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THE RISE OF MERITOCRACY AND THE INHERITANCE OF
ADVANTAGE

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# The Rise of Meritocracy and the Inheritance of Advantage

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## Abstract

We present a model where more accurate information on the background of individuals facilitates statistical discrimination, increasing inequality and intergenerational persistence in income. Surprisingly, more accurate information on the actual capabilities of workers leads to the same result – firms give increased weight to the more accurate information, increasing inequality and fostering discrimination. The rich take advantage of this through educational investments in their children, lowering mobility. Using our model to interpret the data suggests that a country like the US might be a land of opportunity for the sufficiently able but where (for endogenous reasons) ability is strongly correlated with background.

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# 1   Introduction

Intergenerational mobility is usually measured by the correlation between a proxy of the lifetime income of parents and that of their children. With almost no exceptions, this correlation does not control for the abilities of the children, but imagine for a moment that it did. The partial correlation of parental income would then indicate the advantages that a child with richer parents would enjoy vis-à-vis another child with exactly the same abilities but a worse background. Our point in this paper is to remark that a low partial correlation of parental income by no means implies that the unconditional correlation needs to be low. We show that if a society is better able to judge (and reward) individuals according to their true ability, then the *unconditional* correlation (not controlling for ability) will necessarily increase, albeit the *partial* correlation (controlling for it) could decrease. This is, societies that reward ability handsomely – and where as a consequence the children of the rich do not enjoy advantages once ability is accounted for – are bound to show larger inequalities and lower social mobility. The reason is that the process of human capital accumulation implies that in those societies the differences between the abilities of the children of the rich and the poor are bound to be high.

To study these issues, we develop a theory of human capital accumulation stressing the role of statistical discrimination benefiting those from privileged backgrounds and the effects of having imperfect objective information on an individual's productive abilities.

In our model the income of an individual reflects the perception that society has on her productive abilities, summarizing all the information that the market has on her. It partly reflects what it is known about her abilities themselves, but also reflects any independent information available on the agent's background. This is because the market is aware that some backgrounds are more conductive to human capital accumulation than others. In equilibrium, richer parents tend to invest more in their children and, consequently, their children tend to be more productive. Thus, knowing the background of an individual is informative on their abilities, and this information will be used by the market in determining their income *even conditioning on the direct information available about their abilities.*

We present two main results. The first concerns the effects of having more information about the background and abilities of individuals on inequality and mobility. It is not surprising that more accurate information on background translates into a larger degree of inequality and lowers intergenerational social mobility. This is because more information on background allows firms to positively discriminate in favor of those with higher expected human capital. Moreover, and perhaps more surprisingly, we show that the same effects (more inequality and less mobility) appear as a consequence of the market having better information on the agent's productive ability – increasing the degree of *"meritocracy"* (in the sense of allowing firms to judge the abilities of workers more accurately, independently of their backgrounds) has qualitatively the same effects on inequality and mobility as giving more *"advantages"* to the rich (by means of differentiating them more clearly from the poor and thus favoring them via positive statistical discrimination).

The intuition for this result is as follows. By definition, in a *"meritocratic"* society the effect of parental income should be very small when conditioning on the productivity of the agent, but

these two variables (background and productivity) are not independent. More information on the productive abilities of agents increases the differences in rewards that those agents receive as the market is more efficient at differentiating the very productive from the less so. These differences in income translate into differences in investment in the human capital of children, which results in larger differences in their productive abilities and a greater correlation between the income of parents and their children. In equilibrium this gets amplified by a feedback mechanism, as larger inequality increases the value that the market assigns to any available information on an individual's talent.

Our second result concerns the effects of information (again, on background and on productivity) on the incentives of parents to invest in their children.

In a very meritocratic society (where firms will know the productivity of your children and pay them accordingly), the return to educational investment will be large, and thus your investment in your children will be larger than in a less meritocratic one.

In contrast, an increase in the information that the market has on the background of individuals does not have the same effect. As we will see, it helps firms better guess the productivity of agents (via statistical discrimination), but the effects on the incentives to educate children are minimal for the obvious reason that your income level does not depend on the education that your children have. Firms employing your children care more about what income you had, as it helps them guess the education that your children received, but you cannot change their beliefs about your children's background by educating them more or less.

Thus, in societies with better information on productive ability, we should expect more investment in education than in societies where *"advantages"* are more prevalent (i.e. statistical discrimination based on background). But the former and the latter do not need to differ in the degree of inequality and mobility.

Thus, our contribution is to stress that a low level of intergenerational mobility and a high degree of inequality are perfectly compatible with a high degree of *"meritocracy"*, with people being judged and rewarded according to their productive abilities. Moreover, two societies with the same degree of mobility and inequality may have arrived at such a situation from very different routes. One may be characterized by people being judged and rewarded by their abilities, while the other by the prevalence of advantages, the children of the rich being *presumed* to be more productive than the children of the poor. From the point of view of mobility and inequality both societies may look identical, but they differ radically in the incentives they provide to invest in education. In the society where people are rewarded for their productive ability there are strong forces conductive to high effort in educational investment, as this investment is likely to be rewarded with future income. These forces are absent in a society where advantages prevail.

Our contribution is theoretical, but we perform an exercise that aims to provide an example of the type of effects our mechanism could produce. We do the intellectual exercise of looking at what the model implies for the degree of *"meritocracy"* in different societies. In particular, we are interested in knowing if the implied degree of "meritocracy" in the US is large relative to other countries.

When compared with other developed economies the US shows a very large degree of inequal-

ity, a low degree of intergenerational mobility, and a very high level of investment in education.[1] Following the logic of the model, it seems plausible that the US could have a large degree of inequality and low mobility precisely because it has a high degree of *"meritocracy"*. We show that that is indeed the manner in which the model reads the data.

The model is able to replicate the data only if the degree of implied *"meritocracy"* in the US is much larger than in other OECD countries. If this were true, the US would still be a land of opportunity *provided that you have high productive ability.* The low mobility and high inequality in the US would be a reflection of the (endogenous) distribution of those abilities across people and generations, not a failure to reward merit. In order to solve the model we make implausible assumptions on the extent of redistribution and public education that are bound to overestimate the weight of our mechanism. Thus, we are reluctant to interpret these results as evidence of the distribution of meritocracy across societies. It is nevertheless a nice illustration of the mechanism that we model.

The paper is organized as follows. Section 2 reviews the existing literature and discusses semantics. Section 3 describes and sets up the model. Section 4 will look at the agent's problem and human capital investment. Section 5 will consider the firm's problem and pricing the signals on human capital and background. Section 6 will describe the steady state of the model and section 7 will consider some comparative statics exercises on inequality and mobility. Section 8 will describe the equilibrium and comparative statics on educational investment. Section 9 shows the way in which the model filters existing data. Section 10 summarizes and concludes. All proofs are relegated to the appendix.

## 2    Semantics and Related Literature

We are by no means the first to think along these lines. The word *"meritocracy"* itself was coined (as recently as 1958) by sociologist Michael Young in his book *"The Rise of Meritocracy"*[2] in order to put forward some of the ideas that we model. Young's book narrates an imaginary history of the UK up to 2034 (the book is supposedly written in 2033). In the meritocratic society it describes, positions are allocated based solely on merit, not on birth, but the book describes a dystopia, not an utopia. The reasons are in essence the ones we describe. The distribution of talent is endogenous to the workings of society. The high reward for talent that meritocracy entails fosters inequality in the distribution of income. The education system then tailors to the rich, who can afford more and better education for their children. Thus, meritocracy fosters both inequality and persistence in income across generations. Young's book finishes with the death of the supposed author at the hands of an anti-meritocratic revolution when those left behind by history rebel against what they perceive as the unbearable inequalities that meritocracy has created.

Given that the word *"meritocracy"* has today a positive meaning, it is surprising to notice

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[1]It is well known that intergenerational mobility correlates negatively with inequality. See Krueger (2012). This correlation has been documented across countries (Corak (2013)), across US commuting zones (Chetty et al. (2014)) and across Italian provinces (Güell et al. (2015)). The US is at the low mobility, high inequality end of this relationship.

[2]Young (1958)

that not even 60 years ago it was coined as a warning. It was recognized that it would give rise to a feeling of justice and would foster short-run (perhaps even long-run) efficiency, but it was doomed to increase inequality to the point of generating social instability.

We do not want to make a big deal about semantics, but it is necessary to clarify that when we use the word "meritocracy" we will stick to its original meaning: the ability of society to recognize the productive ability of individuals and reward them accordingly. For us it is a statement about the availability of direct information on a worker's productive ability. We like to think of this information as the result of the education system signaling with more or less accuracy the abilities of individuals. This itself could be a consequence of a more or less widespread use of ability tests and the extension of systematic performance measures across the schooling system. Alternatively, it could be the result of stratifying education via school-rankings which select and signal their students abilities.[3] In any case, for us this information is an exogenous parameter characterizing society, and we call it the degree of *"meritocracy"* of a society.

The important thing is the concept, though, not the word by which we name it. We recognize that other people may have different conceptions on what *"meritocracy"* means, and some people have strong opinions on this semantic issue. Clearly, many people associate the word with a reduction of the privileges that some members of society may enjoy. These privileges (which we might call *"cronyism"*) mean that some people may be rewarded far in excess to their contribution to society. Examples would be the corruption in the allocation of public positions via friendship or connections, or the impossibility of accessing higher education and positions of power and substance without the benefit of parental wealth and connections. Notice that the reduction of those privileges is not what we mean by *"meritocracy"*. It could be another perfectly reasonable definition, but is not the one that we use, and it is not what Young meant when he *invented* the word.

Since Becker (1957) we have known that irrational discrimination has negative effects not only on those discriminated against, but also on the discriminator. Competition and market forces should work against the extent of those privileges: a firm that hires a person because he has an aristocratic name rather than talent, is a firm that will lose money and be driven out of a competitive market. Thus, our approach is to model the advantages that the rich enjoy as a result of rational statistical discrimination of firms with limited information.[4] These firms know that *in equilibrium* the children of the rich will have received a better education and are likely to be more productive than the children of the poor. Discrimination is, thus, the rational response to the available information, as in many statistical discrimination models such as Arrow (1973),

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[3]Young's book was a reaction to the extension of the tripartite education system in England, Wales and Northen Ireland at the end of WWII. By this system children were examined at the age of 11 and selected into one of three school types based on their performance. At the top, grammar schools would feed into university (themselves selecting into further categories according to rank and student quality). At the bottom, students would be trained in "practical skills". Young's book became paramount in the abolition of the system in the 70's, with the introduction of the comprehensive school system that, at least in theory, offers the same education to all.

[4]In any case, in appendix K we include "cronyism" (privileges not backed by reason) as an extension of our analysis, and we show that their reduction does indeed decrease the intergenerational persistence of income and (in most cases) the degree of inequality. Still, even in their presence the effects of improving the technology to determine one's ability ("meritocracy" in our sense) are qualitatively the same as in our main analysis.

Coate & Loury (1993), Norman (2003) and Moro & Norman (2004). Simon & Warner (1992) show that "old boys" networks improve information signaling for firms, explaining why referrals from employeees account for a large percentage of hiring. Corak & Piraino (2011) show that children are much more likely to work in the same firms as their parents than chance would suggest.

Inefficient capital markets are another hurdle that the children of the poor endure. The study of this in economics started with Galor & Zeira (1993) and has since created a large literature. Nevertheless few papers, if any, study the effects of this in connection with the degree of meritocracy. We do not take the route of exploiting credit market imperfections as generators of inequality (our agents cannot access capital markets but, as the expected return to education is homogeneous for all agents, they end up investing a fixed proportion of their income in their children's education ), but there is no reason why the effects of *"meritocracy"* would be different, in such a case, from the ones we find.

A final related stream in the literature refers to the study of inequality and intergenerational mobility pioneered by Becker & Tomes (1979). Several of those papers have dealt with understanding the negative relationship between inequality and mobility, a number of which hint at the process of human capital accumulation as the possible cause. Hassler et al. (2007), for instance, show that if the bulk of the difference across economies resides in the workings of the labor market (skill bias or institutions), then the correlation between inequality and mobility across economies would be positive. For the correlation to be negative (as the data shows it is) the main differences across economies needs to be focused in the education system and the process of education acquisition. Likewise, Solon (2004) shows that differences in the degree of progressivity of the educational system generate the negative correlation observed.

A paper to which our work is particularly related is Abbott & Gallipoli (forthcoming). They present a model that aims to explain the geographical differences in intergenerational mobility, extending the Becker-Tomes model by introducing a production sector in which workers human capital inputs are complements. This leads to a negative correlation between progressive public policy and the intergenerational income elasticity. Computing the model, they show that geographic differences in skill complementarity directly account for roughly one fifth of cross-country variation in intergenerational mobility.

# 3   Model Description

We present an overlaping generations model with two different set of actors: families and firms. Agents work and receive a wage. The wage (the only state variable) is different for different agents, and it is a function of their observable characteristics. They care about their own consumption and the well-being of their children (with a certain discount rate). They have no access to capital markets. The only decision they need to make is how much of their income to consume and how much to invest in the education of their children. The only way of moving resources forward in time is by investing in children's education. There is no way of bringing resources back from the future. Obviously, this investment depends crucially on the return to education which is endogenous to the model and which parents take as given.

Firms produce output competitively using only labor. The productive ability of each worker (her *"human capital"*) is an stochastic function that depends positively on the investment that her parents made in her. The production function is linear in the productive ability of workers: output equals human capital one-to-one. Firms, though, cannot observe the productivity of workers. They observe a set of characteristics of the workers, and need to infer the expected human capital of each worker as a function of these characteristics. Firms, thus, pay each worker her expected human capital and make zero profits on average.

The exact manner by which human capital affects income is endogenous to the model. Actually, our main contribution is to articulate the mechanisms by which this process takes place. If firms were able to observe human capital exactly, income would be exactly equal to human capital; but it is observed with noise. That is, the market has two pieces of information about an individual: a noisy signal on her ability and another on her background. Firms determine rationally how much to pay an individual as a function of her two signals.

The quality of each of these signals is exogenous. We say that a society is more *"meritocratic"* if firms have better and more accurate information about the human capital of the individual: if the signal on human capital is more precise. In addition to the signal on human capital, firms use the available information on an agent's background because they know that in equilibrium richer parents invest more in education than poorer ones. Thus, there is statistical discrimination favoring children from better backgrounds, and in a society with better information on an individual's background the circumstances of birth play a larger role in income determination *even when conditioning on individual productive abilities.*

The core of the paper consists of: (i) solving a dynamic general equilibrium model with these three components (educational investment, meritocracy and inherited advantages); and (ii) to show the comparative statics of the endogenous variables to exogenous changes in the precision of the information available on the human capital of the agent (changes in the degree of *"meritocracy"*) and the precision of the information on parental income (which directly facilitate statistical discrimination) in the steady state. We show that better information (irrespective of whether it is on merit or background) always has the effect of increasing inequality and decreasing mobility.

We proceed by first exploring in section 4 the decisions of families given a certain stochastic map from investment to income. The income process is endogenous to the economy, but the family takes it as given.

Once we understand the investment decision of families we look at how much the market values the different characteristics of workers in order to determine wages (in section 5) and the income process.

We finally put both things together in section 8. In equilibrium parents use the resultant relationship between investment and income to infer how much they should invest in their children and how much to consume, thus closing the model.

# 4 Human Capital Investment

We start by setting up the problem of families, which have to choose how much to consume and how much to invest in their children's education.

At each point in time the family consists of a parent and a child. After each period the parent dies and the child becomes a parent.

There are no capital markets, so the resources of the family are exclusively the income generated by the parent in the labor market. There is no way of borrowing in exchange for future income, and there is no way of leaving resources for future generations except by investing in one's children. Thus, the only state variable is the income of the family.

The parent needs to allocate his resources between current consumption and investment in his child's human capital. This investment, *and the income that the parent earned*, will determine the observable characteristics of the child.

Firms produce with human capital only, but they do not observe the human capital of workers, only some informative characteristics about it. Consequently, their role is to decide how to translate those observable characteristics into expected human capital and income.

Thus, the problem which parents confront is:

$$W\left(Y_t^i\right) = \max_{X_t^i} \left\{ \ln C_t^i + \frac{1}{1+\delta} EW\left(Y_{t+1}^i | Y_t^i, X_t^i\right) \right\} \tag{1}$$

s.t.

$$C_t^i = Y_t^i - X_t^i; \quad X_t^i \geq 0 \tag{2}$$

$$Y_{t+1}^i \sim F\left(H_{t+1}^i, Y_t^i\right) \tag{3}$$

$$H_{t+1}^i \sim G\left(X_t^i\right) \tag{4}$$

where $Y_t^i$ is the (lifetime) income of family $i$ at generation $t$, $C_t^i$ is their consumption, $X_t^i$ is their investment in the human capital of their offspring, and $H_{t+1}^i$ is the human capital that the offspring actually achieves. Equation 2 is the budget constraint that parents face given the absence of capital markets. Equation 4 states that human capital is a stochastic function of the investment, without a role for other forms of inheritance (genetic or otherwise).

The investment problem is quite standard. As in Becker & Tomes (1979, 1986), agents decide how much to invest in the human capital of their children. We have assumed that children's innate abilities are stochastic, and parents invest without knowing the realization of their children's abilities. That is, there is a random component to the determination of human capital, and when making the investment decision parents do not know how much human capital their investment will translate into.

The novelty of our theory refers to equation 3. In the next sections we will develop a theory such that in equilibrium the income of an individual is a stochastic function of her human capital and her parental income, which has effects *even controlling for the human capital of the child* . Moreover, the magnitude of the effect of both human capital and parental income is a function of the degree of inequality of the distribution of income, which is itself an endogenous object of the model.

Equations 3 and 4 together imply a relationship between the income of a child and the

income and investment made by their parent. We will begin, in this section, by taking as given the following functional form for this relationship:

$$Y_{t+1}^i = e^{\gamma_0} \left(Y_t^i\right)^{\gamma_1} \left(X_t^i\right)^{\gamma_2} e^{\varepsilon_{t+1}^i} \tag{5}$$

where $\gamma_0$, $\gamma_1$ and $\gamma_2$ are endogenous parameters of this *income determination function* and $\varepsilon_{t+1}^i$ is a stochastic error term with a certain mean $\bar{\varepsilon}$ and variance $V_\varepsilon$. At this stage we are guessing that this is the correct functional form, but in subsequent sections we will show that this is, in fact, the correct form of the income determination function and solve for the equilibrium values of $\gamma_0$, $\gamma_1$ and $\gamma_2$, and the stochastic structure of the random variable $\varepsilon_{t+1}^i$.

The last ingredient which we need to solve the parent's maximisation problem, and find the equation for the accumulation of human capital, is a functional form for equation 4. We assume that it is:

$$H_{t+1}^i = Z \left(X_t^i\right)^\alpha e^{\tilde{\omega}_{t+1}^i}; \quad \tilde{\omega}_{t+1}^i \sim N \left(-\frac{V_\omega}{2}, V_\omega\right) \tag{6}$$

where $Z$ is a constant akin to total factor productivity, and $\tilde{\omega}_{t+1}^i$ is an iid shock normally distributed with a mean such that $E\left(e^{\tilde{\omega}_{t+1}^i}\right) = 1$. We assume that $\alpha \in (0,1)$. Notice that an agent may have very high human capital either because her parents invested a large amount in her (large $X_t^i$), or because she is very good at creating human capital (high $\tilde{\omega}_{t+1}^i$). This second route to excellence may be thought as luck or talent. Our only imposition is that it is not inheritable.

The following result[5] characterizes the optimal investment decisions of dynasties:

**Result 1.** *The solution of the maximization problem in equation 1 requires that investment in education is a fixed proportion of the individual's income: $X_t^i = \lambda Y_t^i$, with:*

$$\lambda = \frac{\gamma_2}{1 + \delta - \gamma_1} \tag{7}$$

*The value function of agents is $W(Y) = A + B \ln Y$, with*

$$A = \frac{\bar{\varepsilon} + \gamma_0 + \ln\left[(1+\delta) - (\gamma_1 + \gamma_2)\right]^{[(1+\delta) - (\gamma_1 + \gamma_2)]} + \ln \frac{\gamma_2^{\gamma_2}}{[(1+\delta) - \gamma_1]^{[(1+\delta) - \gamma_1]}}}{[(1+\delta) - (\gamma_1 + \gamma_2)]^{\frac{\delta}{1+\delta}}} \tag{8}$$

$$B = \frac{(1+\delta)}{(1+\delta) - (\gamma_1 + \gamma_2)} \tag{9}$$

Since parents invest a fixed fraction of their income, $\lambda$, in their child's education, it follows that, in absolute terms, the rich invest more heavily than the poor. Since this is a known feature of the economy, the children of the rich will be perceived to have more human capital than the children of the poor when human capital is unobservable. This leads to statistical discrimination in favour of the children of the rich.

Notice also that as a consequence of investment being a fixed proportion of income (result 1) and human capital having a constant elasticity to investment (equation 6), it follows that:[6]

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[5]Proof in appendix A
[6]Proof in appendix B

**Result 2.** *Log human capital is a linear function of log parental income. The elasticity is an exogenous parameter $\alpha$, while the constant depends on the investment rate $\lambda$:*

$$h_{t+1}^i = \ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha y_t^i + \omega_{t+1}^i \tag{10}$$

*where $\omega_{t+1}^i \sim N\left(0, V_\omega\right)$ is iid noise and $h_{t+1}^i$ and $y_t^i$ represent the log of the child's human capital and the log of parental income respectively.*

This is the equation for the accumulation of human capital. Human capital depends positively on parental income and the fraction of income which parents invest in their children. This equation is known to firms but the level of human capital of a particular worker is not, nor is the parental income of any particular worker (at least not perfectly).

# 5   Wage Determination

We now turn to how the wage of an individual is determined.

Agents can only produce output when working within firms. These firms are competitive in the labor and product markets. The only input is human capital, which produces output on a one-to-one basis.[7]

Our most salient assumption is that firms cannot observe the productivity of the workers. They observe it with noise along with other characteristics of the agent. We call $\Omega_{t+1}^i$ the set of information that the market has on agent $i$. We discuss the composition of this set below. So far we just want to notice that it is public information referring to worker $i$. All firms have the same beliefs on any particular worker. Thus, heterogeneity resides in how different workers are, not in how they are perceived by society.

Competition ensures that firms will make zero expected profits in equilibrium and that the wage of a worker with observable characteristics $\Omega_{t+1}^i$ equals the conditional expectation of her productivity:

$$Y_{t+1}^i = E\left[\exp\left\{h_{t+1}^i\right\} | \Omega_{t+1}^i\right] \tag{11}$$

## 5.1   Information Available to Firms.

The set of information available to firms when determining the wage of worker $i$ is composed of four elements, two specific to the agent, and two summarizing the state of the economy at time

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[7] This view of production omits a role for misallocation in the sense that the productivity of a worker is unaffected by the degree of error in firms' beliefs about their human capital. This is done for simplicity. Presumably though, if firms' beliefs about each worker's human capital were very different to the reality, agents would be assigned to the wrong task and would not realise their full productive potential. In appendix L we consider a production function which includes an allocative cost. This would seem to be more realistic, but the added complexity does not change any of the results presented in the body of the paper.

$t$:

$$\Omega^i_{t+1} = \left\{ a^i_{t+1}, m^i_{t+1}, \mu_{y_t}, V_{y_t} \right\} \tag{12}$$

$$a^i_{t+1} = y^i_t + \varepsilon^{ia}_{t+1}; \quad \varepsilon^{ia}_{t+1} \sim N(0, V_a) \tag{13}$$

$$m^i_{t+1} = h^i_{t+1} + \varepsilon^{im}_{t+1}; \quad \varepsilon^{im}_{t+1} \sim N(0, V_m) \tag{14}$$

where $a^i_{t+1}$ is a public signal on the parental income of agent $i$, and $m^i_{t+1}$ is a public signal on her human capital. $\varepsilon^{ia}_{t+1}$ and $\varepsilon^{im}_{t+1}$ are iid noise shocks, and $\mu_{y_t}$ and $V_{y_t}$ are the mean and variance of the distribution of log-income in the parents' generation. In addition, all firms know how human capital relates to parental income (equation 10).

The two most important parameters for our model are $V_a$ and $V_m$. They determine the amount of information that firms receive on the background of the agents ($V_a$) and their productive ability itself ($V_m$).

$a^i_{t+1}$ is the signal on background, and $V_a$ determines how much information this signal contains. If $V_a$ is very low, firms know with near certainty the income of the parents, and thus how much was invested in the education of the agent. Notice that if $V_a$ is larger, the degree of uncertainty that they have on the amount of education that the agent received depends on the income distribution of the parents: the more inequality among the parents, the more uncertain are the priors and, consequently, the posteriors. This fact will have important consequences.

$m^i_{t+1}$ is the signal on the productivity of the agent, and $V_m$ determines how much *direct* information the market receives on it – how much firms know about the productivity of a worker *independently* of her background and circumstances. It is the amount of "hard" information available, unaffected by prejudice of any sort.

If $V_m$ is very low, the firm knows almost exactly what the productivity of the worker is and she will be paid accordingly *independently of her background*. The firm will pay the productivity and it does not matter if she has a high $h$ because the investment of the parents was high or because she was particularly lucky in the drawing of $\tilde{\omega}$ in equation 6.

If $V_m$ is larger, the firm cares more about the background of the agent as it has predictive value on $h$. In this sense we call *"meritocracy"* the precision of the signal $m^i_t$. If it is high, only the productive ability of agents matters for their wage. If it is low, firms are interested in the background of the agent, which will affect her wage.

### 5.1.1 On critical events

Notice that our model is a model of the first years of the productive life of individuals. We are implicitly assuming that during those years there are critical events in the personal history of agents that determine their income thereafter. It is not an outlandish assumption, as there is a substantial body of evidence suggesting that the first jobs have long lasting effects on her working life.

There is direct evidence that early job stability has persistent wage effects (Neumark (2002)) and initial wage has persistent effects (Oreopoulos et al. (2012), Kahn (2010) or Oyer (2006)). Moreover, Guvenen et al. (2017) show that the first years of labor experience account for an extraordinarily large share of the total variance of lifetime income of individuals (and that it is

the change in what happens over these early years which explains the trend in lifetime income dispersion).

In order to rationalize these facts, a literature (starting with Mincer (1962)) has studied the mechanisms of human capital accumulation and promotion, indicating that tracks are critical in the wage process. Thus, Baker et al. (1994a) and Baker et al. (1994b) (see also Gibbons & Waldman (2006)) show that initial tracks do matter. There are different tracks of jobs, and if you start in a low wage track you tend to be promoted within the track, not towards high wage tracks. As a result, the income differentials across workers with the same tenure are somewhat persistent. Likewise, promotions have persistent effects: if you get promoted first, your wage tends to be persistently higher. The rationale behind this behavior lies in the process of human capital accumulation: you learn things specific to your track which make occupational change costly. Thus, for instance, Pavan (2011) shows that wage growth is critically affected by career and firm tenure, Kambourov & Manovskii (2009) and Sullivan (2010) show that occupation-specific considerations are an important component of human capital, while Gathmann & Schonberg (2010) and Poletaev & Robinson (2008) stress the importance of task-specific considerations. Cortes & Gallipoli (2016) show (and rationalize) that there are large costs of switching occupations. Under this reading our story should be interpreted as a shortcut to modeling the decisions of choosing the first occupation from a menu of careers with persistent wage differences.

University assignment is another obvious way in which agents have permanent shocks on their lifetime incomes in their youth as a consequence of the perceptions that society has on their abilities. It is easy to imagine a model where universities strive to get good students (as they will eventually increase the university's reputation, for instance) while facing imperfect information on their ability. Merit and statistical discrimination would conflict in the same manner as in our model. In the end, only so many students are admitted to MIT, which results in a lifetime income bonus even for those of them who are not as capable as the team dealing with admissions thought they were. In general, the process of admission to educational institutions is, in all certainty, a critical part of the manner in which *"meritocracy"* affects society. In some societies which elementary school you attend affects, in and of itself, the high school you will attend, which itself determines the University you attend, and the prospects in life you will face. Insofar as the process of admission at any of these levels is affected by parental income, *even if by the indirect channel of statistical discrimination*, a process akin to the one we model will appear.

It is by no means infeasible to model such a process, or to include career and occupation-specific considerations. We have opted not to do so. First, because it would enormously complicate the model without adding particular insight in the dimensions in which our theory is novel. Second, because we would be unable to solve it analytically, our arguments losing some of their sharpness. In any case, one could think of our model as a metaphor for those structures, and we plan to explicitly include them in future work.

## 5.2 Updating Beliefs

Firms process $\Omega^i_{t+1}$ in order to generate rational beliefs on agent $i$'s human capital, but before knowing the specific values of the signals of any agent firms have a prior on the distribution of human capital. This prior is generated by (i) the rule by which families invest in their children (equation 10), including the distribution of "luck" in learning ($\omega$) and (ii) the distribution of income among parents. Clearly, if $y_t$ is normally distributed, the prior on $h^i_{t+1}$ will also be normally distributed, and the larger the variance of parental income, the less precise the prior of firms on the human capital of their children.

Once they receive the signals $a^i_{t+1}$ and $m^i_{t+1}$ on a specific agent $i$, the market will update its prior using Bayes rule. We summarize the outcome of this process in the following result:[8]

**Result 3.** *Given the process of human capital accumulation in equation 10 and the information set given by equation 12, the posterior of the log of human capital is given by:*

$$h^i_{t+1}|\Omega^i_{t+1} \sim N\left(\mu_{h^i_{t+1}|\Omega^i_{t+1}}, V_{h^i_{t+1}|\Omega^i_{t+1}}\right) \tag{15}$$

*with*

$$\mu_{h^i_{t+1}|\Omega^i_{t+1}} = \beta_{m_{t+1}} m^i_{t+1} + \left(1 - \beta_{m_{t+1}}\right)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha\left\{\beta_{a_{t+1}} a^i_{t+1} + \left(1 - \beta_{a_{t+1}}\right)\mu_{y_t}\right\}\right] \tag{16}$$

$$V_{h^i_{t+1}|\Omega^i_{t+1}} = \beta_{m_{t+1}} V_m \tag{17}$$

$$\beta_{a_{t+1}} = \frac{V_{y_t}}{V_{y_t} + V_a} \tag{18}$$

$$\beta_{m_{t+1}} = \frac{\alpha^2 \beta_{a_{t+1}} V_a + V_\omega}{\alpha^2 \beta_{a_{t+1}} V_a + V_\omega + V_m} \tag{19}$$

To understand the result it is easiest to first focus on expression 18. $\beta_{a_{t+1}}$ is the weight given to the signal on parental income when the firm updates its prior on the parental income of agent $i$ (determined by $\mu_{y_t}$ and $V_{y_t}$) with the information received in signal $a^i_t$. The posterior on log parental income is: $\left\{\beta_{a_{t+1}} a^i_{t+1} + \left(1 - \beta_{a_{t+1}}\right)\mu_{y_t}\right\}$. Notice that the variance of the *posterior* on log parental income is $(1 - \beta_{a_{t+1}})V_y$ which is smaller than the unconditional variance $V_{y_t}$.

$\beta_{m_{t+1}}$ (equation 19) is the weight given to the signal $m^i_t$ in the firm's posterior on log human capital. The expression $\left[\alpha^2 \beta_{a_{t+1}} V_a + V_\omega\right]$ is the variance of $h^i_{t+1}$ conditional on $a^i_{t+1}$. The variance of the posterior on log human capital is given by equation 17. Notice that $\beta_{m_{t+1}} V_m$ can also be written $\left(1 - \beta_{m_{t+1}}\right)\left[\alpha^2 \beta_{a_{t+1}} V_a + V_\omega\right]$, indicating that the variance of the posterior on log human capital is smaller than the variance of $h^i_{t+1}$ conditional on $a^i_{t+1}$.

The expected value of the productivity of agent $i$ (equation 16) is a convex combination of the realization of the direct signal on productivity ($m^i_t$) and the expected log human capital for an individual with log parental income equal to $\left\{\beta_{a_{t+1}} a^i_{t+1} + \left(1 - \beta_{a_{t+1}}\right)\mu_{y_t}\right\}$, the posterior belief on log parental income given $a^i_{t+1}$. Notice that if the signal on human capital is very accurate ($V_m \to 0$), then $\beta_{m_{t+1}}$ equals one, and income equals human capital exactly, independently of its distribution within the population and independently of the background of the individual.

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[8]Proof in appendix C

But in any other case preconceptions matter for wage determination: the larger is the variance of the merit signal, the larger is the weight given to the "prior" on log human capital. That is, the larger is the weight given to the *average* human capital that you would expect from an individual with the perceived background of the person being evaluated. Thus, $V_m$ is indeed a measure of how much background matters.

We now map these beliefs on $h^i_{t+1}$ into wages. Notice that firms care about expected productivity, not its logarithm (which is normally distributed). We take care of this in the following result:[9]

**Result 4.** *The logarithm of the wage of individual i with signals $a^i_{t+1}$ and $m^i_{t+1}$ is:*

$$y^i_{t+1} = \left(1 - \beta_{m_{t+1}}\right) \left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha \left(1 - \beta_{a_{t+1}}\right) \mu_{y_t}\right] + \frac{\beta_{m_{t+1}} V_m}{2}$$
$$+ \alpha \beta_{a_{t+1}} \left(1 - \beta_{m_{t+1}}\right) a^i_{t+1} + \beta_{m_{t+1}} m^i_{t+1} \tag{20}$$

Thus, log income is a linear function with three components:

- A constant that depends on the technology of human capital acquisition, how much the average person invests in education (remember that $\lambda$ is the share of income invested and $\mu_{y_t}$ the average of the log of income) and a factor that depends on the ex-post variance of the distribution of human capital.[10]

- A term that depends on the information that society has on her specific background ($a^i_{t+1}$). We will call $\hat{\beta}_{a_{t+1}} = \alpha \beta_{a_{t+1}} \left(1 - \beta_{m_{t+1}}\right)$ the "value" that society gives to background.

- A term that depends on the direct ("objective") information that society has on her productive ability ($m^i_{t+1}$).

Large values of $\beta_{m_{t+1}}$ imply that society rewards highly the observed objective measures of productive ability. Large values of $\hat{\beta}_{a_{t+1}}$, on the other hand, imply that society rewards highly the perception of a favored upbringing *even controlling* for the objective measures of ability. Of course, both are endogenous. They depend not only on the precision of both signals, but also on the distribution of income (and thus investment).

# 6 Steady State

We turn to the determination of these weights next, but before doing so it is convenient to rewrite equation 20 as a stochastic income process: its Becker-Tomes representation. Noticing that $a^i_{t+1}$ and $m^i_{t+1}$ are both stochastic functions of $y^i_t$ we can write the law of motion of the log

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[9]Proof in appendix D

[10] This is a property of the variance of the log normal: the larger the variance of the logarithm, the exponentially larger is the expectation. It is the variance of the *posterior* of the distribution of log human capital that is relevant here, not its unconditional distribution. Notice that the variance of the posterior is $\beta_{m_{t+1}} V_m$. A property of the normal distribution is that this variance does not depend on the realization of any of the signals. This would not be true with an arbitrary distribution function.

of income:

$$y^i_{t+1} = \ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha \left(1 - \beta_{a_{t+1}}\right)\left(1 - \beta_{m_{t+1}}\right)\mu_{y_t} + \frac{\beta_{m_{t+1}} V_m}{2}$$
$$+ \alpha\left[\beta_{a_{t+1}}\left(1 - \beta_{m_{t+1}}\right) + \beta_{m_{t+1}}\right]y^i_t + \alpha\beta_a\left(1 - \beta_{m_{t+1}}\right)\varepsilon^{ia}_{t+1} + \beta_{m_{t+1}}\left(\varepsilon^{im}_{t+1} + \omega^i_{t+1}\right) \qquad (21)$$

where the intergenerational income elasticity is then $\rho_{t+1} = \alpha\left[\beta_{a_{t+1}}\left(1 - \beta_{m_{t+1}}\right) + \beta_{m_{t+1}}\right]$.

From here, after some algebra, we can determine the law of motion of the variance of log income.

$$V_{y_{t+1}} = \alpha^2\left[\beta_{a_{t+1}}\left(1 - \beta_{m_{t+1}}\right) + \beta_{m_{t+1}}\right]V_{y_t} + \beta_{m_{t+1}} V_\omega \qquad (22)$$
$$\beta_{a_{t+1}} = \frac{V_{y_t}}{V_{y_t} + V_a}\,; \qquad \beta_{m_{t+1}} = \frac{\alpha^2\beta_{a_{t+1}} V_a + V_\omega}{\alpha^2\beta_{a_{t+1}} V_a + V_\omega + V_m}$$

This system of differential equations completely describes the dynamics of the model. Notice that $V_{y_t}$ is not stochastic, given initial conditions, as there is no aggregate risk.

Before describing the properties of the steady state of the system we find it useful to make two observations.

1. *In our model there is no role for self-fulfilling expectations. Expectations never drive the dynamics.*

   This sets our model apart form the bulk of the literature on race- or gender-based statistical discrimination. In that literature the observed characteristic has no intrinsic value as there is nothing inherently good or bad in belonging to a given race or gender. Nevertheless, the fact that a characteristic is observable may make it acquire informative value: if everybody *expects* people of a certain race or gender to act in a certain way (investing little in education, for instance), it might be optimal to privately act in accordance with such an expectation (you will invest little in education if people expect you to have little education and it is your race, not your education, that is observed).

   Multiple equilibria are natural in such an environment, as the informative content of an observable characteristic depends on how people are expected to act and those expectations feed back into actions. Other expectations would lead to other actions, and those actions could feed those different expectations. Most obviously: the race or gender that is expected to have lower education could be changed and nothing of substance would be altered.

   Our model is very different in this respect because it is objectively good to be the child of rich parents, and it is objectively good to have high productive ability. There is no way of sustaining an equilibrium where low income parents invest in their children as much as rich parents do, as the marginal utility of consumption of poor parents is larger. It is easy then to see that the market will always discriminate against the children from deprived backgrounds. If it did not, the rich would still invest more in their children than the poor, so it would be irrational not to discriminate. Likewise, an individual with a large value of $m_t$ is always going to be paid more than another that differs only in having a lower value of $m_t$. More productivity is more productivity, and there is no other conceivable way of interpreting it.

2. *There is a positive feedback mechanism by which inequality fosters further inequality.*

On the one hand, the more inequality there is (large $V_{y_t}$), the more heterogeneous agents are in their productive ability, and the more uncertainty firms face. Consequently, the market more heavily rewards indications of productive ability, both the direct ones ($m_t^i$), and the suggestive ones via parental investment ($a_t^i$). $\beta_{a_{t+1}}$ and $\beta_{m_{t+1}}$ are increasing in $V_{y_t}$. On the other hand, the larger the value given to the signals ($\beta_{m_{t+1}}$ and $\beta_{a_{t+1}}$), the larger the amount of inequality the next generation will endure, as the differences among agents are more salient.

Thus, the more that society values the observable differences between agents, the more inequality it creates, and because of that, the more that it values the observable differences between agents. Or, looking at it from the other side, inequality fosters further inequality via increasing the extent to which society differentiates among its members. This positive feedback mechanism is ingrained in the process of statistical discrimination. It implies a multiplier effect: the effect of any exogenous change in parameters which leads to an increase in inequality will be amplified.[11]

Thus, our model does not allow for the existence of different sets of strategies and beliefs which could be mutually compatible for a single set of state variables: equilibrium is unique. Nevertheless, the existence of the positive feedback mechanism generates the possibility of multiple path dependent steady states. This happens if different values of the state variables lead you to different steady states in the long run, but there is no possibility of moving form one of those steady states once the economy has settled in it.

In our model multiple steady states arise if the elasticity of human capital to investment is large enough. If $\alpha \geq 1$ there are three steady states, of which two are stable. Of these one has infinite variance, with $\rho$ being not smaller than one and huge responses to $a$ and $m$. The other stable equilibria is qualitatively identical to the one that we present for $\alpha < 1$. We prefer to restrict the parameter space to ensure that this possibility does not arise. There are three reasons for this: (1) because we do not know how to interpret infinite variance , (2) because the restriction necessary ($\alpha < 1$) is eminently reasonable , and (3) because the steady state that results is equivalent with the reasonable (i.e. finite variance) stable steady state if the restriction does not apply. From now on we always assume that $\alpha < 1$.

We characterize the solution of this system of differential equations in the next result.[12]

**Result 5.** *If $\alpha < 1$ equations* (18), (19) *and* (22) *define a system of differential equations with a unique steady state which is globally stable. In the steady state log income is normally distributed*

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[11]Proof in appendix E

[12]Proof in appendix F.

*with variance being characterized by the (unique) solution to the following system of equations:*

$$V_y = \frac{\beta_m V_\omega}{1 - \alpha^2 \left[\beta_a \left(1 - \beta_m\right) + \beta_m\right]} \tag{23}$$

$$\beta_a = \frac{V_y}{V_y + V_a} \tag{24}$$

$$\beta_m = \frac{\alpha^2 \ \beta_a \ V_a + V_\omega}{\alpha^2 \ \beta_a \ V_a + V_\omega + V_m} \tag{25}$$

Unfortunately, the explicit solution to this system of equations is extremely cumbersome and uninformative. We can instead solve analytically the comparative statics on the relevant exogenous variables. That is, we look at how the steady state values of the endogenous variables ($V_y$, $\beta_a$ and $\beta_m$) move as a consequence of exogenous changes in $V_a$ and $V_m$.

# 7 Comparative Statics Exercises

The following section lays out the main theoretical results of our paper.

## 7.1 An Exogenous Increase in the Amount of Information on Background

A decrease of $V_a$ means that the market will have more accurate information on the background of agents. This is not good news for equality. Being better able to differentiate who received more education, the market will be more inclined to discriminate in their favor, providing greater advantages to those from affluent backgrounds. But how much the market chooses to discriminate is a function of the degree of inequality in the economy, and this is a state variable whose path is determined endogenously.

In appendix G we prove the following result, which characterizes the effects of $V_a$

**Result 6.** *An increase in the accuracy of the signal on background (a decrease of $V_a$) results, in steady state, in more inequality, greater persistence of income across generations, more discrimination based on perceptions of the background of an agent, and a smaller elasticity of income to the signal on ability:*

$$\frac{dV_y}{dV_a} < 0; \quad \frac{d\rho}{dV_a} < 0; \quad \frac{d\beta_a}{dV_a} < 0; \quad \frac{d\hat{\beta}_a}{dV_a} < 0; \quad \frac{d\beta_m}{dV_a} > 0 \tag{26}$$

More accurate information on the background of an individual increases the attention that firms give to this signal, increasing the persistence of income across generations and its variance across individuals.

Notice that this result is far from obvious: Decreasing the amount of noise in the economy (i.e., increasing the information that agents have) *increases* the dispersion of incomes. You reduce noise, but as result you increase dispersion.

The reason lies in the increase in the persistency of the income process. Better information on the background of the individuals allows firms to discriminate more accurately between agents,

directly favoring those from better backgrounds. A more persistent income process is bound to have a larger unconditional variance. Thus, inequality increases, which itself increases even further the value of the information on background via the positive feedback mechanism discussed above.

Notice also that the effect on the human capital signal $(\beta_m)$ is the opposite. Better information on background results in a *lower* elasticity of income to the ability of individuals. This might look surprising, given that inequality is larger. More inequality implies that firms are less aware on the abilities of any specific worker, and thus, one could have expected that firms would give more attention to the meritocratic signal of human capital $(m_t^i)$. They do not, and the reason is that, albeit the unconditional variance of income increases, the variance of log income *conditional on the signal $a_t^i$ decreases*. Thus, the dispersion of human capital *conditional on $a_t^i$* decreases, and there is less demand for the meritocratic signal. There is a crowding-out effect, background replacing merit in the determination of one's income and, consequently, a profoundly antipathetic decrease of intergenerational mobility.

## 7.2 An Exogenous Increase in the Degree of Meritocracy

Next we want to consider the effect of exogenously reducing $V_m$. A reduction in $V_m$, all else equal, improves the quality of the human capital signal, providing greater advantages to those with greater human capital. With all the caveats discussed in section 2, we consider it reasonable to say that a society with a lower value of $V_m$ is more *"meritocratic"*.

The first thing to notice is that the same feedback mechanism that is very obvious for information on background, acts in the same manner for the information on merit. The easiest way to see this is by assuming that there is no signal on background $(V_a \to \infty)$. In such a case the law of motion (22) becomes:[13]

$$V_{y_{t+1}} = \alpha^2 \beta_{m_{t+1}} V_{y_t} + \beta_{m_{t+1}} V_\omega; \qquad \beta_{m_{t+1}} = \frac{\alpha^2 V_{y_t} + V_\omega}{\alpha^2 V_{y_t} + V_\omega + V_m} \tag{27}$$

More information on merit (lower $V_m$) induces the market to rely more heavily on such information, increasing $\beta_{m_{t+1}}$. This necessarily increases the spread of incomes, as the differences between agents become more salient. Finally, the increase in inequality makes firms less sure of the human capital of their workers in the following generation, increasing the value that they assign to any information on merit, increasing $\beta_m$ further.

It is easy to see that the unique steady state of (27) is the unique solution to:

$$V_y = \frac{\beta_m V_\omega}{1 - \alpha^2 \beta_m} \tag{28}$$

$$\beta_m = \frac{\alpha^2 V_y + V_\omega}{\alpha^2 V_y + V_\omega + V_m} \tag{29}$$

and that the steady state values of $V_h$, $V_y$ and $\beta_m$ are all decreasing in $V_m$. The intergenerational income elasticity (now equal to $\alpha \beta_m$) increases as $V_m$ is reduced.

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[13]Notice that as $V_a$ approaches infinity, then $\beta_{a_{t+1}} \to 0$ and $\beta_{a_{t+1}} V_a \to V_{y_t}$

Contrary to what could be thought, meritocracy does not increase the degree of intergenerational mobility. It decreases it. More information on people's ability is bound to decrease intergenerational mobility because ability and background are correlated and, by increasing income dispersion, meritocracy *increases the value of any existing information on people's ability.* The children of the rich, being on average more productive than the children of the poor, benefit from the increase in the accuracy of information on merit, leading to more persistent income shocks and further inequality. Meritocracy has very much the same effects as an increase in the information available on background.

It is now easier to consider the effect of an exogenous improvement in the quality of the human capital signal when both signals are available and useful to the firm (i.e. $V_a$ is finite). We do so in the following result.[14]

**Result 7.** *An increase in the accuracy of the signal on ability (a decrease of $V_m$) results, in steady state, in more inequality, greater persistence of income across generations, a larger elasticity of income to the signal on ability and more weight given to the signal on background when evaluating an agent's parental income (which is what $\beta_a$ measures):*

$$\frac{dV_y}{dV_m} < 0; \quad \frac{d\rho}{dV_m} < 0; \quad \frac{d\beta_m}{dV_m} < 0; \quad \frac{d\beta_a}{dV_m} < 0 \tag{30}$$

*Moreover, given a set of values for $\alpha \in (0,1)$ and $V_a \in R^+$ $(V_a < \infty)$, there exists a variance of the error in the signal on ability, $\hat{V}_m$, such that $\left(0 < \hat{V}_m < \infty\right)$*

$$\text{If } V_m < \hat{V}_m, \text{ then } \frac{d\hat{\beta}_a}{dV_m} > 0 \tag{31}$$

$$\text{If } V_m > \hat{V}_m, \text{ then } \frac{d\hat{\beta}_a}{dV_m} < 0 \tag{32}$$

*The value of $\hat{\beta}_a$, the weight given to the signal on background when evaluating an agent's human capital, is maximal if $V_m = \hat{V}_m$.*

If society is better endowed to judge its members' merit, it is doomed to increase the dispersion of their incomes (paying more to those judged to be better). This increased dispersion has effects on both the value assigned to merit, $\beta_m$, and the value assigned to "advantages", $\hat{\beta}_a$.

First, it increases the dispersion of the abilities of the children, thus feeding back into increased underlying uncertainty and the value of the signal on human capital in the following period. Thus, not surprisingly, better information on human capital increases the market value of that signal.

The effects on the weight given to background when determining income $(\hat{\beta}_a)$ are more complicated. First of all, there is a "crowding-out effect" in the opposite direction to that in result 6. Better information on human capital makes you place less weight on background as merit replaces inherited advantages in the determination of human capital. This is clear from the fact that $\beta_m$ enters negatively in $\hat{\beta}_a = \alpha \beta_a (1 - \beta_m)$. However, there is an effect on the other direction too: as income variance increases, the signal on background becomes more valuable in

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[14]Proof in appendix H

judging parental income. This increase in $\beta_a$ acts in the opposite direction to the increase in $\beta_m$. The net effect on $\hat{\beta}_a$ depends on the relative size of the effects on $\beta_a$ and $\beta_m$.

We can understand the net effect by doing the following mental exercise. Imagine that $V_m$ were very low (and thus, firms have good information on ability). In that case $\beta_m$ would be very large (close to one), and the effect of the increase of $\beta_a$ would be very small $\left( \frac{\partial \hat{\beta}_a}{\partial \beta_a} = \alpha \left( 1 - \beta_m \right) \right)$. The net effect of a decrease in $V_m$ would be an increase in the market value of the human capital signal, but a decrease in the value of the parental income one. There would be a trade-off between meritocracy and advantages as the crowding-out effect dominates.

Now imagine the polar opposite case where $V_m$ is very large. In such a case $\beta_m$ would be close to 0 and background information would play the dominant role in the determination of human capital. Any increase in the quality of information on ability would increase the market value of both signals because, in this case, the effect of an increase in $\beta_a$ on $\hat{\beta}_a$ is relatively large.

In any case, notice that the degree of intergenerational mobility *always* decreases whenever the society becomes more meritocratic as a consequence of a decrease of $V_m$. This occurs both when there is a trade-off (and advantages become less important) or when there is not. This is a consequence of inheritance. The talented become richer, and thus incomes are bound to become more dispersed. This increased dispersion of incomes is going to be translated into a further dispersion of abilities as the rich invest more in their children. Abilities then, being better evaluated, translate into more income for the children of the rich *even if it is perfectly possible that firms care less about the background of agents.*

This is one of the main insights of our paper. Meritocracy in and of itself is not going to increase intergenerational mobility or decrease the prevalence of inheritance. And this is bound to happen even if an increase in meritocracy does produce a decrease in the advantages associated with being from a good background, which is by no means a foregone conclusion.

This is not to say that meritocracy is a bad thing. In the next section we show that it increases the share of income invested in human capital. The significance of our result is to notice that there are several roads that lead to countries having low intergenerational mobility and high inequality: one is the "aristocratic" route, where the children of the rich benefit from positive statistical discrimination as the rich are known to invest more heavily in their children's education than the poor; but a very different road leading to the same mobility and inequality is the meritocratic one. If the aspects of reality that one focuses on are limited to the degree of mobility and inequality, meritocracy and the weight of background are almost equivalent. Both of them produce a negative correlation between inequality and mobility, reproducing the observed data. Thus, just looking at the data we cannot say if a society is more or less meritocratic. For doing so we need to find a variable that reacts differently to advantages and to meritocracy. We do so in the next section.

# 8    Equilibrium

The log-normal structure of the model has allowed us to do the rather unusual exercise of solving for the comparative statics in steady state of a set of the endogenous variables ($V_y$, $\beta_a$ and $\beta_m$) without solving the complete model. This facilitates our analysis enormously by making the

pricing decisions independent of the share of investment in education ($\lambda$), insofar as all parents invest the same share of their income, which we saw in section 4 was the case. Now we solve for the complete equilibrium, including investment in human capital.

An equilibrium consists of (i) a rule for parents' investment behavior and (ii) an income determination process such that:

1. The investment behavior of parents is optimal given the income determination process.

2. The income determination process is such that the wage of each worker equals her expected productivity given the information available on the worker ($\Omega_t^i$) *and* the investment behavior of parents.

In section 4 we saw that if parents believe that the income of their children is going to be determined by $Y_{t+1}^i = e^{\gamma_0} (Y_t^i)^{\gamma_1} (X_t^i)^{\gamma_2} e^{\varepsilon_{t+1}^i}$ (for some values of $\gamma_0$, $\gamma_1$, $\gamma_2$ and stochastic process $\varepsilon_{t+1}^i$), then they choose optimally to invest a fixed proportion of their income $\lambda = \frac{\gamma_2}{1+\delta-\gamma_1}$.

In section 5 we saw what the stochastic process of income is if parents invest a fixed proportion $\lambda$ of their income in their children.

Thus, in order to prove that we have located an equilibrium it remains to be shown that there exists values of $\gamma_0$, $\gamma_1$, $\gamma_2$ and a well defined stochastic process $\varepsilon_{t+1}^i$ such that equation (5) is a representation of the stochastic process of income defined by equation (20) in result 4 for the values of $V_y$, $\beta_a$ and $\beta_m$ obtained in result 5. We do so in the following result.[15]

**Result 8.** *Equilibrium. The equilibrium stochastic process of income as a function of parental income and investment is $Y_{t+1}^i = e^{\gamma_0} (Y_t^i)^{\gamma_1} (X_t^i)^{\gamma_2} e^{\varepsilon_{t+1}^i}$ with:*

$$\gamma_0 = \ln Z + \alpha (1 - \beta_m) \left[ (1 - \beta_a) \mu_y + \ln \lambda \right] + \frac{1}{2} \left[ \beta_m V_m - V_\omega \right] \tag{33}$$

$$\gamma_1 = \alpha \beta_a (1 - \beta_m) \tag{34}$$

$$\gamma_2 = \alpha \beta_m \tag{35}$$

$$\varepsilon_{t+1}^i = \alpha \beta_a (1 - \beta_m) \varepsilon_{t+1}^{ai} + \beta_m \left( \omega_{t+1}^i + \varepsilon_{t+1}^{mi} \right) \tag{36}$$

*and, consequently, the equilibrium share of income invested in children's education is:*

$$\lambda = \frac{\alpha \beta_m}{1 + \delta - \alpha \beta_a (1 - \beta_m)} \tag{37}$$

The elasticities of income with respect to parental income and investment are $\alpha \beta_a (1 - \beta_m)$ and $\alpha \beta_m$ respectively. The portion of talent which is not derived from parental income ($\omega_t^i$) plays a larger role in the determination of income when $\beta_m$ is higher, implying it has a more substantial impact in more meritocratic societies.

From results 5 and 8 we derive the following corollary, which describes the equilibrium.

**Result 9.** *If $\alpha < 1$ there exists an unique steady state. In steady state the values of $V_y$, $\beta_a$ and $\beta_m$ are the unique solution to:*

$$V_y = \frac{\beta_m V_\omega}{1 - \alpha^2 \left[ \beta_a (1 - \beta_m) + \beta_m \right]}; \qquad \beta_a = \frac{V_y}{V_y + V_a}; \qquad \beta_m = \frac{\alpha^2 \beta_a V_a + V_\omega}{\alpha^2 \beta_a V_a + V_\omega + V_m}$$

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[15]Proof in appendix I.

*The intergenerational correlation of income equals*

$$\rho = \alpha \left[ \beta_a \left( 1 - \beta_m \right) + \beta_m \right]$$

*and the share of income invested in human capital is in all families identical, and equals*

$$\lambda = \frac{\alpha \beta_m}{1 + \delta - \alpha \beta_a \left( 1 - \beta_m \right)}$$

Given that we have already seen the comparative statics of $V_y$, $\rho$, $\beta_m$ and $\hat{\beta}_a$ with respect to $V_m$ and $V_a$, all that remains to be shown is how the investment rate $\lambda$ responds to those variables.

We start with $V_m$. It is quite intuitive that if firms are very well informed about the productivity of workers, parents will want to invest a large share of their income in their children's education. This is because, in that case, children's income necessarily depends on their productivity and not on other considerations that could be inferred from their background. Consequently, the return to investment in education becomes large: it translates readily into future income.

Thus, the following result is not surprising:[16]

**Result 10.** *An increase in the accuracy of the human capital signal (a decrease of $V_m$) results, in steady state, in an increase in the proportion of income invested in education.*

Notice that in our context parents want to educate their children only insofar as the market values how productive they look. Thus, educational investment is very sensitive to $\beta_m$. If $\beta_m$ is very low, you will not invest in your children not because it does not raise the human capital of your child, nor because it does not raise their human capital signal, but because the market does not value that signal. The low rewards to the human capital signal mean you would rather eat the resources.

This is a strong force by which parents respond to the greater rewards to education arising from a more accurate human capital signal. This force works in exactly the opposite way in response to an increase in the accuracy of the information available on background. As we saw in result 7, a decrease of $V_a$ produces a steady state decrease in $\beta_m$, decreasing the return to investing in education.

There is, though, an additional force at play which complicates the analysis following a decrease of $V_a$: insofar as parental investment still raises agent's incomes, it can be used to generate advantages for more distant generations of the family through the perception that richer parents provide. Specifically, by raising the income of children, educational investment raises the advantages of grandchildren, greatgrandchildren and so on. This occurs through the statistical discrimination on background which firms use to determine wages. Thus, in societies where information on background is more readily available, and perceptions on background are more heavily rewarded by firms, this creates an incentive to invest *more*.

This complicates the analysis of the response of investment to an increase in accuracy of the signal on background, which we summarize in the following result [17]

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[16]Proof in appendix J.
[17]Proof in appendix J.

**Result 11.** *An increase in the accuracy of the signal on background (a decrease of $V_a$) may increase or decrease investment. Moreover, for a value of $V_m$ low enough, it necessarily decreases it.*

Thus, $V_m$ and $V_a$ have different effects on $\lambda$. In the next section we will use these differences to identify the degree of *meritocracy* and the prevalence of advantages in different societies.

# 9 A Numerical Illustration

The existing data on on inequality and intergenerational mobility is often interpreted in public discussion in terms of meritocracy and the prevalence of advantages. Thus, when a society is shown to have relatively low intergenerational mobility it is often understood to be *not meritocratic*, at least in comparison to another society with more intergenerational mobility. Our model contradicts such a view, our main insight being that, given the complexities of the transmission of inheritance, one should be cautious when extrapolating from the existing data on intergenerational mobility and inequality to the causes of the patterns seen in this data.

Thus, while the motivation and the main contribution of this the paper is eminently theoretical, it is interesting to see how the model interprets such data. We have seen that two societies showing similar levels of inequality and mobility could have arrived at such a point via different routes. For instance, Italy and the US have similar levels of mobility and inequality, but it is perfectly possible that one may have a high degree of meritocracy and low prevalence of advantages, while the other may be just the reverse.

Moreover, in the light of the model it is perfectly possible that a society has less intergenerational mobility than another while paying individuals much more according to the objective measures of their productivity and less as a function of their background. Thus in principle, the US could have a higher intergenerational correlation of income than Sweden, while still being more meritocratic.

The objective of this section is to look at how our model maps the existing data into the degree of meritocracy and the prevalence of advantages of different societies. It is by no mean an empirical "test" of the model, and we do not want to read the results in a literal sense. All models are "wrong", in the sense of being a simplification, and when dealing with complex issues (such as inheritance and advantages), in order to be comprehensible, they need to abstract from many issues that are undoubtedly relevant.

Nevertheless, a model is better than no model. The naive reading of the data that is normally made (less mobility equals less meritocracy and more prevalence of advantages) implies a much bigger oversimplification, if not a logical fault. Consequently, the natural next step is to measure the degree of meritocracy and the prevalence of advantages that our model suggests exist in different societies.

## 9.1 Procedure

Obviously, there exists no direct data on $\beta_m$ or $\hat{\beta}_a$, much less on $V_a$ and $V_m$. All that is available is data on intergenerational income correlations, $\rho$, inequality, $V_y$, and human capital investment

rates, $\lambda$. Our goal is to use this data in order to reveal (for any society $j$) the values of $V_a^j$ and $V_m^j$ (and thus $\beta_m^j$ and $\hat{\beta}_a^j$) that would generate values for $\rho^j$, $V_y^j$, and $\lambda^j$ within the model that are the closest possible to the data values. To clarify notation, from now on we will include an index $j \in \{1, ..., J\}$, indicating the society (country) to which we are referring.

We assume that societies differ in the amount of information available for firms to evaluate workers' productivity $(V_m^j)$, and background $(V_a^j)$.[18] The other three parameters in the model $(\alpha, V_\omega$ and $\delta)$ are assumed to take the same value across all societies.

The discount rate $\delta$ is assumed to be 1% annually in our central exercise.[19] We also look at other values for $\delta$ in our sensitivity analysis in Appendix O. It is difficult to postulate, a priori, numerical values for the elasticity of human capital accumulation to educational investment $(\alpha)$ or the variance in the process of exogenous shocks in the acquisition of human capital $(V_\omega)$. Therefore we calibrate these parameters within a very comprehensive and internally coherent data set, due to Chetty et al. (2014), which describes variation across US commuting zones. This exercise is described in Appendix M.[20]

We obtain values of $\alpha = 0.409$ and $V_\omega = 1.038$ for use in our central exercise. We also look at other values for $\alpha$ and $V_\omega$ in our sensitivity analysis in Appendix O.

Location specific values for $V_m^j$ and $V_a^j$, together with the common values of $\alpha$, $\delta$ and $V_w$, generate model values for the variables for which we have counterparts in the data. Specifically, within the model we get the degree of intergenerational mobility $\rho^j$, the degree of inequality $V_y^j$, and the investment in education $\lambda^j$. Our goal is to find the values of $V_a^j$ and $V_m^j$, for all countries $j \in \{1, ..., J\}$, such that the model values of these variables $(V_y^j, \rho^j$ and $\lambda^j, \forall j)$ are the closest possible to their data counterparts $(\hat{V}_y^j, \hat{\rho}^j$ and $\hat{\lambda}^j, \forall j)$.

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[18] In addition to $V_a^j$ and $V_m^j$ societies could differ in the process of human capital accumulation. In our model these differences would be captured in the intercept $Z$ in equation 6. In principle we could also calibrate $Z^j$ for each country $j$, and include average income as another target, but notice that such extension would be irrelevant. The reason is that differences of productivity in the human capital accumulation function would map one to one into differences in average income, and not affect the rest of model variables at all. We would match average income exactly, because we have an extra variable for each country dedicated to that task. Given that this variable does not affect our variables of interest this seems a fairly futile exercise and we ignore cross country heterogeneity in the human capital accumulation process.

[19] The model discounts across periods using a factor equal to $1/(1 + \delta)$. We assume a period is a generation, or approximately 30 years, and so we want $\delta$ such that $1/(1 + \delta) = (1/1.01)^{30}$ i.e. we use $\delta = 0.348$ to be equivalent to a 1% annual discount rate over a period lasting 30 years.

[20] We use the Chetty et al. (2014) data across the approximately 700 US commuting zones to produce estimates for $\alpha$ and $V_\omega$ rather than doing this within the international data set, with 15 observations, for two reasons. Firstly, it is clear that $700 >> 15$ and so better estimates are likely. And secondly, the US commuting zone data is likely much more internally consistent: for example, the share of income invested in education between two commuting zones is likely to measure (however imperfectly) precisely the same conceptual object; conversely, the share of income invested in education between two countries may measure different objects as different statistical agencies are involved in the primary collection of this data and different definitions may be used. As well as using this exercise to produce our central estimates for $\alpha$ and $V_\omega$, it also produces estimates of the precision of the signals on merit and advantages across US commuting zones. These can be mapped and correlated with observable commuting zone characteristics (also from the Chetty et al. (2014) data). The results of this are shown in Appendix M and we believe it is an interesting exercise. Across US commuting Zones the prevalence of merit seems to correlate well with having a large service sector and large foreign communities, while the fraction of African-Americans and racial segregation correlates well with the prevalence of inherited advantages. We do not include this analysis and these results in the main body of the paper as this puts too much emphasis upon empirical applications, which the model is not really capable of supporting (due to its omission of public education and redistribution). The rhetorical point, that a reasonable model can allow high degrees of meritocracy to co-exist with low levels of mobility, and that indeed our model reads the data for certain countries in this way, is best made using the international data.
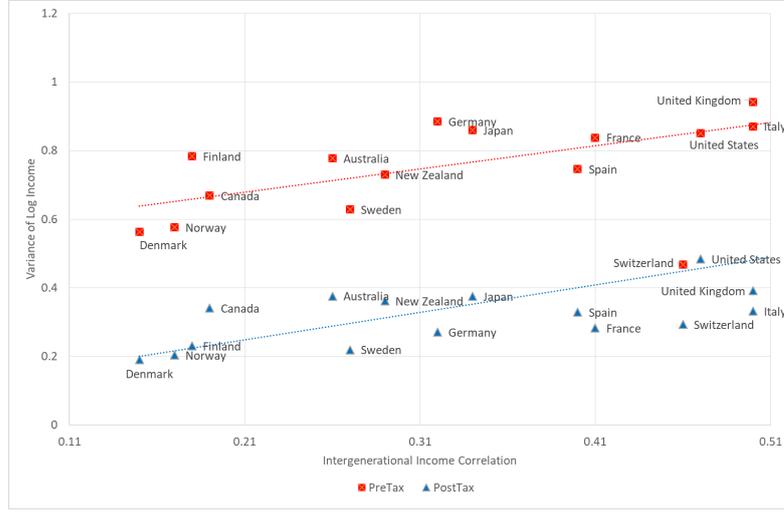
Figure 1: The Great Gatsby Curve

We look for the values of $\{V_m^j, V_a^j\}$ $\forall j$ that minimize the following objective function:

$$
\min_{\{V_m^j, V_a^j\}_{\forall j}} L = \left\{ \begin{array}{l} \left(1 - Corr\left[\hat{\rho}, \rho\right]\right)^2 + \left(1 - Corr\left[\hat{V}_y, V_y\right]\right)^2 + \left(1 - Corr\left[\hat{\lambda}, \lambda\right]\right)^2 \\ + \left(\ln \bar{\rho}^D - \ln \bar{\rho}^M\right)^2 + \left(\ln \bar{V}_y^D - \ln \bar{V}_y^M\right)^2 \end{array} \right\}
$$

where the elements of the first line of the objective function are (one minus) the correlation across countries between the model moments and their data counterparts. Thus, for instance, $Corr\left[\hat{\rho}^j, \rho^j\right]$ is the correlation across all countries between the model generated values for $\rho$ (as a consequence of $\{V_m^j, V_a^j\}_{\forall j}$) and their data counterparts $\hat{\rho}$. The second line of the loss function is the square of the difference between the model and data values of the means of $\rho$ and $V_y$ across all countries.

We maximize the correlations, instead of minimizing the square errors, of the moments because in this way we normalize the magnitudes of the three variables, giving them effectively equal weight. The loss is convex in the square of the correlation mistake, indicating that we prefer to equate the loss across the three moments. Notice that this does not fix either the mean or the variance of the moments. That is, the mean and variance of the moments of the data could be vastly different than the ones in the model, while the correlation could be high. In order to avoid this we also account (in the second line) for the deviations of the mean of $\rho$ and $V_y$ from the ones observed in the data.

Note two things. First, we could also account for deviations in the variances. We choose not to do this in order to have some untargeted moments so as to check how good the fit is. Second, we do not include the mean or the variance of $\lambda$ because, as we discuss later, we believe there are strong reasons why the data we have for this is only indicative of the variation in this variable, and says little about its level.

The optimization algorithm is explained in detail in appendix N.

## 9.2   International Data

The discussion on the pre-eminence of inheritance and the possible end of the American dream has focused a substantial amount of attention in the so-called "Great Gatsby Curve".[21] This is a relationship between the degree of inequality and a measure of the intergenerational persistence of income across countries, which we depict in figure 1. This positive correlation has been documented across countries (Corak (2013)), across US commuting zones (Chetty et al. (2014)) and across Italian provinces (Güell et al. (2015)), among others. The existence of such a relationship across locations within a country, which clearly share the same institutional and redistributive environment, is indicative that this relationship is not caused only by differing levels of redistribution (as exist across countries).

The data on the intergenerational correlation of income between parents and children is from Corak (2013). In Figure 1 we depict its relationship both with the degree of inequality before (red squares in the plot) and after (blue triangles) taxes and redistribution.

This data has received much attention, and the relationship indicates that societies showing less inequality are more likely to show more mobility, which is not in itself particularly surprising, and has been extensively discussed in the literature[22]. What did get the public attention was the fact that the US is at the dismal extreme of the curve: the US has a very low degree of mobility and a large degree of inequality irrespective of how inequality is measured. When looking at pre-tax income inequality, the US is very much in the same place as Italy and much worse than, for instance, the Scandinavian countries. Moreover, when looking at post-tax and redistribution inequality the relationship is unaltered (more inequality, less mobility) while the US remains at the dismal extreme. This has been read as indicating that the US is no longer the land of opportunity, but one where merit is secondary to privilege.

It is naturally appealing to check whether our model reads the data as indicating that the prevalence of merit within the US is relatively large or small. From the point of view of our theory, this data could imply the usual, naive interpretation in which meritocracy is equivalent to mobility (i.e., relatively low values of $\beta_m$ in the US), or just the opposite. We look at the degree of meritocracy and the prevalence of advantages that our model suggests that could have generated the observed data across the different countries. As we shall see, countries that look similar on the "Great Gatsby Curve" can be very different in meritocracy-advantage space. In particular, some countries with very low mobility appear to be highly meritocratic.

As well as using the Corak (2013) values for the intergenerational correlation of income between parents and sons across countries, and OECD data on the distribution of income, we also use OECD data on the level of educational investment. In order to abstract from issues that might depend on the stage of development, we opt to do our exercise only with developed countries.[23] Our data set then consists of 15 countries.

We need to take into account the fact that in the model we have abstracted from institutional

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[21]The origin of the name is obscure and difficult to ascertain. The relationship became prominent following Krueger (2012), though it was based on previous work by Corak (2013).

[22]As mentioned above Hassler et al. (2007) and Solon (2004), for instance, discussed the theoretical relationship between inequality and mobility before this was documented by Corak

[23]We use the intersection of the sets of countries which appear in the Corak (2013) data, and in the OECD data. The only country at a different stage of development which exists in this intersection is Chile, which is then excluded.

| Country | $\rho$ | $V_y$ | | Education share of GDP | |
|---|---|---|---|---|---|
| | | PreTax | PostTax | TotalEd | PrivateEd |
| Australia | 0.26 | 0.777494 | 0.375999 | 5.51% | 1.34% |
| Canada | 0.19 | 0.669317 | 0.340614 | 5.77% | 0.70% |
| Denmark | 0.15 | 0.563195 | 0.190069 | 8.05% | 1.49% |
| Finland | 0.18 | 0.783977 | 0.230952 | 5.60% | 0.40% |
| France | 0.41 | 0.837894 | 0.282429 | 6.07% | 0.63% |
| Germany | 0.32 | 0.885219 | 0.2708 | 4.87% | 0.22% |
| Italy | 0.50 | 0.87078 | 0.332033 | 5.12% | 0.82% |
| Japan | 0.34 | 0.859455 | 0.375552 | 5.91% | 1.93% |
| New Zealand | 0.29 | 0.730179 | 0.362101 | 6.51% | 1.18% |
| Norway | 0.17 | 0.576651 | 0.203958 | 5.16% | 0.09% |
| Spain | 0.40 | 0.746395 | 0.328532 | 4.98% | 0.59% |
| Sweden | 0.27 | 0.628584 | 0.21964 | 5.96% | 0.00% |
| Switzerland | 0.46 | 0.467887 | 0.292865 | 5.23% | 0.08% |
| United Kingdom | 0.50 | 0.941469 | 0.391376 | 5.38% | 1.08% |
| United States | 0.47 | 0.851498 | 0.484567 | 7.00% | 1.47% |

Table 1: International Data

differences across countries. In particular we have abstracted from (i) taxation and redistribution, and (ii) procurement of public education. Both of these are of obvious importance, and both affect the quantitative implications of the model. The only reason for not incorporating these in the model is that it would have impeded us in obtaining analytic results. Including these features while preserving the log-linear nature of the model (which enables analytical tractability), is a complicated task that is part of our future research on this topic. For the present paper, instead of developing such an extended theory, we opt to adapt the data to our model. That is, we fit the model to (1) both the distribution of income before and after taxes, and to (2) both the investment in private education and in total education (i.e. both private plus public education spending). The qualitative results are robust to any of the four possible data configurations. Given that the intention of our exercise is not to provide a numerical value of the degree of "meritocracy", but rather to show that our way to looking at the problem can be potentially important, we believe that this should suffice to convince the reader. Notice, in any case, that the relationship between mobility and inequality seems to be essentially the same irrespectively of whether you look at income before or after taxes and redistribution.

In table 1 we present our data. The Corak (2013) data on mobility ($\rho$) is defined as specified in the model and we use it as it is. We map the GINI coefficient of the distribution of income (provided by the OECD) into the variance of log income under the assumption that income is lognormally distributed.[24] The OECD data allows us to do this for the distributions of income both before and after taxes and redistribution.

In order to proxy for human capital investment we use either the total educational spending, or private educational spending, divided by GDP (all figures from the OECD). However, spending on human capital accumulation for children is larger, perhaps substantially larger, than what is spent in formal schooling. It includes, for instance, the spending on real-estate incurred in order to enjoy the externalities generated from the presence of high income students in the

[24]Specifically, we invert the relationship: $GINI = 2\Phi\left(\frac{\sigma_y}{\sqrt{2}}\right) - 1$. See Aitchison & Brown (1963)

neighborhood, or the spending on extra-curricular activities (from violin lessons to trips to museums). We have no way of including data on all these activities. It is reasonable to expect, nevertheless, that total spending will be related to formal education spending. Thus, we use the variation in this spending but do not fit to the level of this spending.[25]

Thus, we have four data configurations: (pre-tax income, total educational spending), (post-tax income, total educational spending), (pre-tax income, private educational spending) and (post-tax income, private educational spending). In each of them we apply the same calibration procedure for selecting $(V_m^j, V_a^j)$, $\forall j$, but we do not adjust $\alpha$ or $V_\omega$.

## 9.3 Results

In figure 2 we plot the implied rewards to merit and advantage (in panel *(a)*), and the implied precision[26] of both signals (in panel *(b)*) using the income distribution before taxes and the total spending on education.

As can be seen from these charts, there are countries for which the *unconditional* effect of background on income is high, while the effect *conditional* on the individual's observed abilities is very low. For example, we know that parental background matters a lot in the US relative to other countries because that is precisely what it means for mobility to be low. Moreover, in the numerical exercise the imputed precision of the signal on background is among the highest. Nevertheless, the implied reward of having a good background $\hat{\beta}_a$ is among the lowest. The reason is that the implied precision of the direct objective signal on the productive ability of individuals is so much higher in the US than anywhere else. This results in very large rewards to merit $\beta_m$, and a very low rewards to background, $\hat{\beta}_a$. The unconditional effect of background is large, but the effect *conditional* on ability is very small.

This is perhaps our most important insight: rewards to merit may be very large whilst there is a low degree of mobility. Large rewards to merit will cause low mobility and high inequality. Moreover, that is how the model reads the situation in the US.

Italy and the US look very similar in the before tax Great Gatsby curve, but they are very far apart in the maps of imputed rewards and precisions generated using the same before tax data. The reason is that they are very different in the educational effort that their citizens make. The US invests in education much more than the average country, while Italy is unremarkable in this respect. The model interprets this as indicating that the return to educational investment must be perceived as larger in the US, which is what happens if the precision of the merit signal is larger in the US.

---

[25]More precisely, we define a variable $k^j$ for each country $j$ equal to the standardized (i.e. zero mean and unit variance) educational spend. We then define $\hat{\lambda}^j = \mu + \sigma k^j$ as our proxy to the values of the data moment, and our empirical counterpart to $\lambda^j$. $\mu$ and $\sigma$ are chosen by the calibration routine in order to equalize the mean and variance of the "empirical" value of $\hat{\lambda}^j$ with its model counterpart. Our goal in the calibration is to maximize the correlation between empirical and model variables across locations, and therefore the inclusion of $\mu$ and $\sigma$ is innocuous except for the mostly aesthetic effect of equalizing the distribution of empirical and model moments: this does not affect the correlation across countries. We consider the true, total, human capital investment effort, $\lambda$, to be a linear transformation of the data, with the values of the transformation ($\mu$ and $\sigma$) being obtained from the numerical fit to the data. Notice again that $\mu$ or $\sigma$ only adjust the mean and variance of $\lambda$ across countries between model and data, but their value does not affect the correlations, and in this respect they are there for aesthetic value.

[26]Where the precision of a signal $I \in \{A, M\}$, $P_I \equiv -\ln V_I$, is used to make the graph more readable.
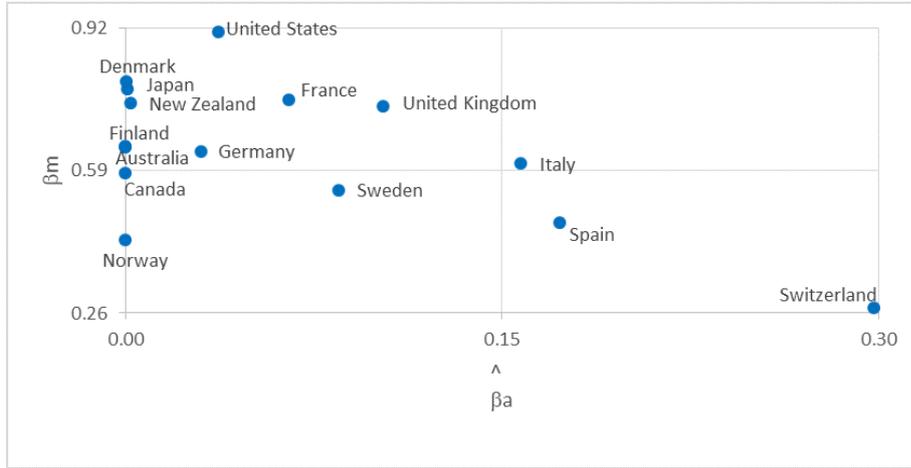
(a) Rewards to merit and background $\beta_m^j, \hat{\beta}_a^j$
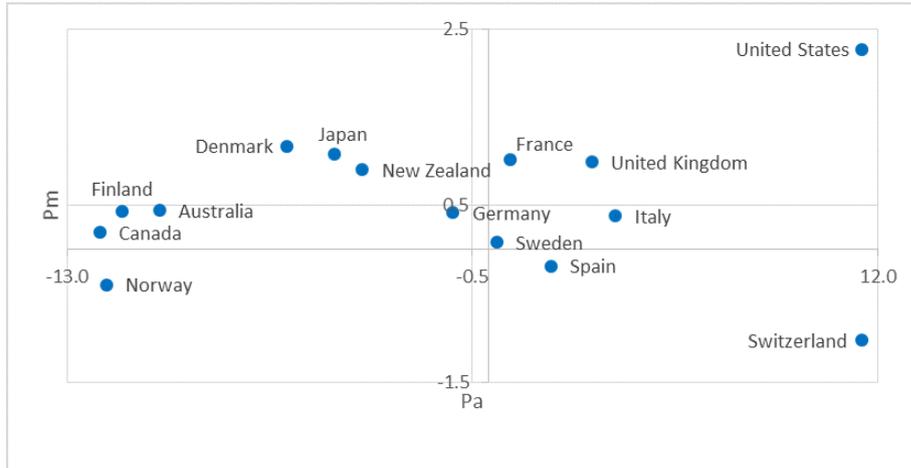


(b) Precisions of signals on merit and background $P_m^j, P_a^j$

Figure 2: Implied Rewards and Precisions of Merit and Background signals for all countries for the income distribution before taxes and total educational investment.

This is what differentiates the US: it invests a lot in education (see in table 1). Two countries which are very close on the Great Gatsby curve may have arrived at this point via very different routes. In particular, the model suggests that more accurate objective information on the ability of workers, larger rewards to merit, and less reward to background *conditional* on merit, is seen in countries that invest more in education. This generates the larger demand for educational investment.

Note however that the model is not a map from educational investment into the quality of the signal on workers' ability. This is very clear when comparing the US and Denmark in the numerical exercise. Denmark is the only country with a larger educational effort than the US. In spite of this, the model imputes lower precision on both signals to Denmark, as this is the only way of making the model results compatible with the low inequality and high mobility observed in the data.

In figure 3 we present the result of the same exercise performed under alternative data configurations. Qualitatively the same results arise, and although the behavior of the US is somewhat less radical, it is always among the countries with the highest precision on the merit signal, and its largest reward. With respect to Italy, the US always puts considerably more value on merit, and there is less prevalence of background conditional on merit. With respect to Denmark, the US always has larger values of the precision of both signals and larger rewards to both signals.

The main point of this section is that, when confronted with data, the suggestions of the paper seem at least plausible. A naive interpretation of intergenerational mobility as the degree of "meritocracy" may be a dangerous oversimplification. The US might still be a land of opportunities *conditional* on having the right ability. Of course, those abilities are to a large extent inherited, and that is the reason why the intergenerational persistence of income is so large. But notice that the reason why they are so inheritable might be precisely that merit is highly rewarded; that, in a deep sense, the US is a very meritocratic society.

# 10  Summary, Conclusion and Further Research

Our main contribution is perhaps to remark that "merit" – understood as rewarding individuals by their productive ability – is not at all in contradiction to having very low intergenerational mobility. Very much the opposite.

Measures of intergenerational mobility do not control for the productive ability of individuals. They simply measure how much children's outcomes are explained by those of their parents. Parental outcomes may be strongly correlated with those of their children when not conditioning on productive ability, but the partial correlation may be very small when conditioning on ability. Obviously, in such a case productive ability is what is inherited from the parents.

It is perhaps not surprising that in the model of statistical discrimination that we have presented, societies endowed with a better ability to judge people's background are bound to have larger inequality and lower mobility than otherwise. This is because more information facilitates discrimination favoring those with more privileged backgrounds.

It is more surprisingly that an increase in the ability of society to judge any individual's
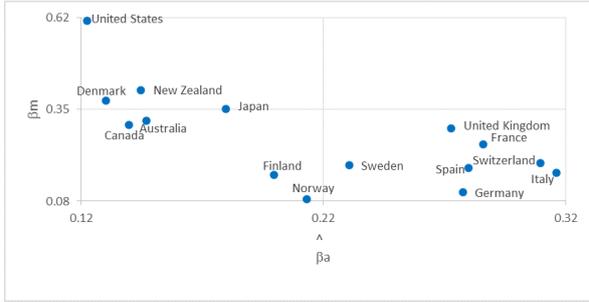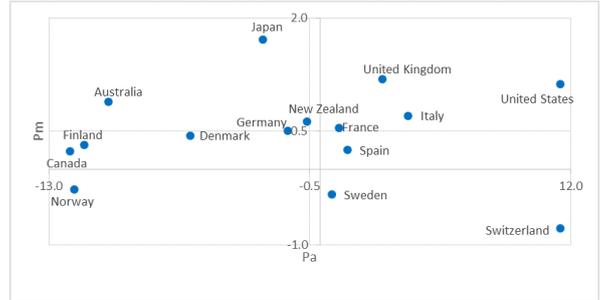
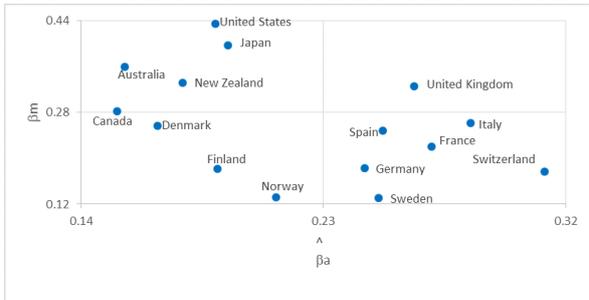(a) $\beta_m$, $\hat{\beta}_a$. After taxes. Total education.
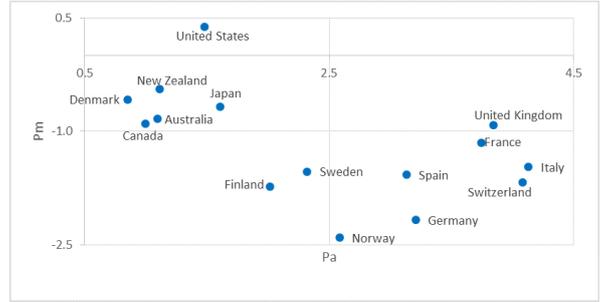
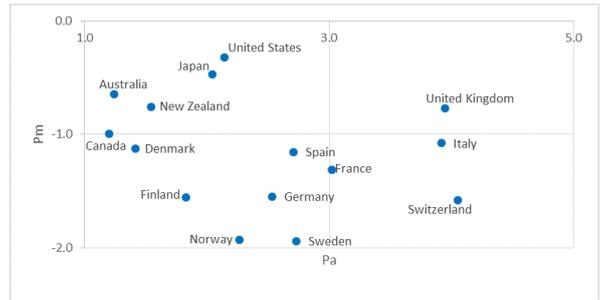(b) $P_m$, $P_a$. After taxes. Total education.

(c) $\beta_m$, $\hat{\beta}_a$. Before taxes. Private Education.

(d) $P_m$, $P_a$. Before Taxes. Private Education.

(e) $\beta_m$, $\hat{\beta}_a$. After taxes. Private education.

(f) $P_m$, $P_a$. After taxes. Private education.

Figure 3: Implied Rewards and Precisions across countries for other data configurations.

productivity also translates into larger differences in the income of agents and lower mobility.

Larger inequality translates across generations for two reasons: (1) human capital investment decisions transform larger differences in parents into larger differences in their children's abilities; and (2) via a feedback mechanism by which the larger the degree of inequality, the more weight society puts on any available information about an individual, generating further inequality and hence further reducing mobility.

Inheritance is ingrained in the process by which inequality translates into lower mobility. It hinges on the fact that more inequality in the income of the parents translates into more inequality in the human capital of their children. Because it does, more access to information not only feeds into more inequality, but also into linkages across generations. Both things (inequality and lack of mobility) positively correlate with each other, but they could be caused either by a prevalence of merit or by a prevalence of inherited advantages.

Nevertheless, there is an important aspect in which the accuracy of the information on merit and background have differential effects: accurate information on human capital increases the return to educational investment. It encourages human capital accumulation because you know that your children are going to be paid according to their productivity, something that you can affect by investing in education. More accurate information on background has a much milder effect on the incentives to invest.

This difference has allowed us to conduct an interesting numerical exercise: seeing how the model reads the data on inequality, intergenerational mobility, and educational investment across countries. Independently of its empirical value, it is a good example of the mechanisms underlying our model.

Conducting this exercise shows that it may very well be the case that in societies where parental income has a very large unconditional effect on a child's income, this same parental income may have a very small effect when controlling for the abilities of the child. Thus, the US shows very little mobility when compared with most other developed countries, while the inferred rewards to merit are among the largest.

Two societies that look similar in the Great Gastby Curve, may look very different in the space defined by the rewards to merit and background, and this may be reflected in the efforts that they make with regard to educational investment. For instance, the US and Italy look very much the same in respect to their levels of inequality and mobility, but the US invests much more in education. Consequently, the model reads the data as saying that merit is much more highly rewarded in the US than in Italy.

A society may have much higher rewards to merit, and lower rewards to advantages, than another that is much more equal and that has much more intergenerational mobility. For instance, the implied prevalence of merit in the US is larger than in Denmark, albeit it is much more unequal and less mobile. This is interesting because Denmark is the one country in the sample that has more investment in education (both public and private) that the US. The reason for getting this ranking is that in order to exhibit the low inequality and high mobility of Denmark, the model cannot reward the human capital signal very highly. When we simulate the model, a consequence of having accurate information on the people's productivity is to be at the dismal end of the Great Gatsby Curve, as the US is in the data.

Of course, many caveats to our model and its implementation have to be made.

We model the first years of active life of individuals in the labor market, and we simply assume that their effects last forever. A more complete model would allow either for task specific learning (which makes initial perceptions have permanent effects), or would allow for path dependence in the human capital accumulation process (e.g. the specific high school attended, or grades achieved, determining the quality of the university attended, itself having permanent labor market effects). We did not include these more complicated mechanisms in our model for simplicity and tractability, but we plan to explore them in further work.

Likewise, we did not include any consideration of public education or redistribution in our analysis. Including them in our model would reduce inequalities in the acquisition of human capital and, by disconnecting human capital from parental income, it would also reduce the persistence of these inequalities across generations. Their introduction would affect the quantitative aspects of the model, which is why we think of our numerical exercise more as an informed example of the workings of the model rather than a measurement exercise. It is natural to think of extensions of the model including both, and any real empirical implementation would need to have both.

Still, notice that including redistribution and public education would not affect any of the points that we made, which are qualitative, not quantitative. Insofar as richer parents invest in their children more than poorer parents, more information (either on background or on merit) will contribute to furthering inequalities and making them more persistent. For our mechanism not to be present, either redistribution should be absurdly large (making everybody equal after taxes) or it should be made impossible for parents to invest in their children's[27] education, which seems equally far fetched.

There are other extensions of the paper that are also naturally appealing. The statistical discrimination structure of the model could be used to model conspicuous consumption: agents could invest in their appearance in order to "look" as though they have good backgrounds. It would also be very interesting to use independent information on individuals' ability (obtained from educational scores or IQ tests) and systematically measure the effect of background on outcomes across countries conditioning and not conditioning on those measures of ability. All these are projects that we plan to develop in further research.

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[27]Including investments in where to live. In the presence of externalities in human capital acquisition, parents will segregate according to income. See for instance the seminal paper by Benabou (1993), and the large literature thereafter.

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# Appendix

## A   Proof of Result 1

*Proof.* We solve the program:

$$W\left(Y_t^i\right) = \max_{X_t^i} \left\{ \ln\left[Y_t^i - X_t^i\right] + \frac{1}{1+\delta} EW\left(e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_t^i}\right) \right\} \tag{38}$$

First we prove that parents invest a fixed percentage of their income in their children:

$$X_t^i = \lambda Y_t^i \tag{39}$$

To do so we guess

$$W\left(Y_t^i\right) = A + B \ln Y_t^i \tag{40}$$

which we will later verify. The Euler equation is:

$$\frac{1}{Y_t^i - X_t^i} = \frac{1}{1+\delta} \frac{\partial EW\left(e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_t^i}\right)}{\partial X_t^i} \tag{41}$$

Since,

$$EW\left(e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_t^i}\right) = A + B\left[\bar{\varepsilon} + \gamma_0 + \gamma_1 \ln Y_t^i + \gamma_2 \ln X_t^i\right] \tag{42}$$

the Euler equation becomes:

$$\frac{1}{Y_t^i - X_t^i} = \frac{1}{1+\delta} B\gamma_2 \frac{1}{X_t^i} \tag{43}$$

implying,

$$X_t^i = \frac{B\gamma_2}{(1+\delta) + B\gamma_2} Y_t^i \tag{44}$$

and

$$C_t^i = \frac{(1+\delta)}{(1+\delta) + B\gamma_2} Y_t^i \tag{45}$$

Now, substituting $X_t^i$ into the expectation we get:

$$EW\left(Y_{t+1}^i | Y_t^i, X_t^i\right) = EW\left(e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_t^i}\right) \tag{46}$$

$$= A + B\left[\bar{\varepsilon} + \gamma_0 + (\gamma_1 + \gamma_2)\ln Y_t^i + \gamma_2 \ln \frac{B\gamma_2}{(1+\delta) + B\gamma_2}\right] \tag{47}$$

and the value function will be:

$$W\left(Y_t^i\right) = \ln \frac{(1+\delta)}{(1+\delta) + B\gamma_2} + \ln Y_t^i + \frac{A}{(1+\delta)} \tag{48}$$

$$+ \frac{B}{(1+\delta)}\left[\bar{\varepsilon} + \gamma_0 + (\gamma_1 + \gamma_2)\ln Y_t^i + \gamma_2 \ln \frac{B\gamma_2}{(1+\delta) + B\gamma_2}\right] \tag{49}$$

So, if the guess is right:

$$A = \ln \frac{(1+\delta)}{(1+\delta) + B\gamma_2} + \frac{A}{(1+\delta)} + \frac{B}{(1+\delta)}\left[\bar{\varepsilon} + \gamma_0 + \gamma_2 \ln \frac{B\gamma_2}{(1+\delta) + B\gamma_2}\right] \tag{50}$$

and

$$B = \frac{B}{(1+\delta)}(\gamma_1 + \gamma_2) + 1 \tag{51}$$

Solving for $B$

$$B = \frac{(1+\delta)}{(1+\delta) - (\gamma_1 + \gamma_2)} \tag{52}$$

which should be positive. We will show that the equilibrium values of $\gamma_1$ and $\gamma_2$ are always such that this happens. Finally, solving for $A$

$$\delta A = (1+\delta)\ln \frac{(1+\delta)}{(1+\delta) + B\gamma_2} + \frac{(1+\delta)}{(1+\delta) - (\gamma_1 + \gamma_2)}\left[\bar{\varepsilon} + \gamma_0 + \gamma_2 \ln \frac{B\gamma_2}{(1+\delta) + B\gamma_2}\right] \tag{53}$$

It is useful to notice that

$$1 - \lambda = \frac{(1+\delta)}{(1+\delta) + B\gamma_2} = \frac{(1+\delta) - (\gamma_1 + \gamma_2)}{(1+\delta) - \gamma_1} = 1 - \frac{\gamma_2}{[(1+\delta) - \gamma_1]} \tag{54}$$

so:

$$\lambda = \frac{\gamma_2}{[(1+\delta) - \gamma_1]} \tag{55}$$

$$\delta A = (1+\delta)\ln\left[1 - \frac{\gamma_2}{[(1+\delta) - \gamma_1]}\right] + \frac{(1+\delta)}{(1+\delta) - (\gamma_1 + \gamma_2)}\left[\bar{\varepsilon} + \gamma_0 + \gamma_2 \ln \frac{\gamma_2}{[(1+\delta) - \gamma_1]}\right] \tag{56}$$

$$A = \frac{\bar{\varepsilon} + \gamma_0 + \ln\left[(1+\delta) - (\gamma_1 + \gamma_2)\right]^{[(1+\delta) - (\gamma_1+\gamma_2)]} + \ln \frac{\gamma_2^{\gamma_2}}{[(1+\delta)-\gamma_1]^{[(1+\delta)-\gamma_1]}}}{[(1+\delta) - (\gamma_1 + \gamma_2)]\frac{\delta}{1+\delta}} \tag{57}$$

$\square$

# B    Proof of Result 2

*Proof.* This follows directly from the human capital accumulation equation 6 and the investment rule, $X_t^i = \lambda Y_t^i$. $\square$

# C    Proof of Result 3

*Proof.* Since $(h_{t+1}^i, a_{t+1}^i, m_{t+1}^i)$ has a multivariate normal distribution, we can appeal to the conditional normal p.d.f. to find the distribution of $h_{t+1}^i | \Omega_{t+1}^i$.

Let $\mathbf{X}$ be a partitioned multivariate normal random vector with $\mathbf{X}^T = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2^T \end{bmatrix}$, where $\mathbf{X}_1 = \begin{bmatrix} h_{t+1}^i \end{bmatrix}$ and $\mathbf{X}_2^T = \begin{bmatrix} a_{t+1}^i & m_{t+1}^i \end{bmatrix}$. The mean of $\mathbf{X}$ is given by:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \tag{58}$$

where $\mu_1 = [\mu_h]$ and $\mu_2^T = [\mu_y \ \mu_h]$.

The variance-covariance matrix of $\mathbf{X}$ is given by:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \tag{59}$$

where:

$$\boldsymbol{\Sigma}_{11} = \left[ \begin{array}{c} V_h \end{array} \right] \tag{60}$$

$$\boldsymbol{\Sigma}_{12} = \left[ \begin{array}{cc} \alpha V_y & V_h \end{array} \right] = \boldsymbol{\Sigma}_{21}^T \tag{61}$$

$$\boldsymbol{\Sigma}_{22} = \left[ \begin{array}{cc} V_y + V_a & \alpha V_y \\ \alpha V_y & V_h + V_m \end{array} \right] \tag{62}$$

Then the distribution of $h_{t+1}^i$ conditional on $\Omega_{t+1}^i$ is univariate normal with the following mean and variance:

$$E\left(h_{t+1}^i | \Omega_{t+1}^i\right) \tag{63}$$

$$= \mu_1 + \boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_{22}^{-1} \left(\mathbf{x}_2 - \mu_2\right) \tag{64}$$

$$= \mu_h + \frac{1}{(V_h + V_m)(V_y + V_a) - \alpha^2 V_y^2} \left[ \begin{array}{cc} \alpha V_y & V_h \end{array} \right] \left[ \begin{array}{cc} V_h + V_m & -\alpha V_y \\ -\alpha V_y & V_y + V_a \end{array} \right] \left[ \begin{array}{c} a_{t+1}^i - \mu_y \\ m_{t+1}^i - \mu_h \end{array} \right] \tag{65}$$

$$= \mu_h + \alpha \beta_a (1 - \beta_m) \left[a_{t+1}^i - \mu_y\right] + \beta_m \left[m_{t+1}^i - \mu_h\right] \tag{66}$$

$$= \beta_m m_{t+1}^i + (1 - \beta_m) \left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha \beta_a a_{t+1}^i + \alpha (1 - \beta_a) \mu_y\right] \tag{67}$$

and,

$$Var\left(h_{t+1}^i | \Omega_{t+1}^i\right) \tag{68}$$

$$= \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21} \tag{69}$$

$$= V_h - \frac{1}{(V_h + V_m)(V_y + V_a) - \alpha^2 V_y^2} \left[ \begin{array}{cc} \alpha V_y & V_h \end{array} \right] \left[ \begin{array}{cc} V_h + V_m & -\alpha V_y \\ -\alpha V_y & V_y + V_a \end{array} \right] \left[ \begin{array}{c} \alpha V_y \\ V_h \end{array} \right] \tag{70}$$

$$= V_h - \alpha^2 V_y \beta_a (1 - \beta_m) - \beta_m V_h \tag{71}$$

$$= \alpha^2 V_y (1 - \beta_m)(1 - \beta_a) + (1 - \beta_m) V_\omega \tag{72}$$

$$= \beta_m V_m \tag{73}$$

$\square$

# D  Proof of Result 4

*Proof.* Given the conditional distribution of the log of human capital is normal, the conditional distribution of human capital is log normal with mean:

$$Y_{t+1}^i = E\left(exp\left\{h_{t+1}^i\right\} | \Omega_{t+1}^i\right) = \exp\left\{\mu_{h_{t+1}^i | \Omega_{t+1}^i}\right\} \exp\left\{\frac{V_{h_{t+1}^i | \Omega_{t+1}^i}}{2}\right\} \tag{74}$$

Substituting and taken logarithms produces expression 20. $\square$

# E  The Multiplier Effect

This appendix proves the existence of the multiplier effect described in section 6 and calculates its magnitude.

*Proof.* Imposing the steady state condition, $V_{y_t} = V_{y_{t-1}} = V_y$ on the law of motion of the variance of log income given in equation 22, gives:

$$V_y = \alpha^2 \left[\beta_a(1 - \beta_m) + \beta_m\right] V_y + \beta_m V_\omega \tag{75}$$

Solving this gives the equation for steady state variance of log income given in result 5. We will call the right-hand side of equation 75: $\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)$.

We can then examine two things: the shift in $\Psi$ from a change in one of the exogenous parameters, keeping $V_y$ fixed; and the total derivative of $V_y$ with respect to the same parameter. The ratio of the latter to the former is the multiplier.

Take $V_a$ as an example. For a given $V_y$, the effect of a change in $V_a$ on the $\Psi$ function is given by:

$$\frac{\partial \Psi}{\partial V_a} \;=\; \frac{\partial \Psi}{\partial \beta_a}\frac{\partial \beta_a}{\partial V_a} + \frac{\partial \Psi}{\partial \beta_m}\left[\frac{\partial \beta_m}{\partial V_a} + \frac{\partial \beta_m}{\partial \beta_a}\frac{\partial \beta_a}{\partial V_a}\right] \tag{76}$$

This is the *partial* effect for a fixed $V_y$. Allowing $V_y$ to fully adjust however, we find the effect of a change in $V_a$ to be:

$$
\begin{aligned}
\frac{dV_y}{dV_a} \;=\;& \frac{\partial \Psi}{\partial V_y}\frac{dV_y}{dV_a} + \frac{\partial \Psi}{\partial \beta_a}\frac{d\beta_a}{dV_a} + \frac{\partial \Psi}{\partial \beta_m}\frac{d\beta_m}{dV_a} \\
=\;& \left[\frac{\partial \Psi}{\partial V_y} + \left[\frac{\partial \Psi}{\partial \beta_a} + \frac{\partial \Psi}{\partial \beta_m}\frac{\partial \beta_m}{\partial \beta_a}\right]\frac{\partial \beta_a}{\partial V_y}\right]\frac{dV_y}{dV_a} + \frac{\partial \Psi}{\partial \beta_a}\frac{\partial \beta_a}{\partial V_a} + \frac{\partial \Psi}{\partial \beta_m}\left[\frac{\partial \beta_m}{\partial V_a} + \frac{\partial \beta_m}{\partial \beta_a}\frac{\partial \beta_a}{\partial V_a}\right] \\
=\;& \frac{1}{1 - \left[\frac{\partial \Psi}{\partial V_y} + \left[\frac{\partial \Psi}{\partial \beta_a} + \frac{\partial \Psi}{\partial \beta_m}\frac{\partial \beta_m}{\partial \beta_a}\right]\frac{\partial \beta_a}{\partial V_y}\right]} \left\{\left[\frac{\partial \Psi}{\partial \beta_a} + \frac{\partial \Psi}{\partial \beta_m}\frac{\partial \beta_m}{\partial \beta_a}\right]\frac{\partial \beta_a}{\partial V_a} + \frac{\partial \Psi}{\partial \beta_m}\frac{\partial \beta_m}{\partial V_a}\right\}
\end{aligned}
\tag{77}
$$

The first term on the right-hand side (i.e the fraction) is the multiplier term. The second term is the shift in the $\Psi$ curve calculated in 76. If the multiplier is greater than 1, it tells us that the total (or long-run) effect on $V_y$ of a change in $V_a$ is larger than the partial effect when $V_y$ is fixed (or the short-run effect, since $V_{y,t-1}$ *is* fixed).

By substitution, the multiplier term is:

$$\frac{1}{1 - \alpha^2\left[1 - (1 - \beta_a)^2 (1 - \beta_m)^2\right]} > 1 \tag{78}$$

Thus any change in $V_a$ is amplified since the additional discrimination which it creates feeds into next period's income variance and the levels of discrimination which individuals in that generation are subjected to.

We can carry out exactly the same exercise for $V_m$, $V_\omega$ and $\alpha$. Although the partial effect of a change in each of the parameters differs, the multiplier is the same in each case. In all cases it is given by equation 78. $\qquad\square$

# F  Proof of Result 5

*Proof.* $\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)$ is defined as in appendix E. It follows that:

$$\frac{d\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y} \tag{79}$$

$$= \frac{\partial \Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{\partial V_y} + \frac{\partial \Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{\partial \beta_a}\frac{d\beta_a}{dV_y} + \frac{\partial \Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{\partial \beta_m}\frac{d\beta_m}{dV_y} \tag{80}$$

$$= \alpha^2\left[\beta_a(1 - \beta_m) + \beta_m\right] + \alpha^2(1 - \beta_m)V_y\frac{d\beta_a}{dV_y} + \left[\alpha^2\beta_a V_a + V_\omega\right]\frac{d\beta_m}{dV_y} \tag{81}$$

Since,

$$\frac{d\beta_a}{dV_y} = \frac{1 - \beta_a}{V_y + V_a} > 0 \tag{82}$$

and,

$$\frac{d\beta_m}{dV_y} = \alpha^2\frac{(1 - \beta_m)(1 - \beta_a)^2}{\alpha^2\beta_a V_a + V_\omega + V_m} > 0 \tag{83}$$
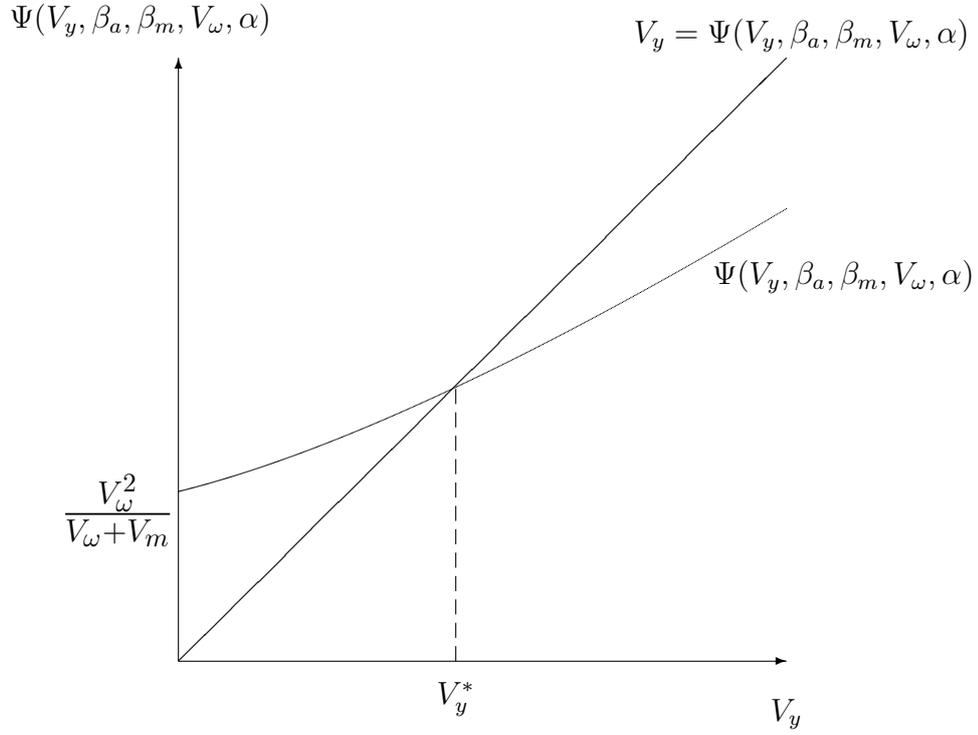
Figure 4: Law of motion of $V_y$

it follows that,

$$\frac{d\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y} = \alpha^2 \left[\beta_a(1 - \beta_m) + \beta_m\right]\{1 + (1 - \beta_m)(1 - \beta_a)\} \tag{84}$$

$$= \alpha\rho \left[1 + (1 - \beta_m)(1 - \beta_a)\right] > 0 \tag{85}$$

while the second derivative is:

$$\frac{d^2\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y^2} = \frac{\partial \left[\frac{d\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y}\right]}{\partial \beta_a}\frac{d\beta_a}{dV_y} + \frac{\partial \left[\frac{d\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y}\right]}{\partial \beta_m}\frac{d\beta_m}{dV_y} \tag{86}$$

$$= 2\alpha^2 (1 - \beta_a)(1 - \beta_m)\left[(1 - \beta_m)\frac{d\beta_a}{dV_y} + (1 - \beta_a)\frac{d\beta_m}{dV_y}\right] > 0 \tag{87}$$

We also know that:

$$\Psi(0, \beta_a, \beta_m, V_\omega, \alpha) = \frac{V_\omega^2}{V_\omega + V_m} \tag{88}$$

$$\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha) \to \infty \quad \text{as } V_y \to \infty \tag{89}$$

$$\frac{d\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)}{dV_y} \to \alpha^2 \quad \text{as } V_y \to \infty \tag{90}$$

Since $\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)$ starts above the 45 degree line, is upward sloping and convex in $V_y$, and has a maximum slope of $\alpha^2 < 1$, it must cut the 45 degree line once from above. This gives the unique, stable, steady state value of $V_y$. The determination of the steady state is illustrated in figure 4.

Note that the shape of the curve implies there will be a multiplier effect from changes in the parameters. Any parameter change which causes a shift in $\Psi(V_y, \beta_a, \beta_m, V_\omega, \alpha)$ will lead to a larger change in $V_y$. This multiplier effect was described in appendix E and will be discussed further when we consider the comparative statics of the model.

□

# G  Proof of Result 6

*Proof.* Notice:

$$\frac{d\beta_a}{dV_a} = \frac{\partial \beta_a}{\partial V_y}\frac{dV_y}{dV_a} + \frac{\partial \beta_a}{\partial V_a} = \frac{1}{V_y + V_a}\left[(1 - \beta_a)\frac{dV_y}{dV_a} - \beta_a\right] \tag{91}$$

and

$$\frac{d\beta_m}{dV_a} = \frac{\partial \beta_m}{\partial \beta_a}\frac{d\beta_a}{dV_a} + \frac{\partial \beta_m}{\partial V_a} = \frac{\alpha^2(1 - \beta_m)}{\alpha^2 \beta_a V_a + V_\omega + V_m}\left[(1 - \beta_a)^2\frac{dV_y}{dV_a} + \beta_a^2\right] \tag{92}$$

So, after some algebra

$$\frac{dV_y}{dV_a} = \frac{-\alpha^2 \beta_a^2 (1 - \beta_m)^2}{1 - \alpha\rho\left[1 + (1 - \beta_m)(1 - \beta_a)\right]} \tag{93}$$

Note also that:

$$1 - \alpha\rho\left[1 + (1 - \beta_m)(1 - \beta_a)\right] = 1 - \alpha^2\left[1 - (1 - \beta_m)^2(1 - \beta_a)^2\right] > 0 \tag{94}$$

Therefore, the effect of a change in $V_a$ on $V_y$ is:

$$\frac{dV_y}{dV_a} = \frac{-\alpha^2 \beta_a^2(1 - \beta_m)^2}{1 - \alpha^2\left[1 - (1 - \beta_m)^2(1 - \beta_a)^2\right]} < 0 \tag{95}$$

The effects of a change in $V_a$ on $\beta_a$ and $\beta_m$ are:

$$\frac{d\beta_a}{dV_a} = -\frac{\beta_a(1 - \beta_a)}{V_a}\left[1 + \frac{\alpha^2 \beta_a(1 - \beta_a)(1 - \beta_m)^2}{(1 - \alpha^2) + \alpha^2(1 - \beta_a)^2(1 - \beta_m)^2}\right] < 0 \tag{96}$$

$$= -\frac{\beta_a(1 - \beta_a)}{V_a}\left[\frac{1 - \alpha^2\left[1 - (1 - \beta_a)(1 - \beta_m)^2\right]}{1 - \alpha^2\left[1 - (1 - \beta_m)^2(1 - \beta_a)^2\right]}\right] \tag{97}$$

$$\frac{d\beta_m}{dV_a} = \frac{\alpha^2 \beta_a^2(1 - \beta_m)^2}{V_m}\left[\frac{1 - \alpha^2}{1 - \alpha^2\left[1 - (1 - \beta_m)^2(1 - \beta_a)^2\right]}\right] > 0 \tag{98}$$

The extent to which firms value and use the signal on background is not measured by $\beta_a$ but by $\hat{\beta}_a = \alpha\beta_a(1 - \beta_m)$. A change in the precision of the advantage signal has the following effect on $\hat{\beta}_a$:

$$\frac{d\hat{\beta}_a}{dV_a} = \alpha(1 - \beta_m)\frac{d\beta_a}{dV_a} - \alpha\beta_a\frac{d\beta_m}{dV_a} < 0 \tag{99}$$

The effect of a change in $V_a$ on $\rho$ is:

$$\frac{d\rho}{dV_a} = \alpha(1 - \beta_m)\frac{d\beta_a}{dV_a} + \alpha(1 - \beta_a)\frac{d\beta_m}{dV_a} \tag{100}$$

$$= -\frac{\alpha\beta_a(1 - \beta_a)(1 - \beta_m)^2}{V_a V_m\left[1 - \alpha^2\left[1 - (1 - \beta_m)^2(1 - \beta_a)^2\right]\right]} \tag{101}$$

$$\cdot\left\{(1 - \alpha^2)(V_\omega + V_m) + \alpha^2(1 - \beta_a)(1 - \beta_m)V_m\right\} < 0 \tag{102}$$

$\square$

# H  Proof of Result 7

*Proof.* Notice that:

$$\frac{d\beta_a}{dV_m} = \frac{\partial \beta_a}{\partial V_y}\frac{dV_y}{dV_m} = \frac{(1 - \beta_a)}{V_y + V_a}\frac{dV_y}{dV_m} \tag{103}$$

and

$$\frac{d\beta_m}{dV_m} = \frac{\partial\beta_m}{\partial\beta_a}\frac{d\beta_a}{dV_m} + \frac{\partial\beta_m}{\partial V_m} \tag{104}$$

$$= \frac{\alpha^2(1-\beta_a)^2(1-\beta_m)}{\alpha^2\beta_a V_a + V_\omega + V_m}\frac{dV_y}{dV_m} - \frac{\beta_m}{\alpha^2\beta_a V_a + V_\omega + V_m} \tag{105}$$

Then,

$$\frac{dV_y}{dV_m} = \frac{-\beta_m^2}{1 - \alpha\rho\ [1 + (1-\beta_m)(1-\beta_a)]} \tag{106}$$

As above:

$$1 - \alpha\rho\ [1 + (1-\beta_m)(1-\beta_a)] = 1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right] \tag{107}$$

Therefore, the effect of a change in $V_m$ on $V_y$ is:

$$\frac{dV_y}{dV_m} = -\frac{\beta_m^2}{1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right]} < 0 \tag{108}$$

The effects of a change in $V_m$ on $\beta_a$ and $\beta_m$ are:

$$\frac{d\beta_a}{dV_m} = -\frac{(1-\beta_a)^2}{V_a}\left[\frac{\beta_m^2}{1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right]}\right] < 0 \tag{109}$$

$$\frac{d\beta_m}{dV_m} = -\frac{\beta_m(1-\beta_m)}{V_m}\left[1 + \frac{\alpha^2\beta_m(1-\beta_m)(1-\beta_a)^2}{1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right]}\right] < 0 \tag{110}$$

The effect of a change in $V_m$ on $\rho$ is:

$$\frac{d\rho}{dV_m} = \alpha(1-\beta_m)\frac{d\beta_a}{dV_m} + \alpha(1-\beta_a)\frac{d\beta_m}{dV_m} \tag{111}$$

$$= -\frac{\alpha\beta_m(1-\beta_a)(1-\beta_m)}{V_a V_m}\left\{V_a + \frac{(1-\beta_a)\beta_m V_m}{\left[1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right]\right]}\right\} < 0 \tag{112}$$

Now turning to the effect on $\hat{\beta}_a$. A change in the precision of the ability signal has the following effect on $\hat{\beta}_a$:

$$\frac{d\hat{\beta}_a}{dV_m} = \alpha(1-\beta_m)\frac{d\beta_a}{dV_m} - \alpha\beta_a\frac{d\beta_m}{dV_m} \tag{113}$$

$$= \frac{\alpha\beta_m(1-\beta_m)}{V_a V_m\left[1 - \alpha^2\left[1 - (1-\beta_m)^2(1-\beta_a)^2\right]\right]}\left\{\beta_a V_a(1-\alpha^2) - V_\omega(1-\beta_m)(1-\beta_a)^2\right\} \tag{114}$$

This is $\lessgtr$ zero if:

$$\beta_a V_a(1-\alpha^2) \lessgtr V_\omega(1-\beta_m)(1-\beta_a)^2 \tag{115}$$

There is a turning point at $\hat{V}_m$ where $\hat{V}_m$ gives values of $\beta_a$ and $\beta_m$ which solve the following equation:

$$\beta_a V_a(1-\alpha^2) = V_\omega(1-\beta_m)(1-\beta_a)^2 \tag{116}$$

The left-hand side of the above equation is decreasing in $V_m$ while the right-hand side is increasing. Therefore $\hat{V}_m$ is a unique turning point and a maximum:

$$\frac{d\left[\beta_a V_a \left(1 - \alpha^2\right)\right]}{dV_m} = V_a \left(1 - \alpha^2\right) \frac{d\beta_a}{dV_m} < 0 \tag{117}$$

$$\frac{d\left[V_\omega \left(1 - \beta_m\right)\left(1 - \beta_a\right)^2\right]}{dV_m} = -V_\omega(1 - \beta_a)\left\{2\left(1 - \beta_m\right)\frac{d\beta_a}{dV_m} + \left(1 - \beta_a\right)\frac{d\beta_m}{dV_m}\right\} > 0 \tag{118}$$

For values of $V_m < \hat{V}_m$, $\beta_a V_a \left(1 - \alpha^2\right) > V_\omega \left(1 - \beta_m\right)\left(1 - \beta_a\right)^2$ and $\frac{d\hat{\beta}_a}{dV_m} > 0$. For values of $V_m > \hat{V}_m$, $\beta_a V_a \left(1 - \alpha^2\right) < V_\omega \left(1 - \beta_m\right)\left(1 - \beta_a\right)^2$ and $\frac{d\hat{\beta}_a}{dV_m} < 0$. $\qquad\square$

# I   Proof of Result 8

*Proof.* From equation 20 we can see that:

$$y_{t+1}^i = \left(1 - \beta_m\right)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha\left(1 - \beta_a\right)\mu_y\right] + \frac{\beta_m V_m}{2} \tag{119}$$

$$+ \alpha\beta_a \left(1 - \beta_m\right) a_{t+1}^i + \beta_m m_{t+1}^i \tag{120}$$

Substituting for $a_t^i$ and $m_t^i$:

$$y_{t+1}^i = \left(1 - \beta_m\right)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha\left(1 - \beta_a\right)\mu_y\right] + \frac{\beta_m V_m}{2} \tag{121}$$

$$+ \alpha\beta_a \left(1 - \beta_m\right)\left(y_t^i + \varepsilon_{t+1}^{ia}\right) + \beta_m \left(h_{t+1}^i + \varepsilon_{t+1}^{im}\right) \tag{122}$$

Equation 6 then allows us to substitute for log human capital:

$$y_{t+1}^i = \left(1 - \beta_m\right)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha\left(1 - \beta_a\right)\mu_y\right] + \frac{\beta_m V_m}{2} \tag{123}$$

$$+ \alpha\beta_a \left(1 - \beta_m\right)\left(y_t^i + \varepsilon_{t+1}^{ia}\right) + \beta_m \left(\ln Z + \alpha \ln X_{t-1}^i - \frac{V_\omega}{2} + \omega_{t+1}^i + \varepsilon_{t+1}^{im}\right) \tag{124}$$

It is then just a matter of rearranging to find $\gamma_0$, $\gamma_1$, $\gamma_2$ and $\varepsilon$

$$y_t^i = \left\{\ln Z + \alpha\left(1 - \beta_m\right)\left[\left(1 - \beta_a\right)\mu_y + \ln \lambda\right] + \frac{1}{2}\left[\beta_m V_m - V_\omega\right]\right\} \tag{125}$$

$$+ \alpha\beta_a \left(1 - \beta_m\right) y_t^i + \alpha\beta_m \ln X_t^i + \alpha\beta_a \left(1 - \beta_m\right)\varepsilon_{t+1}^{ia} + \beta_m \left(\omega_{t+1}^i + \varepsilon_{t+1}^{im}\right) \tag{126}$$

$\lambda$ is found by substitution of $\gamma_1$ and $\gamma_2$ into equation 7. Note that $\gamma_1 + \gamma_2 = \rho < 1 + \delta$ as required to ensure $B$ is positive.

$\qquad\square$

# J   Proof of Results 10 and 11

*Proof.*

$$\lambda = \frac{\beta_m \alpha}{1 + \delta - \left(1 - \beta_m\right)\ \alpha\ \beta_a} \tag{127}$$

The partial derivatives of $\lambda$ with respect to $\beta_m$ and $\beta_a$ are therefore:

$$\frac{\partial \lambda}{\partial \beta_m} = \frac{\alpha\left[\left(1 + \delta\right) - \alpha\beta_a\right]}{\left[1 + \delta - \left(1 - \beta_m\right)\alpha\beta_a\right]^2} > 0 \tag{128}$$

$$\frac{\partial \lambda}{\partial \beta_a} = \frac{\alpha^2 \beta_m \left(1 - \beta_m\right)}{\left[1 + \delta - \left(1 - \beta_m\right)\alpha\beta_a\right]^2} > 0 \tag{129}$$

So:

$$\frac{d\lambda}{dV_m} = \frac{\alpha\left[(1+\delta) - \alpha\beta_a\right]}{\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\frac{d\beta_m}{dV_m} + \frac{\alpha^2\beta_m\left(1-\beta_m\right)}{\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\frac{d\beta_a}{dV_m} < 0 \tag{130}$$

With respect to $V_a$:

$$\frac{d\lambda}{dV_a} = \frac{\alpha\left[(1+\delta) - \alpha\beta_a\right]}{\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\frac{d\beta_m}{dV_a} - \frac{\alpha^2\beta_m\left(1-\beta_m\right)}{\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\frac{d\beta_a}{dV_a} \tag{131}$$

$$= \frac{\alpha^2\beta_a\left(1-\beta_m\right)}{V_a V_m\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\left\{\frac{\alpha(1-\alpha^2)\beta_a V_a(1-\beta_m)\left[(1+\delta) - \alpha\beta_a\right]}{1-\alpha^2\left[1-(1-\beta_m)^2(1-\beta_a)^2\right]}\right\} \tag{132}$$

$$- \frac{\alpha^2\beta_a\left(1-\beta_m\right)}{V_a V_m\left[1+\delta - (1-\beta_m)\alpha\beta_a\right]^2}\left\{\frac{(1-\beta_a)\beta_m V_m\left[1-\alpha^2\left[1-(1-\beta_a)(1-\beta_m)^2\right]\right]}{1-\alpha^2\left[1-(1-\beta_m)^2(1-\beta_a)^2\right]}\right\} \tag{133}$$

This is positive if:

$$\alpha\left[\frac{1-\alpha^2}{1-\alpha^2\left[1-(1-\beta_m)^2(1-\beta_a)^2\right]}\right](1-\beta_m) > \frac{V_m}{V_\omega}\left[\frac{1-\alpha^2\left[\beta_a\left(1-\beta_m\right) + \beta_m\right]}{\left[(1+\delta) - \alpha\beta_a\right]}\right] \tag{134}$$

Note that in the limit, as $V_m \to 0$: $\beta_m = 1$, $\frac{1-\beta_m}{V_m} \to \infty$, and $0 < \beta_a < 1$. The above inequality then holds.

$\square$

# K    Privilege and Meritocracy

In this section we present an extension to the model where the children of the rich get an advantage independently of how productive they are thought to be.

As in the main text, human capital is a function of parental investment in education, and parents invest a fixed percentage of their income in education. Consequently, we can write the process of human capital acquisition as:

$$H_{t+1}^i = A \times \left(Y_t^i\right)^\alpha \times e^{\epsilon_{h,t+1}^i} \quad \to \quad h_{t+1}^i = \ln A + \alpha y_t^i + \epsilon_{t+1}^{ih} \quad ; \quad \epsilon_{t+1}^{ih} \sim N\left(0, V_h\right) \tag{135}$$

Also as in the main text, firms receive a signal on the human capital of agents. But, like in section 7.2 we assume that firms have no signal on the agent's background (i.e., $V_a \to \infty$). Therefore, the information available to the firm when forming an expectation about the agent's human capital is $\Omega_{t+1}^{ip} = \left\{m_{t+1}^i, \mu_{y_t}, V_{y_t}\right\}$, where

$$m_{t+1}^i = h_{t+1}^i + \epsilon_{t+1}^{im} \tag{136}$$

The difference to the main text is that we assume that the pricing mechanism discriminates against the poor. Not statistical discrimination, but raw preference-based discrimination. Firms somehow *prefer* to hire the children of the rich *even when conditioning on what one would rationally think about their human capital*. Formally, an agent with certain observable traits $m_{t+1}^i$ and parental income $Y_t^i$ gets as income:

$$Y_{t+1}^i = \left(E\left(H_{t+1}^i \mid \Omega_{t+1}^{ip}\right)\right)^{1-\pi} \times \left(Y_t^i\right)^\pi \times e^{\epsilon_{t+1}^{iu}} \quad ; \quad \epsilon_{t+1}^{iu} \sim N\left(0, V_u\right) \tag{137}$$

where $\pi$ is a parameter measuring how much *privilege* the children of the rich enjoy. If the elasticity of income to parental income *when conditioning on expected human capital* equals one, the income process is exogenously determined to be a random walk, with maximal correlation between parental and children's income (and unconditional income variance approaching infinity). The polar opposite case is the one that we study in the main text, where $\pi = 0$ and parental income is irrelevant given the expectations on human capital.

Of course parental income could be used to form those expectations (as we do in the main text). The difference now is that there is an advantage above and beyond the news that parental income may

provide on human capital. Firms, thus, are inefficient in the sense of not maximizing profits because they overpay employees who have rich parents. In Becker's parlance they *"prefer"* to employ the children of the rich, and are willing to pay them more for the same expected output. We do not explain how it turns out that these firms exist, and why they are not driven to extinction by competitive firms that do not prefer the children of the rich (like the ones in the main text). Notice that if there was any information on parental income, the firms in the main text would also pay more to the children of the rich, but not beyond their expected human capital.

Given this set up, it is straightforward to notice that if the distribution of log income among the parents generation is $y_t^i \sim N(\mu_{y_t}, V_{y_t})$, the prior (before observing $m_{t+1}^i$) of the log of human capital of a certain individual $i$ is:

$$h_{t+1}^i \sim N(\ln A + \alpha\mu_{y_t}, \alpha^2 V_{y_t} + V_h) \tag{138}$$

and the posterior (after observing a certain realization of $m_{t+1}^i$) is:

$$h_{t+1}^i | \Omega_{t+1}^{ip} \sim N\left(\beta_{t+1} m_{t+1}^i + (1 - \beta_{t+1})(\ln A + \alpha\mu_{y_t}) \quad, \quad \frac{(\alpha^2 V_{y_t} + V_h)\, V_m}{\alpha^2 V_{y_t} + V_h + V_m}\right) \tag{139}$$

with

$$\beta_{t+1} = \frac{\alpha^2 V_{y_t} + V_h}{\alpha^2 V_{y_t} + V_h + V_m} \tag{140}$$

Thus,

$$logE\left(H_{t+1}^i | \Omega_{t+1}^{ip}\right) = \beta_{t+1} m_{t+1}^i + (1 - \beta_{t+1})\left(\ln A + \alpha\mu_{y_t}\right) + \frac{\beta_{t+1} V_m}{2} \tag{141}$$

and

$$y_{t+1}^i = \pi y_t^i + (1 - \pi)\left[\beta_{t+1} m_{t+1}^i + (1 - \beta_{t+1})\left(\ln A + \alpha\mu_{y_t}\right) + \frac{\beta_{t+1} V_m}{2}\right] + \epsilon_{t+1}^{iu} \tag{142}$$

which can be written in a Becker-Tomes form by substituting $m_{t+1}^i$ for its value $h_{t+1}^i + \epsilon_{t+1}^{im}$ (and then $h_{t+1}^i$ by $\ln A + \alpha y_t^i + \epsilon_{t+1}^{ih}$):

$$y_{t+1}^i = (1 - \pi)\left[lnA + \alpha(1 - \beta_{t+1})\mu_{y_t} + \frac{\beta_{t+1} V_m}{2}\right]$$
$$+ \left[\pi + (1 - \pi)\alpha\beta_{t+1}\right] y_t^i + (1 - \pi)\beta_{t+1}(\epsilon_{t+1}^{ih} + \epsilon_{t+1}^{im}) + \epsilon_{t+1}^{iu} \tag{143}$$

Equation 143 determines a law of motion for the variance of log income, the persistence of the income process and $\beta$:

$$V_{y_{t+1}} = (\rho_{t+1})^2 V_{y_t} + (1 - \pi)^2 (\beta_{t+1})^2 (V_h + V_m) + V_u \tag{144}$$

$$\rho_{t+1} = \pi + (1 - \pi)\alpha\beta_{t+1} \tag{145}$$

$$\beta_{t+1} = \frac{\alpha^2 V_{y_t} + V_h}{\alpha^2 V_{y_t} + V_h + V_m} \tag{146}$$

In this extension we solve numerically, not analytically, for the properties of the solution of this system. It is easy to check numerically that

**Result 12.** *As in our main model, in steady state, an exogenous increase in the degree of meritocracy (a decrease of $V_m$) leads to a new steady state with a higher degree of intergenerational correlation ($\rho$), greater weight given to the observable signal ($\beta$), and a higher degree of inequality ($V_y$).*

The reasons are the same as in our main model: more meritocracy increases the differences between agents, as they are paid more accurately according to their productive ability, and given that the children of the rich are indeed on average more capable, this effect only increases the inequalities inherited via the privilege channel. The increase in inequality increases the value that the market assigns to objective information in the determination of the wage, $\beta$, increasing further the degree of inequality, etc...

This is the main result of this section: nothing of interest changes by introducing "irrational" privilege, at least with respect to the effects of meritocracy. It still increases inequality and still decreases mobility.

# L  The Cost of Misallocation

One of the omissions from the original model is a cost associated with the misallocation of workers given that firms do not perfectly observe their human capital. In equation 11, a worker's output is assumed to equal his human capital. This was done for simplicity. However, if a firm cannot perfectly identify that human capital, it is likely that the agent will not be able to realise his full productive potential, and all the moreso as the error in the firm's belief grows. In this appendix, the role of allocating workers to tasks is added into the model. It does not have any effect on the results presented in the body of the paper.

In order to capture the role of allocation in production, a term $A$ is added to the production function. This allocative cost pulls the output of the worker down to below their full level of human capital. The greater the error is firms' beliefs, the greater the degree of misallocation and the lower the level of output. Firms, as before, pay workers their expected output conditional on the available information but taking into account that their output will be lowered by misallocation.

$$Y_{t+1}^i = E\left[A \exp\left\{h_{t+1}^i\right\} | \Omega_{t+1}^i\right] \tag{147}$$
$$A = \exp\left\{-\frac{\theta}{2}\left(h_{t+1}^i - E(h_{t+1}^i)\right)^2\right\}$$
$$\theta \in R^+$$

The allocative cost is given by the square of the difference between actual and expected human capital. An exogenous parameter $\theta$ can control the extent to which this misallocation impacts on production. If $\theta$ is set equal to zero, this returns us to the model in the text.

Intuitively, since the human capital of any individual worker is unknown to the firm they cannot make adjustments to the individual wages paid. They adjust everyone's income by the same amount depending on the aggregate amount of misallocation, which is itself an endogenous function of the income distribution. As a result, the additional of this allocative cost impacts on the mean of income, but not the intergenerational elasticity or inequality presented above. Thus the main results of the paper are preserved without any adjustment.

The additional of $A$ alters result 4 in the following way.

**Result 13.** *Given the posterior belief about the log of human capital follows a normal distribution*

$$h_{t+1}^i | \Omega_{t+1}^i \sim N\left(\mu_{h_{t+1}^i | \Omega_{t+1}^i}, V_{h_{t+1}^i | \Omega_{t+1}^i}\right) \tag{148}$$

*and income is given by equation (147), it follows that:*

$$Y_{t+1}^i = \frac{1}{\sqrt{1 + \theta V_{h_{t+1}^i | \Omega_{t+1}^i}}} \exp\left\{\frac{1}{2}\left(\frac{V_{h_{t+1}^i | \Omega_{t+1}^i}}{1 + \theta V_{h_{t+1}^i | \Omega_{t+1}^i}}\right)\right\} \exp\left\{\mu_{h_{t+1}^i | \Omega_{t+1}^i}\right\} \tag{149}$$

*Proof.* We need to calculate

$$Y_{t+1}^i = E\left[\exp\left\{h_t^i - \frac{\theta}{2}\left(h_{t+1}^i - \mu_{h_{t+1}^i | \Omega_{t+1}^i}\right)^2\right\}\right] \tag{150}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{V_{h_{t+1}^i | \Omega_{t+1}^i}}} \exp\left\{h_t^i - \frac{\theta}{2}\left(h_t^i - \mu_{h_{t+1}^i | \Omega_{t+1}^i}\right)^2 - \frac{1}{2}\frac{\left(h_t^i - \mu_{h_{t+1}^i | \Omega_{t+1}^i}\right)^2}{V_{h_{t+1}^i | \Omega_{t+1}^i}}\right\} dh_t^i \tag{151}$$

After some manipulation, this becomes:

$$Y_{t+1}^i = \frac{1}{\sqrt{1 + \theta V_{h_{t+1}^i|\Omega_{t+1}^i}}} \exp\left\{\frac{1}{2}\left(\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1 + \theta V_{h_{t+1}^i|\Omega_{t+1}^i}}\right) + \mu_{h_{t+1}^i|\Omega_{t+1}^i}\right\} \tag{152}$$

$$\cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1+\theta V_{h_{t+1}^i|\Omega_{t+1}^i}}}} \exp\left\{-\frac{1}{2}\frac{\left(h_t^i - \left[\mu_{h_{t+1}^i|\Omega_{t+1}^i} + \left(\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1+\theta V_{h_{t+1}^i|\Omega_{t+1}^i}}\right)\right]\right)^2}{\left(\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1+\theta V_{h_{t+1}^i|\Omega_{t+1}^i}}\right)}\right\} dh_t^i \tag{153}$$

The second term is equal to one, therefore income of individual $i$ is given by:

$$Y_{t+1}^i = \frac{1}{\sqrt{1 + \theta V_{h_{t+1}^i|\Omega_{t+1}^i}}} \exp\left\{\frac{1}{2}\left(\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1 + \theta V_{h_{t+1}^i|\Omega_{t+1}^i}}\right)\right\} \exp\left\{\mu_{h_{t+1}^i|\Omega_{t+1}^i}\right\} \tag{154}$$

$\square$

By taking logs of equation (149) and substituting from result 3 we can find the log income of individual $i$ with signals $a_{t+1}^i$ and $m_{t+1}^i$:

$$\begin{aligned}
y_{t+1}^i = (1 - \beta_m)&\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha(1 - \beta_a)\mu_y\right] \\
&- \frac{1}{2}\left[\ln(1 + \theta\beta_m V_m) - \frac{\beta_m V_m}{1 + \theta\beta_m V_m}\right] \\
&+ \alpha\beta_a(1 - \beta_m)a_{t+1}^i + \beta_m m_{t+1}^i
\end{aligned} \tag{155}$$

This is equivalent to equation (20) in the main body of the paper. Given that $a_{t+1}^i$ and $m_{t+1}^i$ are both stochastic functions of $y_t^i$ we can write the law of motion of the log of income:

$$\begin{aligned}
y_{t+1}^i = \ln Z &+ \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha(1 - \beta_a)(1 - \beta_m)\mu_y \\
&- \frac{1}{2}\left[\ln(1 + \theta\beta_m V_m) - \frac{\beta_m V_m}{1 + \theta\beta_m V_m}\right] \\
&+ \alpha[\beta_a(1 - \beta_m) + \beta_m]y_t^i + \alpha\beta_a(1 - \beta_m)\varepsilon_{t+1}^{ia} + \beta_m(\varepsilon_{t+1}^{im} + \omega_{t+1}^i)
\end{aligned} \tag{156}$$

This is equivalent to equation (21) in the text. It can clearly be seen that the only role for misallocation is in the intercept. As misallocation plays a larger role ($\theta$ is higher), mean log income is reduced, as would be expected. The steady state equations for $\beta_a$, $\beta_m$, $V_y$ and $\rho$ are the same as those given in result 5.

**Result 14.** *Steady State with misallocation. Given the process of human capital accumulation in equation (10), income given by equation (147), and the information set given by equation (12), there exists a unique steady state which is globally stable. In the steady state log income is normally distributed with variance being characterized by the (unique) solution of the following system of equations:*

$$V_y = \frac{\beta_m V_\omega}{1 - \alpha^2[\beta_a(1 - \beta_m) + \beta_m]} \tag{157}$$

$$\beta_a = \frac{V_y}{V_y + V_a} \tag{158}$$

$$\beta_m = \frac{\alpha^2 \beta_a V_a + V_\omega}{\alpha^2 \beta_a V_a + V_\omega + V_m} \tag{159}$$

*The steady state mean of log income and intergenerational correlation of income are given by:*

$$\mu_y = \frac{\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} - \frac{1}{2}\left[\ln\left(1 + \theta \beta_m V_m\right) - \frac{\beta_m V_m}{1 + \theta \beta_m V_m}\right]}{1 - \alpha} \tag{160}$$

$$\rho = \alpha\left[\beta_a\left(1 - \beta_m\right) + \beta_m\right] \tag{161}$$

Misallocation will push down median income ($e^{\mu_y}$) but will otherwise leave the steady state unaffected. All of the results from sections 7.1 and 7.2 are preserved.

Similarly, misallocation will have a fairly limited impact on the equilibrium presented in section 8. The $\gamma_0$ term will be impacted upon by the degree of misallocation, but otherwise nothing is affected.

**Result 15.** *Equilibrium with misallocation. The equilibrium stochastic process of income as a function of parental income and investment is* $Y_{t+1}^i = e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_{t+1}^i}$ *with:*

$$\gamma_0 = \ln Z + \alpha\left(1 - \beta_m\right)\left[\left(1 - \beta_a\right)\mu_y + \ln \lambda\right] - \frac{1}{2}\left[\ln\left(1 + \theta \beta_m V_m\right) - \frac{\beta_m V_m}{1 + \theta \beta_m V_m} + V_\omega\right] \tag{162}$$

$$\gamma_1 = \alpha\beta_a\left(1 - \beta_m\right) \tag{163}$$

$$\gamma_2 = \alpha\beta_m \tag{164}$$

$$\varepsilon_{t+1}^i = \alpha\beta_a\left(1 - \beta_m\right)\varepsilon_{t+1}^{ai} + \beta_m\left(\omega_{t+1}^i + \varepsilon_{t+1}^{mi}\right) \tag{165}$$

*and, consequently, the equilibrium share of income invested in children's education is:*

$$\lambda = \frac{\alpha\beta_m}{1 + \delta - \alpha\beta_a\left(1 - \beta_m\right)} \tag{166}$$

# M   Calibration using US Data

This appendix describes the procedure used, and results obtained, when we fit the model to US Commuting Zone data from Chetty et al. (2014). This exercise has certain advantages over the fit to international data that we describe in the main text: more data/locations; and likely more consistent data definitions across locations. However, we choose to use the international fit as it makes our rhetorical point more eloquently, and it does not overplay this empirical application, which the model is not really capable of supporting (due to its omission of public education and redistribution).

The results of this exercise are used to provide the central estimates for the global parameters $\alpha$ and $V_\omega$, and are interesting in and of themselves. The measures ($V_m^j$ and $V_a^j$, $\forall j$) obtained from fitting the model seem to have reasonable correlations with other characteristics distributed across locations. Thus, across US commuting Zones the prevalence of merit seems to correlate well with having a large service sector and large foreign communities, while the fraction of African-Americans and racial segregation correlates well with the prevalence of inherited advantages.

We describe the data, procedure, and results in order.

## M.1   Data

We look at the configuration of meritocracy and advantages across the geography of the US using the extraordinary data made available by Chetty et al. (2014).[28] They use administrative data in order to estimate intergenerational mobility in 741 US Commuting Zones. Moreover they report the degree of inequality and education investment in each of them (along with many other variables).

**Data on Inequality**

Chetty et al. (2014) document the GINI index of the distribution of income at Commuting Zone level. This is a different measure of inequality than the variance of log income, which is the one we use in the model. In order to run the calibration procedure we translate the GINI index into the variance

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[28]Data available from *"Chetty, Hendren, Kline and Saez (2014): Descriptive Statistics by County and Commuting Zone"*. Section of `http://equality-of-opportunity.org/index.php/data`

of log income assuming that income is distributed lognormally.[29] This data value for the variance of log income across all locations, $j$, is labelled $\hat{V}_y^j$.

**Data on Intergenerational Mobility**

The preferred measure of intergenerational mobility in Chetty et al. (2014) is the rank-rank correlation, which looks at the persistence in the relative position in the income distribution, rather than the parent-child income correlation. There are good reasons for centering in the rank-rank correlation, as it controls better for the relatively scarce information on lifetime income of the children. Nevertheless, this measure does not map directly into our model (or into any formal model that we know of), and it seems reasonable to translate it into parent-child income correlation before calibrating the model.

We do so by determining which rank-rank correlation would be generated by an AR(1) process for log income with normal shocks, and with exogenous mean, variance and autocorrelation. For given mean and variance, we generate a map between the autocorrelation (which can be interpreted as the parent-child income correlation) and the rank-rank correlation. Crucially, the relationship seems to be both bijective and independent of the mean and variance, so we can then invert the relationship to map rank-rank correlations into parent-child income correlations.

The relationship however is almost perfectly one for one (as it can be seen in figure 5), though this will be conditional on the assumption of an AR(1) process for log income with normal shocks (as exists in the model). This implies that there is almost no difference between using the rank-rank correlation and the intergenerational correlation implied by the rank-rank correlation (under the assumptions that hold in the model).[30]

Specifically, we have the Rank-Rank Slope, $\beta_1$ from the Chetty et al. (2014) data, which is the regression coefficient from $r_i^{t+1} = \beta_0 + \beta_1 r_i^t + \epsilon_i^r$, where $r_i^{t+1}$ and $r_i^t$ are the ranks in the income distribution of children and parents respectively. What we would like to have is the intergeneration income correlation, $\rho$, which is the regression coefficient from $y_i^{t+1} = (1-\rho)\bar{y} + \rho y_i^t + \epsilon_i^y$. If we assume a lognormal distribution of income,

$$y = lnY \sim N\left(\bar{y}, V_y\right)$$

and, given large populations, we can assume perfect sampling from this distribution. The rank of a draw, $r(y)$, is therefore 1 minus the probability that any other draw is less than that i.e. $r(y) = 1 - \Phi\left(\frac{y-\bar{y}}{\sqrt{V_y}}\right)$.

We can then simulate the income process using correlated random variables:

- Let $Z_1, Z_2 \sim N(0,1)$

- Define a sample of $\{y^t, y^{t+1}\}$ based on a sample of $\{Z_1, Z_2\}$ using the relationships:

$$y^t = \bar{y} + V_y^{\frac{1}{2}} Z_1$$
$$y^{t+1} = \bar{y} + \rho V_y^{\frac{1}{2}} Z_1 + \sqrt{V_y\left(1 - \rho^2\right)} Z_2$$

- Then $y^t$ and $y^{t+1}$ are distributed as required

- We can simulate a large number of realisations of $\{y^t, y^{t+1}\}$, and for each calculate the realisation of $\{r^t, r^{t+1}\}$ under the assumption of large populations.

- We can then measure the Rank-Rank correlation over the whole sample for given values of $\{\bar{y}, V_y, \rho\}$

- We produce a map between $\rho$ and $\beta_1$, $\beta_1\left(\bar{y}, V_y, \rho\right)$. This relationship seems to be independent of $\bar{y}$ and $V_y$, and it appears very close to a linear relationship i.e. $\beta_1 = \beta_1\left(\bar{y}, V_y, \rho\right) \approx \beta_1\left(\rho\right) \approx \rho$. See figure 5

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[29]Specifically, we invert the relationship: $GINI = 2\Phi\left(\frac{\sigma_y}{\sqrt{2}}\right) - 1$. See Aitchison & Brown (1963)

[30] We performed the same exercise directly using the rank-rank correlations instead of our transformation. The calibrated parameters are almost identical, and the correlation of the values of $\beta_m^j$ and $\hat{\beta}_a^j$ with our benchmark is greater than 97%.
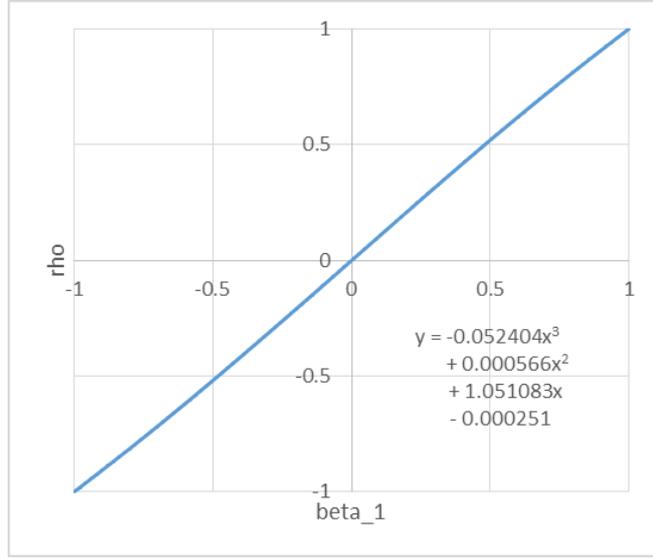
Figure 5: Mapping from rank-rank correlation ($\beta_1$ in the x axis) into intergenerational income correlation ($\rho$ in the y axis)

- Given this independence upon $\bar{y}$ and $V_y$, and the apparent bijectivity, we can invert the relationship to obtain $\rho = \rho(\beta_1)$ as require (though as discussed, this is almost exactly the same as just using $\rho \approx \beta_1$).

- This data value for the intergenerational income correlation across all locations, $j$, is labelled $\hat{\rho}^j$.

**Educational Investment**

As an empirical counterpart to the share of income spent on human capital accumulation, $\lambda$, we use a transformation of the ratio of spending per student on school education to the mean household income for each Commuting Zone. The transformation, and its rationale, is as follows.

Spending on human capital accumulation for children is larger, perhaps substantially larger, than what is spent in formal schooling. It includes, for instance, the spending in real-estate incurred in order to enjoy the externalities generated from the presence of high income students in the neighborhood, or the spending in extra-curricular activities (from violin lessons to trips to museums). We have no way of including data on all this activities, and Chetty et al. (2014) only report on spending per pupil.

It is reasonable to expect, nevertheless, that total spending will be related to formal spending per pupil. Thus, we define a variable $k^j$ for each commuting zone $j$ equal to the standardized (i.e. zero mean and unit variance) ratio of spending per student divided by mean household income. We then define $\hat{\lambda}^j = \mu + \sigma k^j$ as our empirical counterpart to $\lambda^j$. $\mu$ and $\sigma$ are chosen by the calibration routine (see below) in order to equalize the mean and variance of the "empirical" value of $\hat{\lambda}^j$ with those of its model counterpart.

## M.2 Procedure

As described in the main text, we assume a discount rate of $1\% p.a.$ and assume that a period/generation is of the order of 30 years. This translates into an assumed value for $\delta = 0.348$.

We further assume that all US Commuting zones share the same value of $\alpha$ and $V_\omega$, and we use the following routine to fit values for $\alpha$ and $V_\omega$, as well as values for $V_a^j$ and $V_m^j$ for all locations, $j$, that make the model closest to the data.

The routine also chooses values for $\mu$ and $\sigma$ to equalize the mean and variance of the "empirical" value of $\hat{\lambda}^j$ with those of its model counterpart. As described below, our goal in the calibration is to maximize the correlation between empirical and model variables across locations, and therefore the inclusion of $\mu$ and $\sigma$ is innocuous except for the mostly aesthetic effect of equalizing the distribution of empirical and model moments: this does not affect the correlation across commuting zones.

Not all cells in the data are populated, thus we eliminate all commuting zones on the basis of any missing values in the variables needed to construct $\left\{ \hat{V}_y^j, \hat{\rho}^j, k^j : \forall j \right\}$. This leaves a data set of 701

50

commuting zones.

**Initial Conditions**

- To generate initial conditions to start the calibration, we set:

$$\mu = 0, \sigma = 1$$
$$\alpha = 0.5$$
$$V_a^j = V_m^j = V_\omega = 1, \ \forall j$$

- Then change $\alpha$ and $V_a^j = V_m^j = V_\omega, \forall j$, so that $\rho$ and $V_y$ (the results for the model values across locations will all be identical since we are setting all the variances equal to the same value) are equal to the means of $\hat{\rho}^j$ and $\hat{V}_y^j$.

- Then keeping $\alpha$ and $V_\omega$ fixed, we choose, for each $j$, $\left(V_a^j, V_m^j\right)$ to minimise $\left(\ln \rho^j - \ln \hat{\rho}^j\right)^2 + \left(\ln V_y^j - \ln \hat{V}_y^j\right)^2$. This ensures that correlations between data and model $\rho$'s and $V_y$'s are well defined rather than being divide by zero errors. i.e. in each location we have an initial local calibration that matches local inequality and mobility but pays no attention to human capital investment.

**Model to data fitting algorithm**

1. Ennumerate CZs from 1 to 701 (based simply on their ordering in the Chetty et al. (2014) dataset)

2. Let $tol = 100$

3. Starting from $j = 1$ and optimising individually up to $j = 701$, choose $\left(V_a^j, V_m^j\right)$ to minimise the following objective function[31]

$$Obj = (1 - Corr\,[\hat{\rho}, \rho])^2 + \left(1 - Corr\left[\hat{V}_y, V_y\right]\right)^2 + \left(1 - Corr\left[\hat{\lambda}, \lambda\right]\right)^2$$
$$+ \left(\ln \bar{\rho}^D - \ln \bar{\rho}^M\right)^2 + \left(\ln \bar{V}_y^D - \ln \bar{V}_y^M\right)^2$$

4. Repeat 3. until $Obj$ is stable (this should mean that the ordering of the CZs does not matter particularly much), then save the matrix $V_{MAT}^{agg} = \left\{\left(V_a^j, V_m^j\right) : \forall j\right\}$

5. Choose $\mu$ & $\sigma$ so that the mean and standard deviation of the $\hat{\lambda}$ matches the modelled equivalents, subject to the constraint that $\min\left\{\hat{\lambda}^j\right\} \geq 0$. NB It is always the case, independent of $\mu$ & $\sigma > 0$, that $Corr\left[k, \hat{\lambda}\right] = 1$.

6. This step attempts to reduce the size of any outliers by minimising square errors for any large outliers. Starting from $j = 1$, check if $Obj^j > tol$, and if so choose $\left(V_a^j, V_m^j\right)$ to minimise $Obj^j$. Continue up to $j = 701$.

$$Obj^j = (\ln(\hat{V}_y^j) - \ln(V_y^j))^2 + (\ln(\hat{\rho}^j) - \ln(\rho^j))^2 + (\ln(\hat{\lambda}^j) - \ln(\lambda^j))^2$$

Then save the matrix $V_{MAT}^{out} = \{\left(V_a^j, V_m^j\right) : \forall j\}$ and corresponding $Obj\left(V_{MAT}^{out}\right)$.

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[31] As described in the main text, we maximize the correlations, instead of minimizing the square errors, of the moments to normalize the magnitudes of the three variables, giving them effectively equal weight. We can therefore justify this choice, but there was a practical reason for choosing this in the first place. The routine described in this appendix chooses $\{V_a^j, V_m^j : \forall j\}$ conditional on $\alpha$ and $V_\omega$; it then looks for the movement in $(\alpha, V_\omega)$ space that produces the biggest fall in the value of the objective function; this has the effect of prioritising a calibration that reduces the biggest errors, but does not produce particularly high correlations across all those locations that fit the data relatively well; targeting the correlations directly produces much higher correlations across all locations at the expense of a global parameter set $(\alpha, V_\omega)$ that leaves several large outliers in which model and data values do not match very well.

7. Count the number of rows in which $V_{MAT}^{agg} \neq V_{MAT}^{out}$, repeat from 3. until this number is constant.

8. Let $tol = \frac{1}{10}tol$, then repeat from 3. until $tol = 0.1$

9. Let $Inc = 10\%$

10. Repeat steps 2 to 8 four times for each of $\alpha \to \min\{0.9, \max\{0.1, \alpha \times \text{ or } / (1 + Inc)\}\}$ and $V_\omega \to \max\{0.001, V_\omega \times \text{ or } / (1 + Inc)\}\}$, each time recording the value of $V_{MAT}^{out} = \{\left(V_a^j, V_m^j\right) : \forall j\}$ and corresponding $Obj\left(V_{MAT}^{out}\right)$.

11. Change global parameters to those corresponding to the lowest value of $Obj\left(V_{MAT}^{out}\right)$ from the last stage and repeat from stage 2.

12. If the lowest $Obj\left(V_{MAT}^{out}\right)$ is the original (unaltered global parameters), or if we see any cyclical sequences (i.e. where a parameter is increased by $Inc$ followed by a decrease by $Inc$) then let $Inc = \frac{1}{10}Inc$ and repeat from stage 2 (omitting 9) until $Inc = 0.01\%$.

## M.3   Calibration Results

In table 2 and figure 6 we report the calibrated parameters and the goodness of fit for the targeted moments.

Each scatter plot draws the value of the data and the moment value for one of the targeted moments in each of the CZs. In spite the high degree of freedom, the correlations are high and, less surprisingly, the mean of $\rho$ and $V_y$ across CZ's is pinpointed very accurately. Nevertheless, being targeted moments, it is difficult to interpret this as goodness of fit.

The variances of $\rho$ and $V_y$ across CZ's are the only untargeted moments that we have left. On this we do quite well in the first, but less so in the second. The standard deviation of $\rho$ in data is 0.060, and in the model it's remarkably close: 0.063. On the other hand, we do very poorly in the variance of inequality across CZs, its standard deviation in the data is 0.304, and in the model it's 0.095.

The calibrated values of $\alpha$ and $V_\omega$ are 0.409 and 1.038 respectively.

We determine their geographical distribution, and we correlate the results with other macroeconomic variables across commuting zones. Therefore we can look not only at the correlation of meritocracy with advantages, but also at which other variables are correlated with each of them.

Instead of looking at the of $V_m^j$ and $V_a^j$ it is more convenient to look at the precisions. We define $P_m^j = -\ln\left(V_m^j\right)$ and $P_a^j = -\ln\left(V_a^j\right)$. Then, in figure 7 we plot the distribution of the implied precision of the merit and background signals across all CZs, which visually suggests that they are negatively correlated across CZs. This is indeed the case. In table 2 we show that more meritocratic places have on average lower advantages: the correlation of $P_a$ with $P_m$ is negative, which translates into a negative correlation of $-.54$ between the rewards on merit and background. This covariance between the two sources of exogenous variation across CZs explains why when comparing across CZs:

- CZs with more information on background have less mobility, but also less inequality (positive correlation between $P_a$ and both data and modelled $\rho$, negative correlation between $P_a$ and both data and modelled $Gini$).

- CZs with more information on merit suffer more inequality but have almost no difference in mobility ($P_m$ is very positively correlated with both data and modelled $Gini$, but only very marginally positively correlated with the model $\rho$, and very marginally negatively correlated with the data $\hat\rho$).

In figure 8 we plot the geographical distribution of our proxies for merit and reward, along with the data values of mobility and inequality for comparison. Data is in green in quintiles: darkest (lightest) green is highest (lowest) quintile of variable being mapped. We plot in red the CZs with insufficient data. Here the same pattern is apparent. Due to the negative correlation between the two precisions, the precision of merit signals seems to explain inequality better, while the precision of background signals seems to explain mobility better.

It is also of interest to correlate the implied precision of both signals and the rewards to merit and background to a relatively large set of Commuting Zone characteristics available from Chetty et al.

| Calibrated parameters | |
| --- | --- |
| $\alpha$ | = 0.409 |
| $V_\omega$ | = 1.038 |
| $\mu$ | = 0.1694 |
| $\sigma$ | = 0.0219 |
| *Loss Fn* | = 0.285 |

**Targeted Moments**

| Correlation at CZ level data and model | |
| --- | --- |
| $Corr\left(\hat{\rho}, \rho\right)$ | = 0.84 |
| $Corr\left(\hat{V}_y, V_y\right)$ | = 0.64 |
| $Corr\left(\hat{\lambda}, \lambda\right)$ | = 0.64 |

| | Model | Data |
| --- | --- | --- |
| $E\left(\rho\right)$ | 0.309 | 0.308 |
| $E\left(V_y\right)$ | 0.613 | 0.616 |

**Correlations**

| | $P_a$ | $P_m$ | $\hat{\beta}_a$ | $\beta_m$ | $\hat{\rho}$ | $\hat{V}_y$ | $\hat{\lambda}$ | $\rho$ | $V_y$ | $\lambda$ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $P_a$ | 100% | | | | | | | | | |
| $P_m$ | $-22\%$ | 100% | | | | | | | | |
| $\hat{\beta}_a$ | 87% | $-34\%$ | 100% | | | | | | | |
| $\beta_m$ | $-42\%$ | 82% | $-54\%$ | 100% | | | | | | |
| $\hat{\rho}$ | 68% | $-5\%$ | 77% | $-12\%$ | 100% | | | | | |
| $\hat{V}_y$ | $-19\%$ | 66% | $-18\%$ | 62% | 22% | 100% | | | | |
| $\hat{\lambda}$ | $-2\%$ | 41% | $-17\%$ | 60% | $-10\%$ | $-18\%$ | 100% | | | |
| $\rho$ | 81% | 3% | 90% | $-12\%$ | 84% | 11% | 12% | 100% | | |
| $V_y$ | $-26\%$ | 83% | $-37\%$ | 98% | 3% | 64% | 63% | 7% | 100% | |
| $\lambda$ | $-9\%$ | 79% | $-18\%$ | 92% | 20% | 64% | 64% | 26% | 98% | 100% |

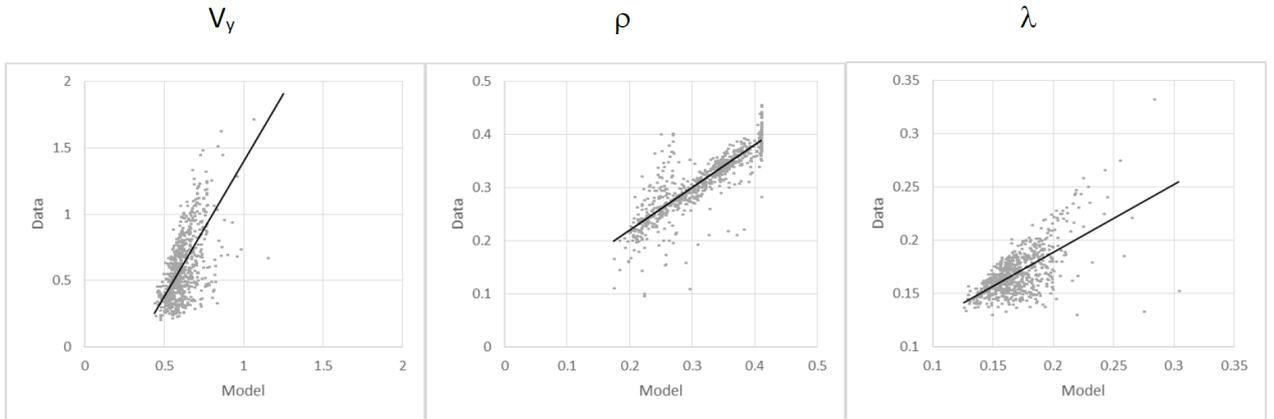Table 2: Calibrated Parameters and Fitness of Targeted Moments.



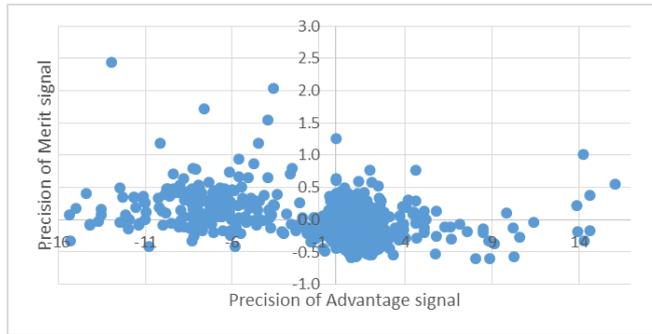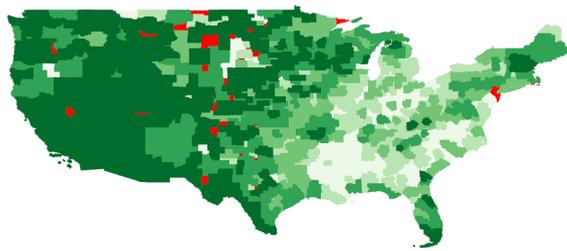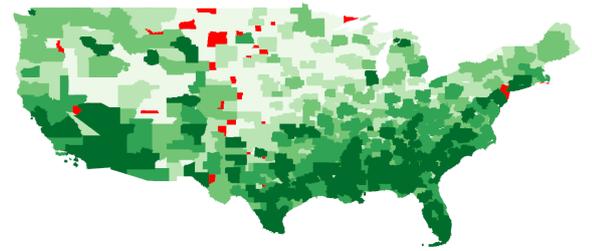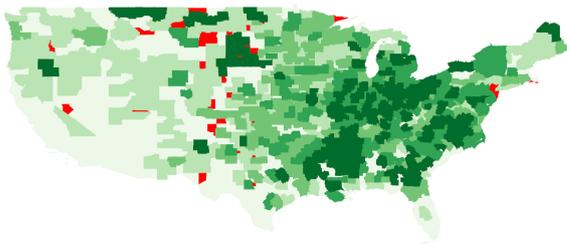Figure 6: Calibrated Common parameters & Model and Data Moments for all CZ.



Figure 7: Distribution of calibrated values of the precission of the signals: $P_m^j$ and $P_a^j$ for all CZ's.
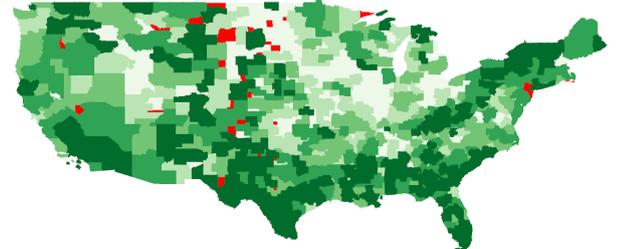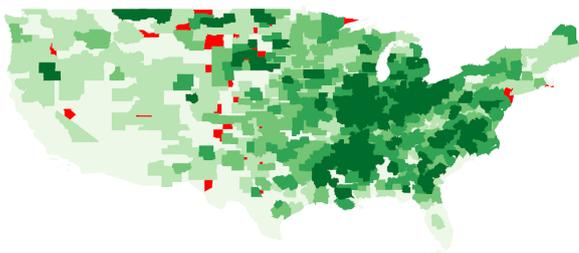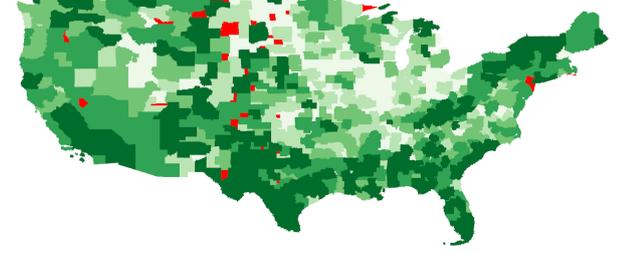
(a) Mobility, $1 - \rho$

(b) Inequality, Gini Index.

(c) Precision of Advantage Signal

(d) Precision of Merit Signal

(e) Reward to Advantage Signal, $\hat{\beta}_a$

(f) Reward to Merit Signal, $\beta_m$

Figure 8: Geographical patterns in the US

| Sample Correlations. | | | | | | |
|---|---|---|---|---|---|---|
| | Mobility | Gini | $P_a$ | $P_m$ | $\hat{\beta}_a$ | $\beta_m$ |
| fraction black | -62% | 58% | 24% | 9% | 25% | 18% |
| racial segregation | -43% | 30% | 12% | 6% | 19% | 15% |
| fraction with commute 15 mins | 47% | -58% | -12% | -11% | -21% | -14% |
| household income per capita | 13% | -8% | -10% | -20% | -2% | -35% |
| state income tax progressivity | 4% | -19% | 6% | -10% | 6% | -18% |
| state eitc exposure | 14% | -31% | 3% | -7% | 0% | -9% |
| teacher student ratio | 2% | 16% | -5% | -12% | -3% | -18% |
| high school dropout rate income adjus | -35% | 47% | 10% | 17% | 9% | 23% |
| college graduation rate income adjus | 4% | -17% | -2% | -4% | -2% | -2% |
| labor force participation rate | 32% | -35% | -11% | -32% | -7% | -44% |
| manufacturing employment share | -40% | -3% | 40% | -16% | 52% | -32% |
| teenage labor force participation rate | 52% | -64% | -13% | -23% | -16% | -38% |
| migration inflow rate | 24% | 8% | -24% | -7% | -25% | -5% |
| migration outflow rate | 27% | 2% | -22% | -8% | -27% | -1% |
| fraction foreign born | 29% | 22% | -31% | 13% | -30% | 25% |
| social capital index | 32% | -66% | 4% | -29% | 4% | -46% |
| fraction of children with single mother | -65% | 62% | 25% | 17% | 23% | 31% |
| fraction of adults divorced | -13% | 20% | -2% | -5% | 4% | -5% |
| income growth | 51% | -35% | -32% | 2% | -36% | 2% |

| Partial Correlations. | | | | | | |
|---|---|---|---|---|---|---|
| | Mobility | Gini | $P_a$ | $P_m$ | $\hat{\beta}_a$ | $\beta_m$ |
| fraction of black | -13% | 11% | -4% | -22% | 3% | -22% |
| | -2.26 | 1.69 | -0.42 | -1.85 | 0.36 | -2.93 |
| racial segregation | -30% | 0% | 9% | -7% | 19% | -1% |
| | -9.68 | -0.10 | 1.75 | -1.14 | 4.08 | -0.23 |
| fraction with commute 15 mins | 17% | -21% | -19% | -18% | -21% | -12% |
| | 3.37 | -3.39 | -2.26 | -1.65 | -2.75 | -1.81 |
| household income per capita | 5% | 17% | -21% | 8% | -15% | -8% |
| | 0.98 | 2.63 | -2.38 | 0.70 | -1.91 | -1.16 |
| state income tax progressivity | -1% | -6% | 5% | -12% | 5% | -18% |
| | -0.30 | -1.47 | 0.86 | -1.74 | 1.08 | -4.14 |
| state eitc exposure | 6% | -7% | -4% | 7% | -10% | 17% |
| | 2.19 | -2.01 | -0.82 | 1.15 | -2.43 | 4.42 |
| teacher student ratio | 23% | -7% | -8% | -14% | -12% | -20% |
| | 6.33 | -1.51 | -1.28 | -1.90 | -2.24 | -4.13 |
| high school dropout rate income adjusted | -5% | 13% | 3% | 18% | 0% | 13% |
| | -1.55 | 3.40 | 0.63 | 2.68 | -0.04 | 3.11 |
| college graduation rate income adjusted | -1% | 2% | -12% | 2% | -9% | 11% |
| | -0.24 | 0.71 | -2.62 | 0.41 | -2.24 | 3.00 |
| labor force participation rate | 6% | -8% | 0% | -30% | -1% | -18% |
| | 1.29 | -1.48 | 0.01 | -3.02 | -0.14 | -2.82 |
| manufacturing employment share | -24% | -21% | 32% | -21% | 48% | -36% |
| | -6.49 | -4.64 | 5.12 | -2.63 | 8.66 | -7.31 |
| teenage labor force participation rate | 15% | -17% | 9% | 10% | 4% | -10% |
| | 2.53 | -2.49 | 0.93 | 0.81 | 0.42 | -1.27 |
| migration inflow rate | -8% | 14% | -11% | 7% | -9% | 9% |
| | -1.51 | 2.18 | -1.17 | 0.57 | -1.03 | 1.28 |
| migration outflow rate | 12% | -24% | 10% | -21% | 7% | -21% |
| | 2.55 | -4.12 | 1.22 | -2.05 | 0.91 | -3.24 |
| fraction foreign born | 18% | 19% | -23% | 8% | -16% | 22% |
| | 4.65 | 4.02 | -3.51 | 0.99 | -2.73 | 4.24 |
| social capital index | -25% | -21% | 31% | -37% | 38% | -32% |
| | -4.50 | -3.18 | 3.37 | -3.20 | 4.67 | -4.36 |
| fraction of children with single mother | -24% | 20% | 21% | 22% | 12% | 30% |
| | -3.93 | 2.81 | 2.10 | 1.69 | 1.28 | 3.72 |
| fraction of adults divorced | -1% | -9% | -8% | -34% | 4% | -27% |
| | -0.26 | -1.86 | -1.16 | -3.96 | 0.71 | -4.98 |
| income growth | 13% | 2% | -19% | 15% | -12% | 14% |
| | 3.35 | 0.43 | -2.99 | 1.85 | -2.14 | 2.70 |
| $R^2$ | 75% | 68% | 34% | 27% | 46% | 59% |

Table 3: 418 CZ without blank data. In the Partial Correlations panel each column is a different LHS variable. In the first line of each row we place the coefficient of the variable (in %), in the second line the t-statistic.

(2014). In table 3 we do so for the 418 Commuting Zones that have information on all the properties that we look at. We run a regression[32] of these characteristics against both mobility and inequality in each CZ, but also on $P_a$, $P_m$, $\hat{\beta}_a$ and $\beta_m$. We only report the values for those characteristics that are significant in at least one regression.

Income level is negatively related to the precision of the signal on advantages, but is unrelated to the precision of the signal on merit; while income *growth* is negatively related to the prevalence of advantages and positively related to the prevalence of merit.

Racial segregation and the share of manufacturing, particularly the latter, seem to be among the variables that correlate most highly with the precision of the background signal (and with its reward $\hat{\beta}_a^j$). It also correlates negatively with the size of the foreign community in the CZ, and with educational achievement indicators (the teacher student ratio and college graduation rate).

The share of manufacturing is also one of the better correlates with the precision of the merit signal, but negatively. The fraction of foreign born strongly correlates positively. This suggest that areas with relatively large service sectors and more migrants tend to be more meritocratic. The reward to merit also decreases with the proportion of African Americans. Educational achievement (share of college graduates) correlates positively with it, while tax progressivity does so negatively.

Finally, social capital seems to correlate positively with the incidence of background and negatively withc the incidence of merit.

In table 4 we perform the same exercise in a sample including all the commuting zones but only those characteristics for which there is data in all 701 locations. It shows the same general patterns.

Summarizing. When we calibrate our model to US data, the implied values of the precision of the merit signal are negatively correlated with the implied precision of the background signal. The prevalence of advantages due to background is very negatively correlated with the degree of mobility, not so much to inequality. High rewards to merit, on the contrary, seem correlated to inequality but only weakly with mobility. Racial characteristics of a CZ have the expected effects: segregation and racial diversity increase the prevalence of advantage and decrease meritocracy. State tax progressivity is negatively related to the precision of the merit signal. The productive structure of the commuting zone is what most clearly correlates with both merit and advantages: manufacturing associates possitively with advantages and negatively with merit. Finally, foreign migrants tend to live in CZs where the implied precision of merit signals is high and background signals is low.

# N    Algorithm used for international data

As described in the main text, we assume a discount rate of $1\%p.a.$ and assume that a period/generation is of the order of 30 years. This translates into an assumed value for $\delta = 0.348$.

We further assume that all countries share the same value of $\alpha$ and $V_\omega$.

The routine also chooses values for $\mu$ and $\sigma$ to equalize the mean and variance of the "empirical" value of $\hat{\lambda}^j$ with those of its model counterpart. As described below, our goal in the calibration is to maximize the correlation between empirical and model variables across locations, and therefore the inclusion of $\mu$ and $\sigma$ is innocuous except for the mostly aesthetic effect of equalizing the distribution of empirical and model moments: this does not affect the correlations across countries.

**Initial Conditions**

- To generate initial conditions to start the calibration, we set:

$$\mu = 0, \sigma = 1$$
$$V_a^j = V_m^j = V_\omega, \ \forall j$$

- Choose, for each $j$, $\left(V_a^j, V_m^j\right)$ to minimise $\left(\ln \rho^j - \ln \hat{\rho}^j\right)^2 + \left(\ln V_y^j - \ln \hat{V}_y^j\right)^2$. This ensures that correlations between data and model $\rho$'s and $V_y$'s are well defined rather than being divide by zero errors. i.e. in each location we have an initial local calibration that matches local inequality and mobility but pays no attention to human capital investment.

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[32]All variables in all exercises have been standardized by subtracting mean and dividing by standard deviation.

| Sample Correlations. | | | | | | |
|---|---|---|---|---|---|---|
| | Mobility | Gini | $P_a$ | $P_m$ | $\hat{\beta}_a$ | $\beta_m$ |
| fraction black | -57% | 54% | 25% | 10% | 25% | 14% |
| racial segregation | -41% | 28% | 14% | 3% | 22% | 6% |
| fraction with commute 15 mins | 41% | -56% | -8% | -6% | -18% | -5% |
| household income per capita | 17% | 7% | -17% | -18% | -9% | -25% |
| local government expenditures per capita | 31% | -10% | -21% | 6% | -26% | 12% |
| state income tax progressivity | 14% | -8% | -11% | -7% | -8% | -9% |
| state eitc exposure | 13% | -25% | 1% | -4% | -4% | -4% |
| labor force participation rate | 22% | -26% | -6% | -29% | -2% | -35% |
| manufacturing employment share | -44% | -2% | 39% | -20% | 51% | -34% |
| teenage labor force participation rate | 44% | -58% | -10% | -22% | -13% | -30% |
| fraction foreign born | 32% | 30% | -40% | 18% | -37% | 30% |
| fraction religious | 6% | -29% | 7% | -11% | 5% | -17% |
| fraction of children with single mother | -59% | 58% | 24% | 19% | 23% | 27% |

| Partial Correlations. | | | | | | |
|---|---|---|---|---|---|---|
| | Mobility | Gini | $P_a$ | $P_m$ | $\hat{\beta}_a$ | $\beta_m$ |
| fraction black | -12% | 17% | 4% | 1% | 1% | 0% |
| | 2.80 | 3.88 | 0.77 | 0.09 | 0.26 | 0.03 |
| racial segregation | -27% | -3% | 15% | -1% | 23% | -1% |
| | -9.89 | -1.15 | 4.00 | -0.36 | 6.49 | -0.30 |
| fraction with commute 15 mins | 9% | -25% | 3% | -7% | 0% | -1% |
| | 2.11 | -5.33 | 0.58 | -1.09 | 0.04 | -0.19 |
| household income per capita | 3% | 18% | -10% | -13% | -2% | -20% |
| | 0.75 | 4.75 | -1.98 | -2.41 | -0.48 | -4.33 |
| local government expenditures per capita | 6% | -3% | -4% | 9% | -8% | 12% |
| | 2.11 | -0.91 | -1.06 | 2.09 | -2.24 | 3.41 |
| state income tax progressivity | 5% | -6% | -4% | -10% | 0% | -14% |
| | 1.92 | -2.38 | -1.27 | -2.80 | 0.03 | -4.70 |
| state eitc exposure | 6% | -5% | -3% | 12% | -13% | 21% |
| | 2.29 | -1.78 | -0.75 | 2.98 | -3.63 | 6.11 |
| labor force participation rate | 4% | -12% | 1% | -22% | 1% | -19% |
| | 1.18 | -3.30 | 0.17 | -4.40 | 0.22 | -4.48 |
| manufacturing employment share | -29% | -15% | 35% | -19% | 47% | -33% |
| | -10.50 | -5.11 | 9.19 | -4.60 | 13.66 | -9.29 |
| teenage labor force participation rate | 2% | -16% | 12% | -2% | 12% | -15% |
| | 0.35 | -2.87 | 1.72 | -0.24 | 1.86 | -2.23 |
| fraction foreign born | 27% | 21% | -27% | 17% | -26% | 27% |
| | 9.68 | 7.00 | -7.18 | 4.20 | -7.43 | 7.68 |
| fraction religious | -8% | 3% | 5% | -6% | 6% | -10% |
| | -2.92 | 0.85 | 1.25 | -1.46 | 1.64 | -2.85 |
| fraction of children with single mother | -29% | 25% | 18% | 7% | 14% | 12% |
| | -6.44 | 5.11 | 2.93 | 1.08 | 2.44 | 2.15 |
| $R^2$ | 65% | 59% | 34% | 19% | 44% | 42% |

Table 4: 701 CZ. In the Partial Correlations table each column is a LHS variable. In the first line of each row we place the coefficient of the variable (in %), in the second line the t-statistic.

**Model to data fitting algorithm**

1. Ennumerate countries from 1 to 15 alphabetically.

2. Let $tol = 100$

3. Starting from $j = 1$ and optimising individually up to $j = 15$, choose $\left(V_a^j, V_m^j\right)$ to minimise the following objective function.

$$Obj = (1 - Corr\,[\hat{\rho}, \rho])^2 + \left(1 - Corr\left[\hat{V}_y, V_y\right]\right)^2 + \left(1 - Corr\left[\hat{\lambda}, \lambda\right]\right)^2$$
$$+ \left(\ln \bar{\rho}^D - \ln \bar{\rho}^M\right)^2 + \left(\ln \bar{V}_y^D - \ln \bar{V}_y^M\right)^2$$

4. Repeat 3. until $Obj$ is stable (this should mean that the ordering of the countries does not matter particularly much), then save the matrix $V_{MAT}^{agg} = \left\{\left(V_a^j, V_m^j\right) : \forall j\right\}$

5. Choose $\mu$ & $\sigma$ so that the mean and standard deviation of the $\hat{\lambda}$ matches the modelled equivalents, subject to the constraint that $\min\left\{\hat{\lambda}^j\right\} \geq 0$. NB It is always the case, independent of $\mu$ & $\sigma > 0$, that $Corr\left[k, \hat{\lambda}\right] = 1$.

6. This step attempts to reduce the size of any outliers by minimising square errors for any large outliers. Starting from $j = 1$, check if $Obj^j > tol$, and if so choose $\left(V_a^j, V_m^j\right)$ to minimise $Obj^j$. Continue up to $j = 15$.

$$Obj^j = (\ln(\hat{V}_y^j) - \ln(V_y^j))^2 + (\ln(\hat{\rho}^j) - \ln(\rho^j))^2 + (\ln(\hat{\lambda}^j) - \ln(\lambda^j))^2$$

Then save the matrix $V_{MAT}^{out} = \left\{\left(V_a^j, V_m^j\right) : \forall j\right\}$ and corresponding $Obj\left(V_{MAT}^{out}\right)$.

7. Count the number of rows in which $V_{MAT}^{agg} \neq V_{MAT}^{out}$, repeat from 3. until this number is constant.

8. Let $tol = \frac{1}{10}tol$, then repeat from 3. until $tol = 0.1$
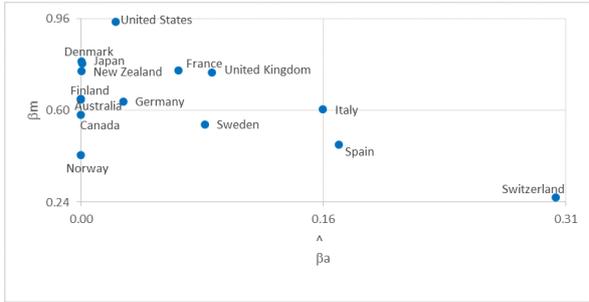
# O   Sensitivity Analysis

Here we show the results obtained for the fit to international data, when the global parameters $\delta$, $\alpha$ and $V_\omega$ are varied from their central assumptions. As can be seen the qualitative results appear relatively insentive to changes in $\delta$, $\alpha$ and $V_\omega$.
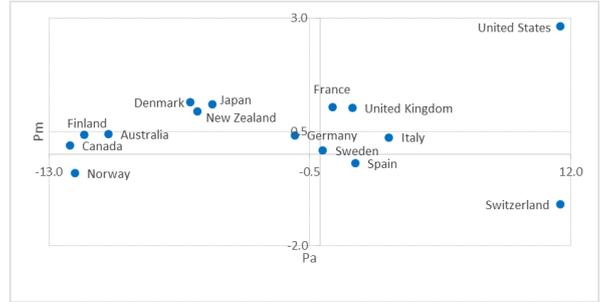
(a) $\beta_m$, $\hat{\beta}_a$. Central assumptions: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 1.038$.
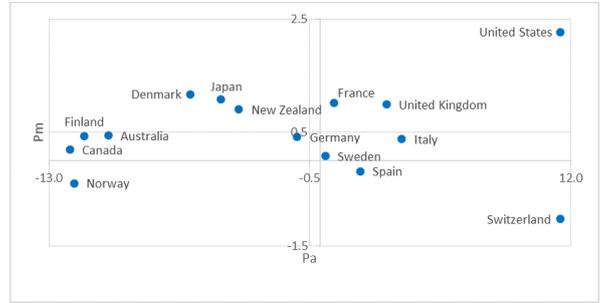


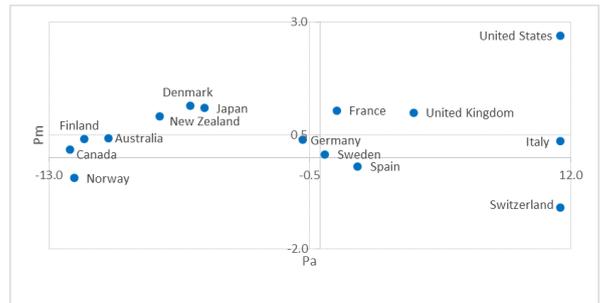(b) $P_m$, $P_a$. Central assumptions: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 1.038$.



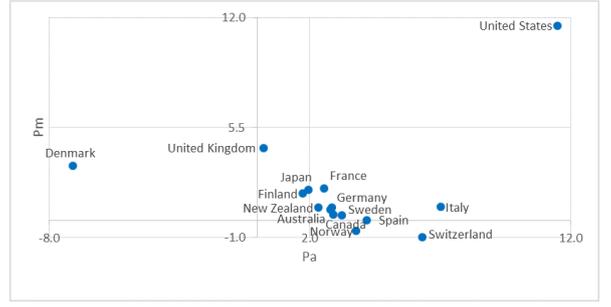(c) $\beta_m$, $\hat{\beta}_a$. Low $\delta$: $\delta = 0.25$, $\alpha = 0.409$ and $V_\omega = 1.038$.



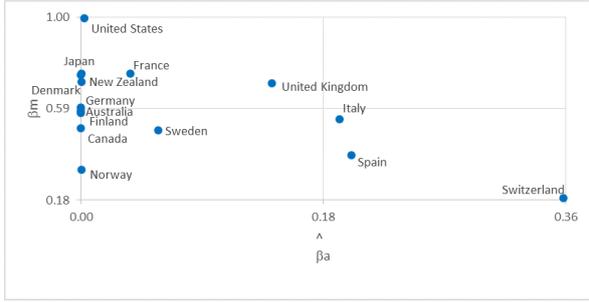(d) $P_m$, $P_a$. Low $\delta$: $\delta = 0.25$, $\alpha = 0.409$ and $V_\omega = 1.038$.



(e) $\beta_m$, $\hat{\beta}_a$. High $\delta$: $\delta = 0.45$, $\alpha = 0.409$ and $V_\omega = 1.038$.



(f) $P_m$, $P_a$. High $\delta$: $\delta = 0.45$, $\alpha = 0.409$ and $V_\omega = 1.038$.

Figure 9: Implied Rewards and Precisions across countries for other global parameter sets.

(a) $\beta_m$, $\hat{\beta}_a$. Low $\alpha$: $\delta = 0.348$, $\alpha = 0.3$ and $V_\omega = 1.038$.
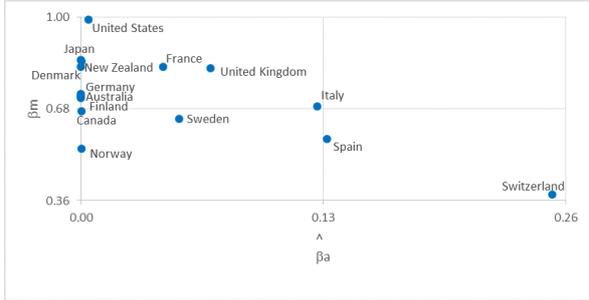
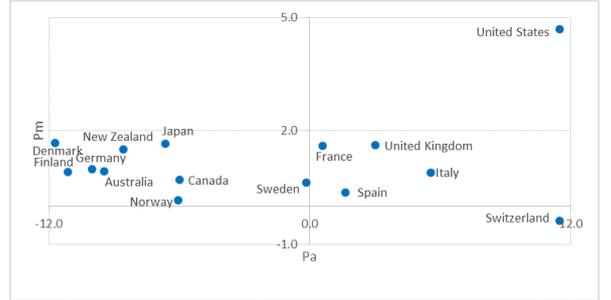(b) $P_m$, $P_a$. Low $\alpha$: $\delta = 0.348$, $\alpha = 0.3$ and $V_\omega = 1.038$.

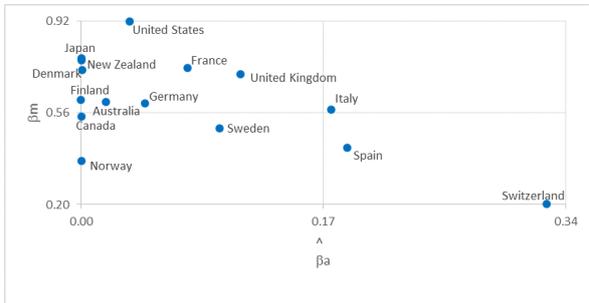(c) $\beta_m$, $\hat{\beta}_a$. High $\alpha$: $\delta = 0.348$, $\alpha = 0.5$ and $V_\omega = 1.038$.
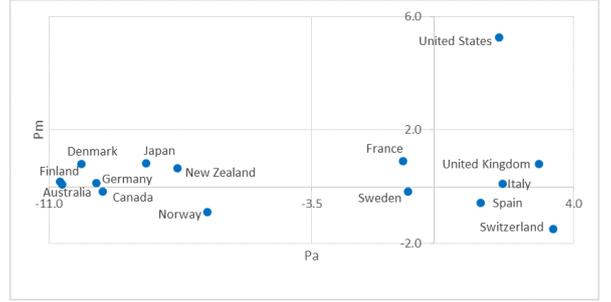
(d) $P_m$, $P_a$. High $\alpha$: $\delta = 0.348$, $\alpha = 0.5$ and $V_\omega = 1.038$.
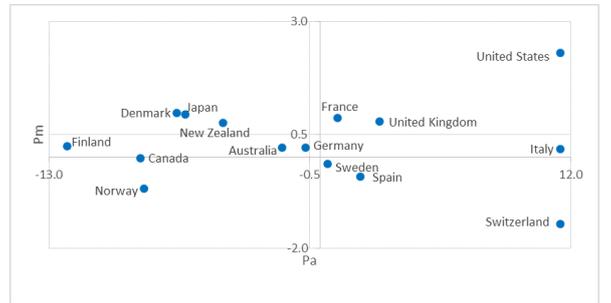
(e) $\beta_m$, $\hat{\beta}_a$. Low $V_\omega$: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 0.9$.

(f) $P_m$, $P_a$. Low $V_\omega$: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 0.9$.

(g) $\beta_m$, $\hat{\beta}_a$. High $V_\omega$: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 1.1$.

(h) $P_m$, $P_a$. High $V_\omega$: $\delta = 0.348$, $\alpha = 0.409$ and $V_\omega = 1.1$.

Figure 10: Implied Rewards and Precisions across countries for other global parameter sets.

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