t^L = (1-\gamma)\phi(K_t - \lambda K_{t-1})$, in steady state this value is $I^{L\star} = (1-\gamma)\phi(1-\lambda)K^\star$. Therefore, investment in zero carbon energy capital can be expressed as

$$I^{L\star} = (1-\gamma)\phi(1-\lambda) \left\{ \frac{\pi \left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} \right] - \phi(1-\lambda)(1 - R + R\pi)}{\pi\lambda + (1-\lambda)(1 - R + R\pi)} + \nu \right\}.$$

Differentiating it with respect to γ gives

$$\frac{\partial I^{L\star}}{\partial \gamma} = \phi(1-\lambda) \left\{ \frac{\frac{\pi(1-\gamma)(\tau+\varsigma)}{1+m} - \pi a + \phi(1-\lambda)(1 - R + R\pi)}{\pi\lambda + (1-\lambda)(1 - R + R\pi)} - \nu \right\}.$$

It is then easy to see that under certain conditions (depending on e.g. the difference between the productivities of the two technologies, the fraction of entrepreneurs with respect to savers, the net private productivity), $I^{L\star}$ increases for low levels of γ before starting to decrease, as shown by the solid line in Figure A.2.
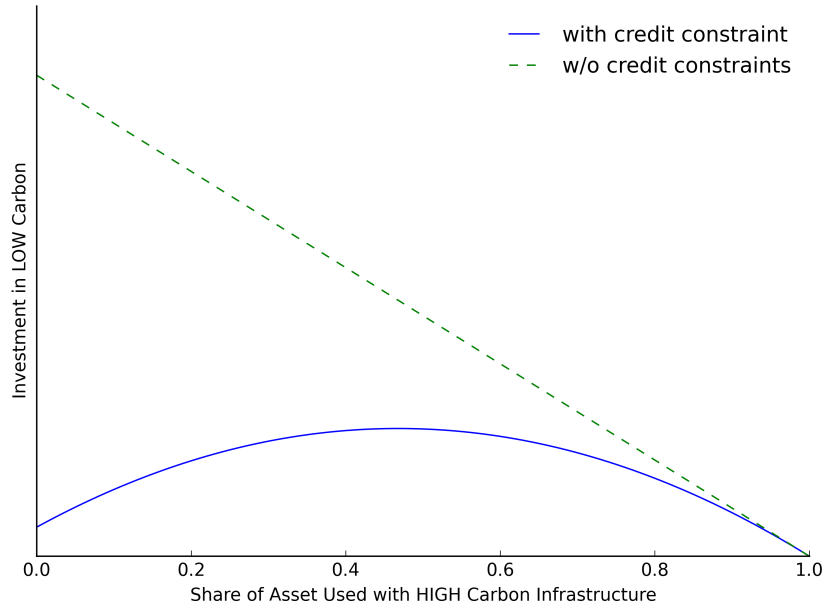

Figure A.2: Absolute investment in zero carbon energy capital as a function of γ

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[55]Assumption C implies $K^\star = u^\star + \nu$, where $u^\star$ is given by Equation (15c).

36

We now want to show that this result is a consequence of the presence of the credit constraint. Consider an economy in which there are no debt enforcement problem so that capital can be optimally allocated. In such an allocation the marginal products of the two technologies would be equalised and the fixed capital price would be given by the discounted gross return from using the entrepreneurs' technology, $q^0 = (a + c)/(R - 1)$. It follows that $u^0 = (a + c)/R$ and, given Assumption C, $K^0 = (a + c)/R + v$. Therefore, without the inefficiency caused by the presence of borrowing constraint, investment in zero carbon energy capital would be given by the following relationship

$$I^{L0} = (1 - \gamma)\phi(1 - \lambda)\left\{\frac{a + c}{R} + v\right\}$$

which is strictly increasing in $1 - \gamma$, the proportion of fixed capital used by entrepreneurs in conjunction with zero carbon energy capital, as shown by the dashed line in Figure A.2. Since the policymaker equalises the private return from using fixed capital with either high or zero carbon energy capital, which is optimally set equal to the returns from the savers' use of fixed capital, it is clear that the proportion γ of high carbon energy capital use cannot affect the amount of fixed capital used overall by the entrepreneurs. Therefore, in steady state, the flow of zero carbon energy capital investment is monotonically decreasing in the high carbon share, γ.

To the extent that the policy target is to maximise investment in zero carbon energy capital, this result shows that the optimal policy may be counter-intuitive: we may get more zero carbon investment if we allow high carbon investment to continue.

A.6   Calibration Strategy

The "calibrations" presented in Kiyotaki and Moore (1997, Chapter III) are not suitable for our exercise because the economy can only return to steady state for extremely small negative shocks: as shown in Figure A.3, the maximum write-off of energy capital that the model can sustain is only approximately 0.2%. Any write-off exceeding this amount requires the introduction of a debt renegotiation mechanism to ensure that the economy can converge to the interior steady state.

Not only we introduce a debt renegotiation mechanism based on Cordoba and Ripoll (2004), but also we define an alternative calibration strategy. This is explained in Section 4, but it broadly consists of the following steps. Firstly, we define the parameters R, λ, a^H, a_L, and a, based on definitional convenience, normalization, and to maintain the interpretation of periods as quarter years.

Secondly, we set δ and μ such that the optimal subsidy and the optimal debt transfer are zero in the decarbonised steady state, and γ using energy data. Thirdly, we try to broadly match the experience of the 2008-09 Financial Crisis. In order to do so, we modify the model slightly and set $\gamma = 0$. This means that m (and the constant in the savers' production function) do not influence this stage of the calibration since $a = a^L = 0.9$ and $g(m) = [\gamma\tau - (1 - \gamma)](1 + m) = 0$. We choose ν, ϕ, and π such that a loss in wealth comparable to the total value of the subprime mortgages causes a decline of approximately 20-25% in value of total capital. In doing so, we try to find a combination of these parameters such that the model can withstand a large write-off of energy capital before needing debt renegotiation. The reason being that relying too heavily upon debt renegotiation ruins the story that models of this sort tell, as when debt renegotiation is introduced, this absorbs most of the impact of the shock, while prices
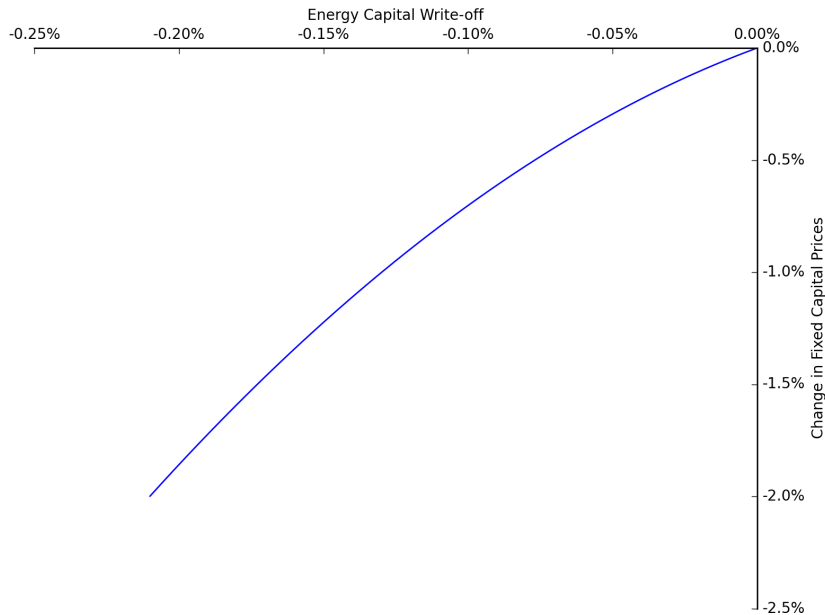
Figure A.3: Maximum shock under the original Kiyotaki and Moore's (1997) parametrisation

change only marginally. See Figure A.4. Additionally, we choose parameters such that we observe that debt holdings are positive in the maximum write-off run. Most calibrations satisfying these conditions produce extremely long economic cycles (when interpreting a period as a quarter). We therefore also try to minimise cycle length subject to satisfying these other conditions. We tried many different parameter combinations in generating this calibration and we assessed the marginal impact of changing each parameter upon various aspects of the solution.

Fourthly, we contemporaneously choose $\bar{K}$, $const$, m, and c such that consumption of individual saver and entrepreneur are equal in the new decarbonised steady state, the maximum output impact in the Financial Crisis run is around 6%, and energy expenditure over GDP is 7.5%. Figure A.5 shows the dynamics on the variable of interest following a shock comparable to the 2008-09 Financial Crisis. The calibrated parameters are presented in Table 2. Targeted and model moments are given in Table 3.

Table 2: Parameters values

$R = 1.01$	$\lambda = 0.975$	$\pi = 2\%$	$\nu = 1.5$
$\phi = 33$	$a^H = 1.10$	$a^L = a = 1.0$	$c = 1.14$
$\bar{K} = 15$	$\gamma = 0.8$	$m = 2.08$	$const = 87$

Figure A.6 shows that the effect of moving from the initial steady state, where more productive high energy capital is used, to the decarbonised steady state, without any write-off, is important but significantly smaller that the effect caused by the write-off, as one can see comparing Figure A.6 with Figure 2 in the main text.
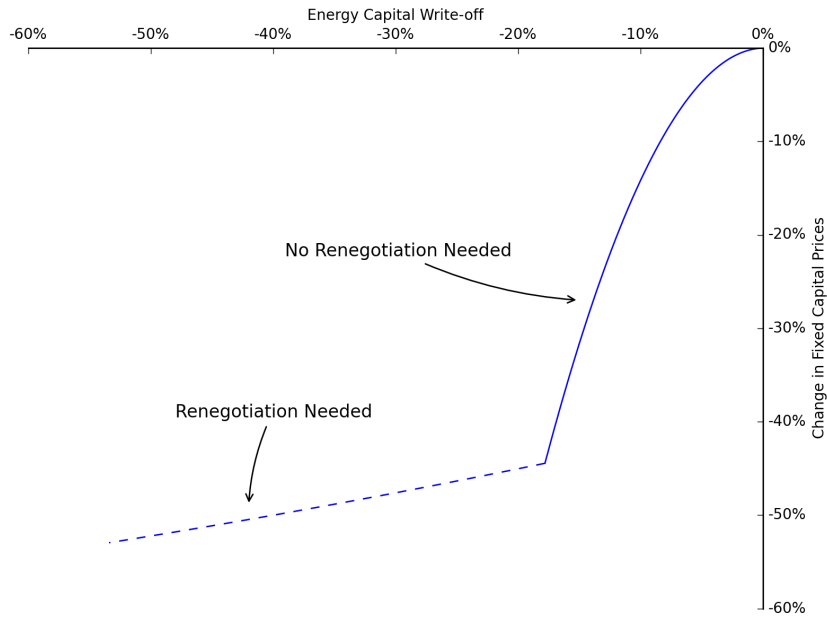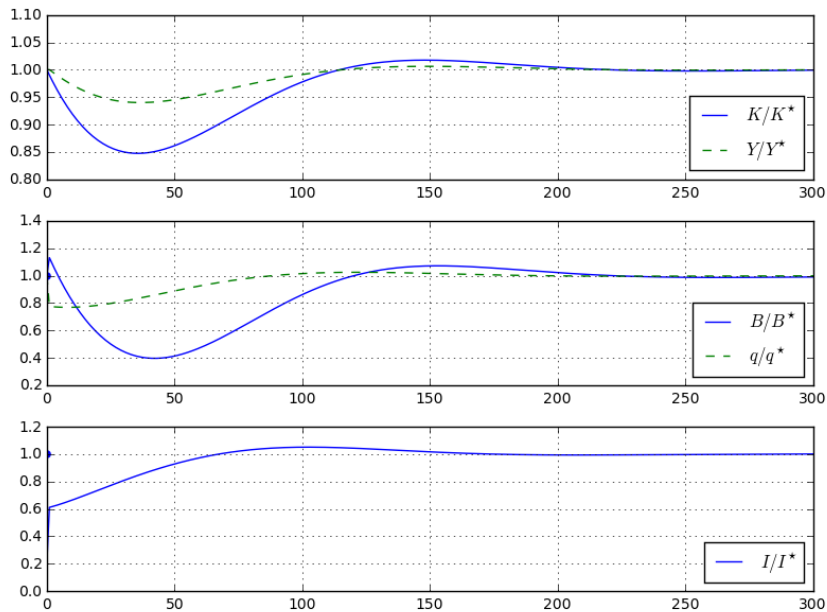
Figure A.4: Current parametrisation

Figure A.5: Dynamics induced by the financial crisis shock

A.7 Dynamics and Timing

In the first period, $t = 0$, timing is as described in Figure A.7. At the time of the announcement (the very start of the $t = 0$ period), asset and credit markets have just closed, so that $K_0 = K_F^\star$ and $B_0 = B_F^\star$ are state variables. The announcement affects prices which hugely impair entrepreneurs' balance sheets. However, after the announcement there is a debt renegotiation opportunity which changes B_0 to $B_{0+} \leq B_F^\star$, but cannot alter K_0: savers and entrepreneurs adjust their credit positions given the entrepreneurs' net worth

Table 3:   Calibrated values

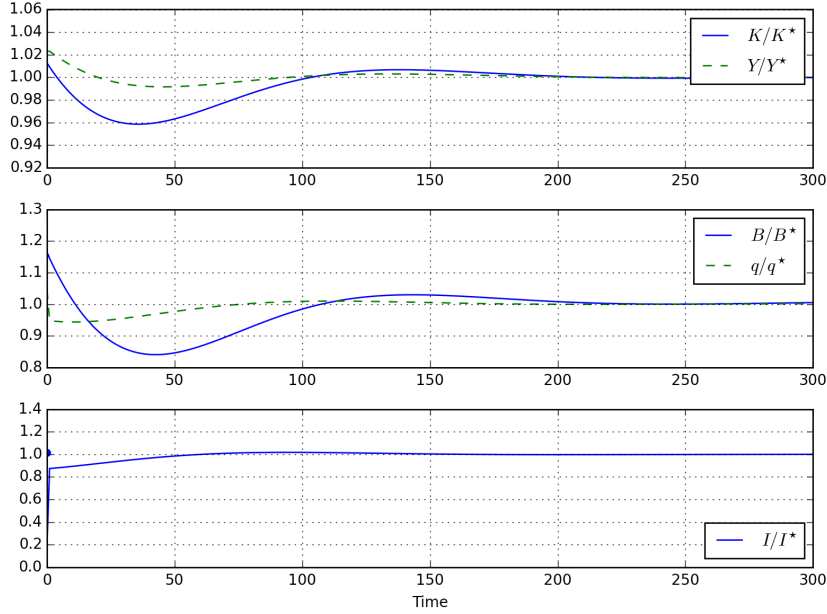|	Target	Model
Energy Expenditure	7.50%	7.50%
Asset Impact of FinCrisis	-20/25%	-22.47%
Output Impact of FinCrisis	-6.00%	-6.00%
Difference in Individual Consumption	0.00%	0.00%

Figure A.6:   Dynamics induced by the productivity differential

implied by their fixed real holdings of the asset. Renegotiation does not take place if the economy can converge back to the steady state given $B_{0+} = B_F^\star$, because savers have no incentive to renegotiate. However, if the economy cannot converge back to its steady state then both parties have the incentive to renegotiate the outstanding value of the debt:[56] the debt level retained by the entrepreneurs is then reduced to $B_{0+} < B_F^\star$, where this B_{0+} value is the maximum value for debt levels consistent with the economy being able to reach steady state. Production then takes place with entrepreneurs using $Z_0^H = \gamma K_F^\star$ high carbon energy capital.[57]

---

[56]Entrepreneurs would always like to reduce their debt. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs. Savers prefer to reduce their debt rather than accept this, and so engage in renegotiation. They write-off the minimum quantity of debt such that the interior steady state can be reached. We refer an interested reader to Appendix A.6 for how renegotiation influences the dynamics following a shock, and to Appendix A.9 for more information about the renegotiation process.

[57]Note that, alternatively, entrepreneurs could leave some of the fixed asset unused in this first period, since markets are closed and it cannot be traded back to the savers. We checked and verified that this option is not optimal for the entrepreneurs, as the reduced consumption in the first period (and the lower net worth) is enough to offset the positive effect of an increased remaining carbon budget.
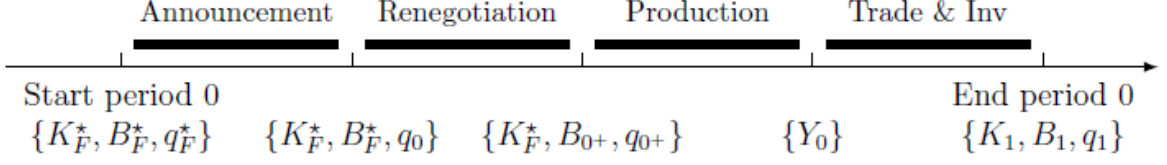
Figure A.7: Timing

At the end of the period, savers and entrepreneurs receive their output, the asset and credit markets open, and agents make consumption and investment decisions. Entrepreneurs must also decide the share $\rho \in [0,1]$ of the remaining $\lambda \gamma K_F^\star$ high carbon energy capital they will use, and the rate, $1 - \lambda_H \in [1-\lambda, 1]$, at which they will retire these goods. These choices are a function of prices, q_1, and determine the values of the state variables for the next period: K_1 and B_1. These choices, which are not independent, must satisfy the carbon budget set by the planner and are optimally taken by the entrepreneurs: it turns out that it is optimally for the entrepreneurs to delay the retirement of the remaining carbon goods to the last period, i.e. $\rho = 1$ and $\lambda_H = \lambda$.

A.8 Stability

In this section we follow Kiyotaki and Moore (1995, Appendix) to linearise the model around the steady state in order to examine the dynamics. The procedure requires using the laws of motion of aggregate entrepreneurs' asset holdings in (13) and borrowing in (14), together with the asset market equilibrium condition in (12), to find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t).

By combining Equations (11) and (12), we find $q_{t+s} = R\big(q_{t+s-1} - u(K_{t+s-1})\big)$ and then substitute this value in Equation (13). Together with (14), we now have the following system of "transition equations" for $s \geq 1$:

$$q_{t+s} = Rq_{t+s-1} - Ru(K_{t+s-1})$$
$$B_{t+s} = q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) + RB_{t+s-1} - aK_{t+s-1} +$$
$$- \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t+s-1}$$
$$K_{t+s} = (1-\pi)\lambda K_{t+s-1} +$$
$$+ \frac{\pi}{\phi + u(K_{t+s})} \left[(q_{t+s} + \phi\lambda + a) K_{t+s-1} - RB_{t+s-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t+s-1} \right].$$

Consider taking a first-order Taylor series expansion to this system around the steady state,

$$\frac{q_{t+s} - q^\star}{q^\star} \approx \frac{\partial q_{t+s}}{\partial q_{t+s-1}}\bigg|_{SS} \frac{q^\star}{q^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \frac{\partial q_{t+s}}{\partial K_{t+s-1}}\bigg|_{SS} \frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} =$$
$$= R\frac{q_{t+s-1} - q^\star}{q^\star} - Ru'(K^\star)\frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} =$$
$$= R\frac{q_{t+s-1} - q^\star}{q^\star} - (R-1)\frac{u'(K^\star)K^\star}{u(K^\star)} \frac{K_{t+s-1} - K^\star}{K^\star}$$

where the last line follows from using $u^\star = q^\star\left(1 - \frac{1}{R}\right)$;

$$
\frac{B_{t+s} - B^\star}{B^\star} \approx \frac{\partial B_{t+s}}{\partial B_{t+s-1}}\bigg|_{SS} \frac{B^\star}{B^\star} \frac{B_{t+s-1} - B^\star}{B^\star} + \frac{\partial B_{t+s}}{\partial q_{t+s-1}}\bigg|_{SS} \frac{q^\star}{B^\star} \frac{q_{t+s-1} - q^\star}{q^\star} +
$$

$$
+ \frac{\partial B_{t+s}}{\partial K_{t+s-1}}\bigg|_{SS} \frac{K^\star}{B^\star} \frac{K_{t+s-1} - K^\star}{K^\star} =
$$

$$
= \left[R + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right] \frac{B_{t+s-1} - B^\star}{B^\star} + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}} \frac{q^\star}{B^\star} \frac{q_{t+s-1} - q^\star}{q^\star} +
$$

$$
+ \left[-\left(q^\star + \lambda\phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right) + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right] \frac{K^\star}{B^\star} \frac{K_{t+s-1} - K^\star}{K^\star}
$$

$$
\frac{K_{t+s} - K^\star}{K^\star} \approx \frac{\partial K_{t+s}}{\partial q_{t+s-1}}\bigg|_{SS} \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \frac{\partial K_{t+s}}{\partial B_{t+s-1}}\bigg|_{SS} \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} +
$$

$$
+ \frac{\partial K_{t+s}}{\partial K_{t+s-1}}\bigg|_{SS} \frac{K^\star}{K^\star} \frac{K_{t+s-1} - K^\star}{K^\star} =
$$

$$
= \left[\frac{R\pi K^\star}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\right] \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} +
$$

$$
- \left[\frac{\pi R}{\phi + u(K^\star)} + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right] \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} +
$$

$$
+ \left\{(1 - \pi)\lambda + \frac{\pi\left(q^\star + \phi\lambda + a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right)}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}} \right.
$$

$$
\left. - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right\} \frac{K_{t+s-1} - K^\star}{K^\star}.
$$

From the last approximation, it follows that

$$
\frac{\partial K_{t+s}}{\partial q_{t+s-1}} = \frac{R\pi K^\star}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial q_{t+s-1}} =
$$

$$
= \frac{R\pi K^\star}{\phi + u(K^\star)}\left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}
$$

$$
\frac{\partial K_{t+s}}{\partial B_{t+s-1}} = -\frac{\pi R}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial B_{t+s-1}} =
$$

$$
= -\frac{R\pi}{\phi + u(K^\star)}\left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}
$$

$$
\frac{\partial K_{t+s}}{\partial K_{t+s-1}} = (1 - \pi)\lambda + \frac{\pi(q^\star + \phi\lambda + a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m})}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}} +
$$

$$
- \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial K_{t+s-1}} =
$$

$$
= \left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}
$$

$$
\left[(1 - \pi)\lambda + \frac{\pi(q^\star + \phi\lambda + a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m})}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}}\right].
$$

The system can be expressed more compactly as

$$
\begin{pmatrix} \hat{q}_{t+s} \\ \hat{B}_{t+s} \\ \hat{K}_{t+s} \end{pmatrix} = J \begin{pmatrix} \hat{q}_{t+s-1} \\ \hat{B}_{t+s-1} \\ \hat{K}_{t+s-1} \end{pmatrix}
$$

where an hatted variable indicates proportional deviation from the steady state and J is the Jacobian in elasticity form. An element of the Jacobian is indicated with J_{mn}, where $m, n = (q, B, K)$, so that $J_{mn} = (\partial m_{t+s})/(\partial n_{t+s-1}) \times (n^\star)/(m^\star)$. More specifically,

$$
J_{qq} = R \qquad J_{qB} = 0 \qquad J_{qK} = -(R-1)\frac{u'(K^\star)K^\star}{u(K^\star)}
$$

$$
J_{Bq} = (q^\star + \phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{q^\star}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{B^\star}
$$

$$
J_{BK} = \left[-\left(q^\star + \lambda\phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right) + (q^\star + \phi)J_{KK}\right]\frac{K^\star}{B^\star}
$$

$$
J_{BB} = R + (q^\star + \phi)J_{KB}\frac{K^\star}{B^\star} \qquad J_{Kq} = \frac{R^2\pi}{\phi + u(K^\star)}\frac{u^\star}{R-1}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}
$$

$$
J_{KB} = -\frac{R\pi}{\phi + u(K^\star)}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}\frac{B^\star}{K^\star}
$$

$$
J_{KK} = \left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}
$$
$$
\left[(1-\pi)\lambda + \frac{\pi\left(q^\star + \phi\lambda + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right)}{\phi + u(K^\star)} - R\pi\frac{u^\star K^\star}{\phi + u(K^\star)}\frac{u'(K^\star)}{u(K^\star)}\right].
$$

By renaming the variables accordingly, we can refer the interested reader to Kiyotaki and Moore (1995, Appendix) for the analysis of the stability of the system around the steady state.

A.9   Shooting Algorithm and Renegotiation Mechanism

The simulations are obtained using the shooting algorithm. By using the laws of motion of aggregate entrepreneurs' asset holdings in (13) and borrowing in (14), together with the asset market equilibrium condition in (12), we can find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t). From Equations (11) and (12), we find $q_{t+1} = R(q_t - u(K_t))$. We now impose $u(K_t) \equiv K_t - \nu$: the previous becomes $q_{t+1} = R(q_t - K_t + \nu)$. The next step is to substitute this value in Equation (13) and solve for K_t. We then have the following

system of "transition equations" that we can iterate:

$$K_{t+s} = \frac{1}{2} \left[\nu - \phi + (1 - \pi) \left(\lambda_{t+s} \rho_{t+s} \gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1}) \right) K_{t+s-1} \right] +$$

$$+ \frac{1}{2} \left\{ \left[\phi - \nu - (1 - \pi) \left(\lambda_{t+s} \rho_{t+s} \gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1}) \right) K_{t+s-1} \right]^2 + \right.$$

$$+ 4 \left[(\phi - \nu)(1 - \pi) \left(\lambda_{t+s} \rho_{t+s} \gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1}) \right) K_{t+s-1} + \right.$$

$$+ \pi K_{t+s-1} \left[q_{t+s} + \phi \left(\lambda_{t+s} \rho_{t+s} \gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1}) \right) + a \right] +$$

$$\left. \left. - \pi R B_{t+s-1} + \pi \frac{\gamma_{t+s-1} \tau_{t+s} - (1 - \gamma_{t+s-1}) \varsigma_{t+s}}{1 + m} K_{t+s-1} \right] \right\}^{0.5} \tag{A.2a}$$

$$q_{t+s} = R(q_{t+s-1} - K_{t+s-1} + \nu) \tag{A.2b}$$

$$B_{t+s} = q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - (\lambda_{t+s} \rho_{t+s} \gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1})) K_{t+s-1}) +$$

$$+ R B_{t+s-1} - a K_{t+s-1} - \frac{\gamma_{t+s-1} \tau_{t+s} - (1 - \gamma_{t+s-1}) \varsigma_{t+s}}{1 + m} K_{t+s-1}. \tag{A.2c}$$

When the available carbon budget is announced at t, the amount of entrepreneurs' energy capital, after depreciation, reduces to $[\rho\gamma + (1 - \gamma)]\lambda K_{t+s-1}$, where $(1 - \rho) \in [0, 0.5]$ is the percentage of the stock of high carbon energy capital that it is optimally written off by the entrepreneurs at $t = 1$.[58] When this shock hits, Equation (A.2b) does not hold because the asset price jumps in response to the shock and entrepreneurs experience a loss on their asset holdings. In the original Kiyotaki and Moore's (1997) model, a shock of the magnitude we are interested in would throw the economy out of the basin of attraction of the interior steady state. To prevent this, we follow Cordoba and Ripoll (2004) and allow for renegotiation of the debt.

In Cordoba and Ripoll (2004), who introduce renegotiation in the basic Kiyotaki and Moore's (1997, Chapter II) model, markets are open during the day, shocks occur at dusk, and then there is a window of opportunity for debt renegotiation to take place, before production occurs overnight. If entrepreneurs want to, they can default on the debt, crucially, before production takes place: the lender gets the ownership of the fixed capital but loses the outstanding value of the debt. They may be able to do better by renegotiating the outstanding value of the debt repayments, RB_{t+s-1}, down to the new value of the collateral, $q_{t+s}K_{t+s-1}$, and incentivising the entrepreneurs to engage in production. This shares the burden of the fall in fixed capital values with the lenders and ultimately limits the decrease in fixed capital prices and output with respect to Kiyotaki and Moore (1997).

Conversely, we use the full Kiyotaki and Moore's (1997, Chapter III) model, where the aggregate value of debt is not exactly aligned with the value of the collateral, since a fraction of entrepreneurs cannot invest and thus repay part of their debt obligations.

---

[58]Note that λ and ρ have a time subscript here, to reflect that the actual depreciation rate must take into account the optimal path of withdrawing the remaining stock of high carbon energy capital. It turns out that $\lambda_{t+s} = \lambda$ and $\rho_{t+s} = 1$ for all $s \geq 1$. Also γ has a time subscript: in the simulation, the social planner has banned investment in high carbon energy capital, therefore depreciation and the optimal path of withdrawing the remaining high carbon energy capital imply that the share of fixed capital used with high carbon energy capital will change over time and eventually go to zero. In particular, $\gamma_{t+s} = \lambda_H^{t+s-1} \lambda \rho \gamma K_0 / K_{t+s}$.

Moreover, rather than allowing renegotiation for any negative shock as in Cordoba and Ripoll (2004), we allow for renegotiation only if the economy cannot converge back to the interior steady state following a shock, as in this case both parties have an incentive to renegotiate the outstanding value of the debt.[59] However, for any level of the shock, there are infinite combinations of changes in prices and debt levels such that the economy converges back to its steady state. See, for example, Figure A.8 where we plot these combinations for a write-off of high carbon energy capita equal to 35% of the current stock. The debt level retained by the entrepreneurs and the price of capital in our simulations are the maximum value for debt levels, B_{t+s-1}^+, and corresponding price q_{t+s}^+, consistent with the economy being able to reach its interior steady state. These correspond to the dot in Figure A.8. This choice is the one that makes the downturn less severe (and thus reduces the welfare increase induced by our policies). Analytically, when renegotiation takes place, debt repayments RB_{t+s-1} are pushed down to RB_{t+s-1}^+, and prices jump to q_{t+s}^+.
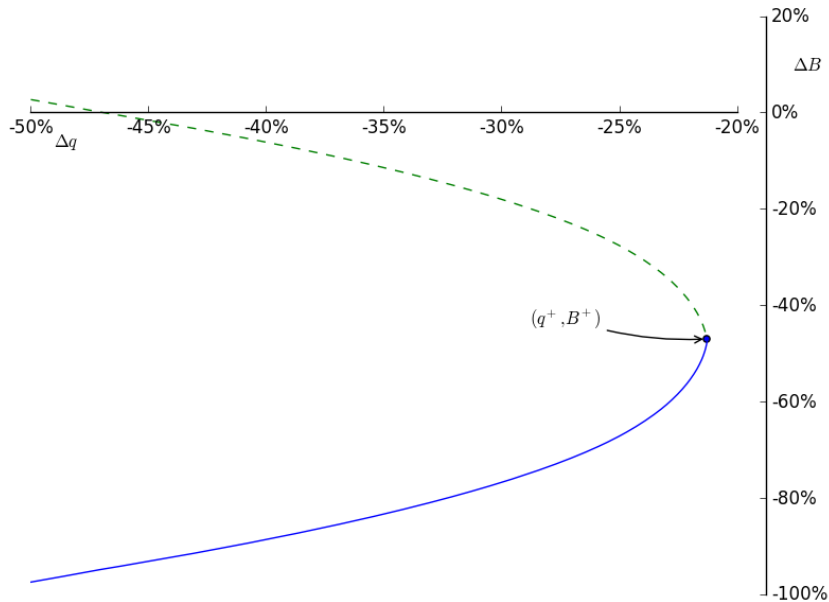

Figure A.8: The (q, B) arms for a write-off of 35% of high carbon energy capital

Given the transversality condition in Assumption H, we know that $q_t = q^\star$ for large t. But since Equations (11) and (12) define the asset price variation as a function of K_t, we can project the asset values back from steady state. So, the rough ideas is to guess the initial variation in asset price given the shock and then iterate the economy forward through time to see if it converges again to the steady state. If the level of asset price eventually explodes, the initial guess is revised downward; if it is forever smaller then the initial guess is revised upward. This "guess and check" procedure is repeated until the asset price is close to the steady state (i.e. within the arbitrary level of tolerance).

---

[59]This is because, while entrepreneurs would always like to reduce their debt, this is not always the case for savers. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs.

When we allow the social planner to take over a fraction ω of debt from the entrepreneurs, the following additional changes are required in the transition equations. Between the period in which the shock is announced and the following period, the value of the entrepreneurs' debt is further reduced to $(1-\omega)B_{t+s-1}^{+}$. If the transfer of entrepreneurs' debt is funded with a constant tax τ^{G} over T periods, for T periods we add τ^{G} in the right-hand side of (A.2c) and subtract $\pi\tau^{G}$ in the right-hand side of (A.2a) (inside the square root). Additionally, the budget constraint of the saver now includes debt repayments and new debt from the social planner, RB_{t+s-1}^{G} and B_{t+s}^{G}. While this does not directly influence the transition equations, it changes the consumption of the savers in each period, thus influencing the social welfare level reached by the economy. Finally, at $t = T+1$, there is no tax any more and the social planner holds no debt, so for $t \geq T+1$, the system of transition equations in (A.2) holds.[60]

---

[60]Simulations are run using Numpy (Walt et al., 2011); graphs are drawn in Matplotlib (Hunter, 2007). We used Python 2.7.