STRATHCLYDE

DISCUSSION PAPERS IN ECONOMICS

TOWARDS INCORPORATING NATURAL CAPITAL INTO A COMPUTABLE GENERAL EQUILIBRIUM MODEL FOR SCOTLAND

BY

GRANT ALLAN, DAVID COMERFORD AND PETER MCGREGOR

NO 18-08

DEPARTMENT OF ECONOMICS
UNIVERSITY OF STRATHCLYDE
GLASGOW

Towards Incorporating Natural Capital into a Computable General Equilibrium model for Scotland.

RESAS1.4.2ciii D4

Authors: Grant Allan, David Comerford*, & Peter McGregor

*Corresponding author: david.comerford@strath.ac.uk

Suggested citation: Allan, G., Comerford, D., & McGregor, P. (2018) “Towards Incorporating Natural Capital into a Computable General Equilibrium model for Scotland”

March 2018

Key words: Natural Capital, Computable General Equilibrium models.   **JEL Classification**: Q57, Q1, C68

Acknowledgements

This report was funded by the Rural & Environment Science & Analytical Services Division of the Scottish Government.

Abstract

Natural capital encompasses those assets which are provided by nature and which are valued by economic actors. As such, there is a clear analogy between natural and other assets, such as physical capital, which are routinely included in models of national economies. However, the valuation of natural assets, to the extent that they are included in such economic models, is typically wrapped up in physical capital along with land values or not valued at all. This could be simply a measurement problem – natural capital might be difficult to appropriately disaggregate from other capital - or because they provide non-market goods which are not included within traditional measures of economic output. The purpose of this paper is to set out – both conceptually and practically – how natural capital can be added to a computable general equilibrium (CGE) model. We focus on: the conceptual differences that should be reflected in such an extension; the challenges of implementing the extension in practice; and identifying the value added generated by an appropriately augmented model.

We explore the empirical implementation of our approach through the addition of carbon emissions and an agricultural biomass ecosystem service flow to our CGE model of the Scottish economy. This working paper specifies this CGE model development, but does not go as far as fully implementing it in the CGE model. When fully implemented in the context of a CGE with a disaggregated agriculture sector, this will allow us simultaneously to track the impact of disturbances, including policy changes, on the economy and the environment and therefore on sustainable development. In the longer-term comprehensive coverage of natural capital stocks and ecosystem services will allow us to track the impact of disturbances, including policy interventions, on Green GDP and Genuine Savings, as well as on aggregate and sectoral economic activity, energy use and emissions.

1 Introduction

Environmental policy is increasingly prominent, especially given the realities of climate change, but usually not at the expense of economic well-being. Many countries, in effect, have a policy of "Sustainable economic growth", which seeks to take into account the environmental impacts of economic development. Given environmental inputs to production and environmental goods that we all enjoy, it is impossible to assess whether the economy is sustainable if we do not monitor the state of the environment. To undertake policy analysis given this, we need to link macroeconomic models capable of evaluating impacts of policy interventions and scenarios with natural capital and ecosystem services within a single coherent framework.

Natural capital is the stock of natural resources or assets, which provide a wide range of goods and services, often called ecosystem services (UK NEA, 2011). Analogously to physical capital, ecosystem services can be conceived of as the dividend or interest rate flow that natural capital yields, while the natural capital value is the value of the stock of the asset. Like physical, human or social capital, it is possible to invest in natural capital, and to see natural capital depleted or depreciated if it is overused without investment.

Linking ecosystems and economy-wide models offers the potential to identify and quantify multiple benefits and the impacts upon, and trade-offs between, economic indicators and the management of natural assets. It offers the potential to examine, for instance, the economic consequences of changes in the availability or quality of ecosystems services. Such information would be particularly useful to understand the effects of possible future trajectories in natural capital, such as climate-related change.

One set of models which are particularly useful for addressing the economic system are Computable General Equilibrium (CGE) models. These are large scale models of the macro-economy, in which budget constraints are satisfied and all markets clear (hence general equilibrium). Typically the productive sectors combine labour and capital with other intermediate inputs in order to make their output, which is demanded by industry as intermediate inputs, and by final consumers. Such models have been particularly usefully applied at varying geographic levels to undertake economic analysis of environmental policies.

The impact of changes in natural capital on economic performance, the impact of economic changes on the use or level of natural capital, and the feedbacks between these, are poorly understood and not typically modelled within existing applied CGE models. To improve our understanding of these relationships, we seek to explicitly link natural capital to the wider economy within a system-wide, economy-environment model.

Much of the work to date in this area has involved incorporating natural capital and ecosystem services within a simple Input-Output (IO) system (see Anger et al, 2014). However, in IO systems, prices are fixed and changes in production follow any changes in demand in a mechanical way that reflects existing supply chains.

We argue that there is likely to be considerable value-added, in terms of contributions to both the academic literature and to policy analysis, in incorporating natural capital and ecosystem services in general equilibrium macroeconomic models. For the former, such models offer a greater degree of internal consistency as the definitions of capital used within the economy are expanded. For the latter, price-based instruments (such as taxes) in environmental policy can be analysed within this expanded framework. While it is perhaps conceptually straightforward to extend these modelling frameworks to natural capital stocks and ecosystem flows (e.g. productive sectors make use of both physical and natural capital stocks and ecosystem service flows in production, and consumers demand some provision of natural capital stocks and ecosystem service in consumption) there are significant practical challenges.

Such challenges have both empirical and theoretical aspects. First, there is the issue of how, in principle, natural capital stocks and ecosystem services should be specified and integrated with a model of the economic system. Second, is the issue of what information is available on natural capital stocks and ecosystem service flows, and how this relates to existing measures of capital. Finally, there is the issue of how this information may be used to calibrate a system-wide, general equilibrium model that incorporates natural capital and ecosystem services. The paper ends by setting out a brief empirical example of a partial equilibrium framework which makes clear some of the issues around model specification, data, model calibration and the importance of key model parameters.

The paper is structured as follows. Section 2 discusses how, in principle, natural assets may be incorporated within general equilibrium models: the analogues between natural assets (or natural capital) and any other asset; the types of “good” (or ecosystem service flow) produced by natural assets and how they compare with the goods and services which are typically modelled; the pricing of these assets and flows; and the assumptions which need to be made on the ownership of assets as they are incorporated into an appropriate system-wide model.

Section 3 discusses in more detail the specific data on natural capital and ecosystem services that are available for Scotland. In Section 4 a modelling framework for the Agricultural Biomass ecosystem service is developed and some illustrative simulation results are presented and analysed. Section 5 concludes with a brief discussion of the future capabilities of a system that embeds the analysis of this paper within a fully specified, economic-environmental CGE model of Scotland.

2 Adding Natural Capital to general equilibrium macroeconomic models

Computable general equilibrium models are typically large scale economic models calibrated to one or more national economies, in which all factors are paid their marginal product, firms maximise profits, consumers maximise utility, budget constraints are satisfied, and all markets clear simultaneously (subject to any explicitly modelled frictions e.g. labour market frictions which allow equilibrium unemployment). Such models are multi-sectoral in nature, with many different final goods produced (whose total value gives Gross Domestic Product, on an expenditure basis) and a complicated production structure which makes use of many intermediate goods. Typically, there are two factors of production: labour and capital, payments to which also total to GDP (on an income basis).

Such models therefore typically fail to accommodate key features that seem crucial to an informed analysis of the environment. In particular, production processes in practice also feature the use of natural assets (or natural capital) or some notion of “inputs” from nature (in the form of “ecosystems service flows”) to generate outputs. (We distinguish different forms of ecosystem services below.) This use of natural inputs in production is analogous to the use of physical capital stocks and intermediate goods in production. Additionally, natural capital and/or ecosystem service flows may be enjoyed by consumers, analogously to how they enjoy consumer goods. It is clear therefore that we might usefully extend economic models, by analogy with how assets and goods are already treated within them, to encompass natural capital stocks and ecosystem service flows.

To do this we need to consider different cases.

Natural capital is usually divided into four types (see Millennium Ecosystem Assessment, 2005) according to what ecosystem services these assets provide: provisioning services are goods or services, usually already included in economic accounts, and provided by nature – water environment for fisheries production is a good example; supporting and regulating services are maintenance services that nature provides for free e.g. the action of a landscape can provide clean water that a water company takes advantage of, and which would be costly to replicate without the natural asset; and cultural services are goods valued by people for their existence, and which are typically not traded – a beautiful landscape is a good example.

From an economic modelling view however, natural capital broadly produces either (final) goods which are not already counted in economic output, or it produces inputs to production of (either intermediate or final) goods that are already counted in economic output.

Including missing final goods means that a comprehensively environmentally-augmented macroeconomic model would naturally then produce estimates of Green GDP (Hartwick, 1990). Tracking all the capital stocks (natural as well as physical) which underlie an economy’s productive capacity produces an estimate of Genuine Savings (Hanley et al, 2015). Green GDP and Genuine Savings are two highly influential attempts in the literature which adjust standard metrics of economic performance for environmental sustainability. A truly comprehensive treatment of natural capital and ecosystem services in the context of a system-wide model would therefore, in principle at least, allow identification of the impact of any policy intervention on Green GDP and Genuine Savings: these would become additional endogenous variables within the augmented system, the values of which would be dependent upon the entire general equilibrium of that system.[1]

2.1 “Free” final goods from nature

---

[1] Currently Green GDP and Genuine Savings are typically simply accounting frameworks. Of course, this is not to deny the challenges of implementing the appropriate accounting practices, but in themselves these are merely *indicators* of sustainable development. Appropriate policy formulation and evaluation requires an understanding of the determinants of these indicators and the transmission mechanisms that link policy interventions to them. That would be the contribution of an appropriately – and comprehensively - augmented CGE model.

In principle this is the easier case.

To illustrate this category, consider the value of a beautiful view. If we are told that this asset has a value of V, and that steady state interest rates are r, then for equilibrium in asset markets, the steady state value of consumption of this "good" must be rV. Relative to a model without this good, GDP is higher by rV.

In a CGE model however, consumers' expenditure decisions are typically the result of maximising utility subject to a budget constraint. This good must enter the utility function in such a way that consumers choose to spend rV on it.

For example, suppose utility is of the commonly used Constant Elasticity of Substitution[2] (CES) form, then the demand for any particular good, i, is given by:

$$Exp_{i,t} = Exp_t \delta_i^{\rho} \left(\frac{P_t}{p_{i,t}}\right)^{\rho-1} \tag{1}$$

where $Exp_{i,t}$ is the expenditure on good i in time period t, Exp_t is the overall expenditure on consumption goods in time period t (related to other periods through an intertemporal Euler Equation), δ_i is the share in demand on good i, ρ is the elasticity of substitution across goods in the utility function, $p_{i,t}$ is the price of good i in time period t, and P_t is the overall price level (across all goods) faced by consumers in time period t.

So the observation that consumers are spending rV on this good implies that each "unit" has some price, p, related to the marginal utility it provides, and that there is some underlying "real quantity" or "number of units" that is supplied and consumed, F = rV/p. If the price were zero, then optimising consumers would consume in infinite quantities, the fact that they do not do so implies a positive price.

In the case of a beautiful landscape, it may be that the "quantity consumed" does not affect the quantity remaining i.e. the real natural capital stock is unaffected by the ecosystem service flow that is drawn from it (though it could be damaged by e.g. development). The nominal value of this stock, V, will vary with the price. Therefore, for example, a positive income shock will boost the value of this natural asset because consumers will now be more willing to spend on it.

What does all this imply for budget constraints? The new expenditure on this good implies that consumers had more money to spend, even though their wages and savings income etc. is unchanged. It must be the case that the same consumers who pay rV in order to consume this good, are also the owners of this good, so that their income is higher by the same amount, and budget constraints are satisfied.

2.2 Natural inputs to goods already measured in the market economy

---

[2] CES includes Leontief, Cobb-Douglas, and linear perfect substitution as special cases for $\sigma = 0$, $\sigma = 1$, and $\sigma \rightarrow \infty$, respectively

To illustrate this second category, consider a fishery which relies on many natural inputs not included in a traditional economic model, but which sells fish that are always included in any model of the national economy. The final goods sold in the economy are unchanged with the addition of natural capital here, and so GDP is unaltered.

In this case the production function of the sector which uses the natural input has to be altered beyond the use of physical capital, labour and intermediate inputs. For example, Lecca et al (2011) discuss how energy should enter the production function, a discussion that extends naturally to ecosystem services and natural capital. CES production functions are widely used in CGE modelling. When output, Y, is produced using a simple combination of labour, L, and capital, K, the basic CES production function is given by:

$$Y = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where α and β are the shares that must sum to 1 for a constant returns to scale production function, and σ is the elasticity of substitution between the factors. This can be extended to more inputs, say ecosystem service flows, F, but if this is done as in equation (3), then we are saying that the elasticity of substitution is the same, σ, between any two of these inputs.

$$Y = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta L^{\frac{\sigma-1}{\sigma}} + \delta F^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

The alternative is to nest the production function based on the elasticity of substitution between the various inputs. Suppose the elasticity between labour and capital was estimated at σ, but that the elasticity of substitution between this capital-labour composite and natural resources was estimated at ν. This would suggest a nested structure of the form of equation (4).

$$[KL] = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$Y = \left(\gamma [KL]^{\frac{\nu-1}{\nu}} + \delta F^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \tag{4}$$

Alternatively the estimated elasticities could be best matched by nesting a natural resources-capital composite below the contribution of labour, or by nesting a natural resources-labour composite below the contribution of capital. The point Lecca et al (2011) make is to show that this nesting structure matters quantitatively, and that the implications of the modelling choices made should be tested.

These modelling choices relate to the concepts of Weak and Strong Sustainability in the environmental economics literature. Weak Sustainability is the idea that other capital can substitute for natural capital (see Hartwick, 1977), while Strong Sustainability has natural capital and other factors of production as complements. A value for the elasticity of substitution between natural resources and other factors, of between zero and one, means that natural resources and other factors are complementary. In this case a scarcity of natural resources (low quantity used) implies that their value in production (price times quantity used) approaches the total value of the output. This is because with an elasticity of substitution less than one, natural resources are an essential input to production – we are in the realm of Strong Sustainability – and if the quantity of this input falls, then the price (marginal product) rises to such an extent that total expenditure on natural resources rises. In the Weak Sustainability case by contrast, with an elasticity of substitution greater than one, other inputs can be used as substitutes for natural resources. In this case, the use of a low quantity, though still associated with a rise in price (marginal product), implies that total expenditure on natural resources falls.

The production function tells us how many real units of output can be created with a given combination of real input quantities. However, marginal product theory is used to determine prices and the distribution of the income that arises from production, so we must also consider the ownership of natural capital stocks and ecosystem service flows.

Production processes which feature natural resources can be modelled by creating new sectors which supply natural resources to existing sectors – making profits themselves while reducing the profits of the existing sectors (this does not imply any changes in the ownership of any profit streams) – which implies new intermediate goods are added to the national accounts (for which there is no market transaction) and so Gross Outputs would differ from published IO tables. These payments may cycle entirely within individual firms if, say, these firms effectively pay themselves as the owners of these assets. This is similar to the argument above in Section 2.1 in which the consumers own the payments that they themselves make for the ecosystem service flows, and it is also similar to the “Imputed rent” that appears in IO tables as homeowners’ consumption of housing services, in which the payment for these services is almost entirely profit, the entitlement to which is owned by these same homeowners (see ONS (2016b)).

The value of natural capital is a combination of the real quantity of useful inputs to production available, and the price that these inputs fetch in the market – which will be a function of their marginal products. A fall in the value of natural capital could therefore be caused either by a fall in the available stocks of natural capital (a negative environmental shock, say) or by a fall in the price (caused by e.g. reduced demand for the output of a sector that relies on these natural inputs).

We now note that the inclusion of natural capital into the production function of a good will typically affect the measure of physical capital which would otherwise be deemed to have been used. There are two main methods by which physical capital within a general equilibrium model can be measured: externally, and internally. If capital is measured internally, within the model, then it can be inferred from the interest rate and from the profits paid to the owners of capital. In steady state general equilibrium, profits are a fair return on capital and so the initial value of the capital stock in sector i, is given by:

$$K_i(1) = \frac{Profits_i(1)}{r(1)} \tag{5}$$

with investments made going forward such that:

$$K_i(t+1) = I_i(t) + (1-\delta_i)K_i(t) \tag{6}$$

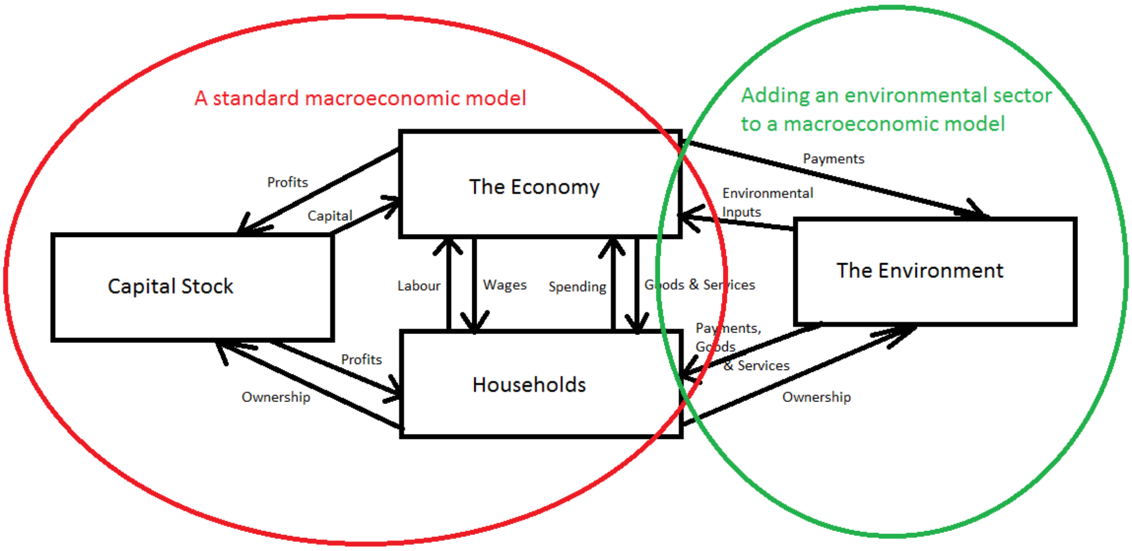
Conversely, if capital is measured externally to the model, then it is an input which is imposed on the model from external data. For example, capital stock could be taken from the Penn World Tables which builds up capital stocks through observed past investment expenditures along with some assumed depreciation. In effect, capital within each sector, i, would behave like the basic accumulation equation above even before the initial, calibration, period for the model, and $K_i(1)$ would be imposed.

In steady state general equilibrium profits are a fair return on capital, and so with externally measured capital we need either: (i) to acknowledge that the calibrated economy is not in steady state equilibrium (supernormal profits are yet to attract new entrants, say); (ii) to allow an endogenous interest rate equating observed profits with the return on capital (this endogenous interest rate will not necessarily equal the observed interest rate in the data); or (iii) to recognise that the externally measured capital stock does not represent the entirety of the economy's asset base.

Irrespective of which of these methods of measuring the capital stock in the economy is used, it will usually be the case that any data on natural capital will be in the form of an externally derived figure for the natural capital base – akin to the externally measured capital discussed above (though possibly with a very different capital accumulation equation) – which will be additional to the physical capital stock. If capital is measured externally then we simply keep track of these two different capital stocks (with quantitative implications for (i) how far the economy is from steady state equilibrium; or (ii) the calculated value of the return of the return on capital). If however capital is measured internally, we must recognise that the capital stock calculation performed within the model is for the total capital stock, and this figure less the externally measured natural capital figure is our estimate for physical capital.

Figure 1 shows a schematic of an environmentally augmented general equilibrium macroeconomic model. Such a model can be used to generate scenarios for analysis in which: changes in natural capital have an impact upon economic performance; policy and/or economic activity have an impact upon the use or level of natural capital; and which shows the feedbacks between these. We discuss the analysis that can be conducted on such a model more fully in Section 4 with the help of specific examples.

Fig 1



In summary, so long as we are considering the production of the same final goods (e.g. augmenting fish production to include payments for the use of ecosystem service flows), a Natural Capital - augmented computable general equilibrium model output for GDP should match published GDP figures. However, if there is no market transaction for the rent of natural capital stocks or for the purchase of ecosystem service flows, then the measures of Gross Output produced by a Natural Capital augmented macroeconomic model will not match published figures for Gross Output, which exclude the "hidden" payments for these goods – "hidden", since no money changes hands. If we are augmenting the final goods considered (e.g. increasing economic activity in respect of the flow value of beautiful landscapes that consumers enjoy), then the measure of GDP produced by a Natural Capital augmented macroeconomic model will also be altered[3].

New economic agents (sectors or actors) may need to be created, with these agents receiving payments for supplying environmental goods and services, and distributing these payments on to the owners of these environmental assets. The data required to implement such a Natural Capital - augmented computable general equilibrium and quantify each approach is a challenge – and the focus of section 3. In this paper we consider what is required for model augmentation using a single example of a natural capital stock and ecosystem service, which does not impact GDP, and explore its integration within an illustrative numerical model. Subsequently, we shall integrate this analysis into a fully-specified CGE model of the Scottish economy, and will focus on incorporating a very limited set of natural assets. In the longer-term we intend to extend the range of natural assets included within the model.

3. Data availability for Natural Capital stocks and Ecosystem Service flows

The primary data source that we can draw upon is ONS (2016) who produce Natural Capital accounts for the UK covering some natural capital stocks and ecosystem service flows. Other data sources include: the World Bank's Wealth Accounting and the Valuation of Ecosystem Services (WAVES)[4]; and the System of Environmental-Economic Accounting (SEEA) Experimental Ecosystem Accounting (EEA) being developed by The United Nations Statistical Division (UNSD), the United Nations Environment Programme (UNEP) TEEB Office, and the Secretariat of the Convention on Biological

---

[3] If we are altering the suite of final goods in a GDP-type measure, then depending upon the final goods considered, it could perhaps be benchmarked against estimates of Green GDP (Hartwick, 1990). Model output will also include estimates of Genuine Savings (Hanley, 2015) which can, depending upon the consistency of natural capital stocks in the model with external data, be benchmarked against these data. See e.g. http://data.worldbank.org/indicator/NY.ADJ.SVNG.GN.ZS?view=chart for World Bank data on what it calls "adjusted net savings". However, while a truly comprehensive treatment of natural capital and ecosystem services within a CGE model would be able to track both Green GDP and Genuine Savings responses to disturbances, our shorter-term objectives in this paper are much more modest.

[4] See https://www.wavespartnership.org/en

Diversity[5]. In furthering this research, we hope to also be able to draw upon new work being produced under the Scottish Government's Rural Affairs, Food and the Environment, Strategic Research Programme, Theme 1: Natural Assets[6].

ONS (2016) provides a valuation of various ecosystem service flows for the UK, and calculates the value of the natural capital stock as the net present value of these flows projected into the future. This framework is appropriate for considering the value of an ecosystem service flow that is provided by some putative "environment" sector or institution. For example, ONS (2016) reports that "16% of the profits from agricultural production can be attributed to services provided by the UK's natural assets". One approach then could be to augment the expenditure of the agricultural sector to include payments to the environment sector (equal to 16% of gross operating surplus), thus reducing agricultural sector profits. The environment sector would sell its output to agriculture and make corresponding profits of its own (as discussed in Comerford, 2017).

Such a framework makes sense and can form a basis for the creation of a Natural Capital augmented CGE model. Note however that such a framework does not necessarily correspond to what may be our common sense notions of the value of the natural environment. In particular, the value of the ecosystem service flow is determined by the market value of the products produced with these services. This means that an increase in the price of a good, say agricultural produce, is associate with an increase in the value of natural capital – without there being any implication of environmental improvement. This is not necessarily a conceptual problem, but it may be a problem in communicating results, since a growth in the value of natural assets is easily confused with environmental improvement.

ONS (2016) has valuations for a number of ecosystem service flows, each of which is used in the production of traded goods and services: oil and gas; coal and peat; minerals; timber; water; agricultural biomass; fish; hydro power; wind power; and recreational services[7]. It also has valuations for air filtration and pollution removal, and carbon sequestration[8], which are currently "free goods", the inclusion of which will be additive to GDP.

There are many other examples of ecosystem services so an environmentally augmented CGE model will always struggle to make any claims of having fully captured everything. The best that we can do is use the data that is available. And data creation, collection and interpretation is difficult. For

---

[5] See https://unstats.un.org/UNSD/envaccounting/eea_project/default.asp

[6] See http://www.gov.scot/Topics/Research/About/EBAR/StrategicResearch/strategicresearch2016-21/srp2016-21/naturalassets

[7] The value of Recreation Services are estimated using expenditures on admission, parking, and transport, and so are already captured in GDP.

[8] Note also that the carbon sequestration ecosystem service flow is another case where communication issues could be problematic. Consider a mature forest or peatland that stores a large quantity of carbon, but in which current sequestration rates are zero. In this framework, the value of the carbon sequestration services provided is zero, and hence the value of the natural capital - as the net present value of these flows projected into the future - is also zero – despite its stores of carbon (of course if these were released then the carbon emissions should be counted as a negative ecosystem service flow). Any model which includes carbon sequestration should therefore also include carbon emissions on the same basis i.e. the social cost of carbon times total emissions will represent a negative ecosystem service flow that will lower the GDP produced by the augmented CGE model. Carbon emissions are available at sectoral level for Scotland from Scottish Government (2016).

example, consider the flood prevention services provided by forest cover. Perhaps this can be inferred from house price differentials across locations, combined with insurance premiums for those in locations which do face high flood risks, but once the ecosystem service of flood prevention is being provided in a region, there is no payment made for that service[9].

A further example which impinges upon both the production and consumption sides of the economy is all those ecosystem services that promote health. For example, the pollution removal services provided by urban trees, and the recreation opportunities provided by biodiverse amenity spaces and by beautiful landscapes, may promote health outcomes in the population. Clearly people value their own health outcomes and so this is a consumption good (albeit one which suffers from the same valuation difficulties, and difficulties in incorporation within a general equilibrium model which satisfies demand optimisation under a budget constraint). However, a healthy population is also a more productive population with a lower rate of inactivity, and higher human capital. It may therefore be ultimately desirable[10] to incorporate health promoting ecosystem services into an economy wide model in order to satisfy the basic objective of improving the understanding of the impact of changes in natural capital on the conventionally measured economic performance, the impact of economic changes on the natural capital, and the feedbacks between these.

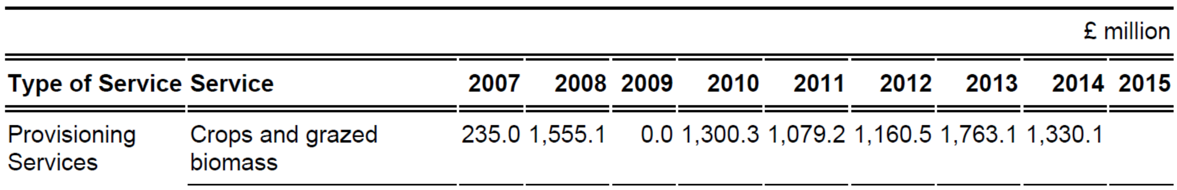
Given these points, in this paper we focus on exploring the introduction of the agricultural biomass ecosystem service, which we have data on from ONS (2016), into a CGE model. Initially we do this in illustrative form which demonstrates how this can be done in principle, but the final version of this paper will implement this in a CGE model of the Scottish economy.

4. Augmenting a CGE model with the Agricultural Biomass Ecosystem Service

Data

ONS (2017) reports that the value of ("provisioning services") ecosystem service flows provided to agricultural production in the UK in 2013-14 was around £1.5bn. This figure can be apportioned in some way to Scotland (perhaps using a population share to reflect Scotland's larger land per capita, but poorer agricultural land quality). The following table is from ONS (2017) UK natural capital ecosystem accounts for freshwater, farmland and woodland.

Table 13: UK farmland annual monetary value by service, 2007 to 2015



£ million

Type of Service	Service	2007	2008	2009	2010	2011	2012	2013	2014	2015
Provisioning Services	Crops and grazed biomass	235.0	1,555.1	0.0	1,300.3	1,079.2	1,160.5	1,763.1	1,330.1	

---

[9] Though payments for Flood Risk Management (e.g. an urban catchment paying a rural location to allow flood waters) will enter GDP.

[10] Though data availability versus data requirements mean that this is not something that will feature in any modelling initially.

These figures are calculated on a *Resource Rent* basis. That is, agricultural profits (which can be observed in the UK IO tables) are deemed to be a fair return on capital invested in agricultural activities (which presumably the ONS have some estimate of, despite this not being observed in IO tables). However, it seems that agricultural profits typically outstrip the fair return on the physical capital stock in agriculture, and the ONS attribute the balance of the profits, over the fair return to physical capital, to a return on environmental assets.

The values in the table above show large fluctuations in the value of this ecosystem service calculated in this manner. These fluctuations are largely caused by fluctuations in the price of agricultural output, which causes fluctuations in agricultural profits, which feed directly through to the value of the ecosystem service flow, since the fair return on physical capital will not fluctuate so much.

ONS (2017) also provide associated natural capital stock values, equal to the discounted present value of future ecosystem service flows. Future ecosystem service flows are estimated as the average of the previous 5 flow values, assumed constant in into the future.

Natural Capital Monetary Estimates - Asset Values, £ million, 2014 Prices, 2007 - 2014									
	2006 (£m)	**2007 (£m)**	**2008 (£m)**	**2009 (£m)**	**2010 (£m)**	**2011 (£m)**	**2012 (£m)**	**2013 (£m)**	**2014 (£m)**
Provisioning Services									
Agriculture[1]		14,859.7	20,860.1	12,992.6	15,462.2	16,760.4	21,274.5	22,288.9	32,352.9

If we model these assets as perpetuities, and label as $\overline{[pF]}$ the annual flow value of ecosystem services (equal to the average flow value over the previous 5 years), V as the asset value, then we have $r = \frac{\overline{[pF]}}{V}$ relating these two data series from the ONS. Doing this (starting in 2011 since that is first year for which we can calculate a 5 year average) produces a series of implied interest rates:

2011	2012	2013	2014
4.9%	4.8%	4.7%	4.1%

The consistency of these implied interest rates is indicative of the fact that the asset values are less subject to the fluctuations produced by the calculation method for this ecosystem service. Given that when calibrating the CGE model, we assume the base year calibrated position of the economy is a steady state, it is appropriate to use the asset value series from the ONS for the value of this ecosystem service. Therefore the data series can be based on the equation $\overline{[pF]} = rV$ where V is the natural capital asset value based on ONS data, adjusted for Scotland's "share", and r is the market interest rate in the CGE base year/calibration.

Creating an Environmental Sector in the CGE Model

The resulting input to production must be paid for i.e. the nominal value of the inputs to agriculture, $\overline{pF}$, are paid by the agriculture sector to the environment sector, reducing agriculture profits, and realising environmental profits (the owners of these two profit streams are the same agents however, so budget constraints are unaffected). This is straightforward.

What is perhaps not so straightforward is choosing how this environmental sector optimally supplies inputs to the agricultural sector based on the prices it sees and the physical characteristics of the good it is supplying.

We firstly need to make modelling decisions about how this natural capital stock responds to its exploitation in production. We imagine that natural inputs to agriculture can be over-used if production is too intensive, but that if allowed to regenerate, then they do so. By analogy with the depletion and accumulation processes usually assumed for renewable resources (and in particular for fisheries), we postulate a logistic process for the real stock of environmental inputs available, S_t:

$$S_{t+1} = S_t + gS_t\left(\frac{\bar{S}-S_t}{\bar{S}}\right) - F_t \tag{7}$$

where F_t is the real quantity of inputs supplied to the agricultural sector, g is the regeneration rate for this environmental asset, and $\bar{S}$ is the available stock that the system would tend towards in the absence of any use of this asset.

At time t, the environmental sector choses a sequence $F_s,\ s \geq t$, to maximise the present value of supplying ecosystem services, taking prices as given (this is consistent with there being a myriad of suppliers in this sector, none of which is big enough to influence the price of environmental inputs), $\Pi_t = \sum_{s=t}^{\infty} \beta_t(s) E_t[p_s] F_s$, where $\beta_t(s)$ is the discount factor that applies over the interval $(t, s \geq t)$, and $E_t[.]$ denotes expectations formed at time t.

It can be shown (see Appendix) that the environmental sector follows a policy rule for deciding the real quantity to supply as a function of prices and the available stock:

$$F_t = S_t + gS_t\left(\frac{\bar{S}-S_t}{\bar{S}}\right) - \frac{\bar{S}}{2g}\left(1 + g - \frac{p_t}{\beta_t E_t[p_{t+1}]}\right) \tag{8}$$

Which has steady state values for the stocks and flows:

$$S^* = \frac{\bar{S}}{2g}\left(1 + g - \frac{1}{\beta^*}\right) \tag{9}$$

$$F^* = gS^*\left(\frac{\bar{S}-S^*}{\bar{S}}\right) \tag{10}$$

Interaction with the Agricultural Sector

The initial specification of the CES production function for the agricultural sector is:

$$X = B\left(\gamma A^{\frac{\sigma-1}{\sigma}} \alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}} (1-\alpha) L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)[VV]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where the capital input K, labour input L, intermediate goods aggregate input [VV], the efficiency of value added A, the efficiency of gross output B, the share of value added in gross output γ, and the share of capital in value added α, are all determined in the calibration or given directly from data; and the elasticity of substitution σ=0.3 is set by assumption.

When augmented to include natural capital the production function becomes:

$$X = B\left(\gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}\delta F^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}(1-\alpha-\delta)L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)[VV]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{11}$$

Profit maximisation in the agricultural sector will then mean maximising:

$$\pi = p_{Ag}X - rK - pF - wL - p_I[VV]$$

Which has the following condition for F implied by the first order condition with respect to F:

$$F = \left(\frac{p_{Ag}}{p}\gamma\delta\right)^{\sigma}(AB)^{\sigma-1}X \tag{12}$$

In general, the price of the ecosystem service will be determined as the marginal product of this factor, and production will be simply a dynamic version of Eqn (11):

$$p_t = \frac{\partial}{\partial F}\left[p_{Ag}(t)X_t\right] = p_{Ag}(t)\gamma\delta(AB)^{\frac{\sigma-1}{\sigma}}\left(\frac{X_t}{F_t}\right) \tag{13}$$

$$X_t = B\left(\gamma A^{\frac{\sigma-1}{\sigma}}\alpha K_t^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}\delta F_t^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}(1-\alpha-\delta)L_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)[VV]_t^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{14}$$

Calibration

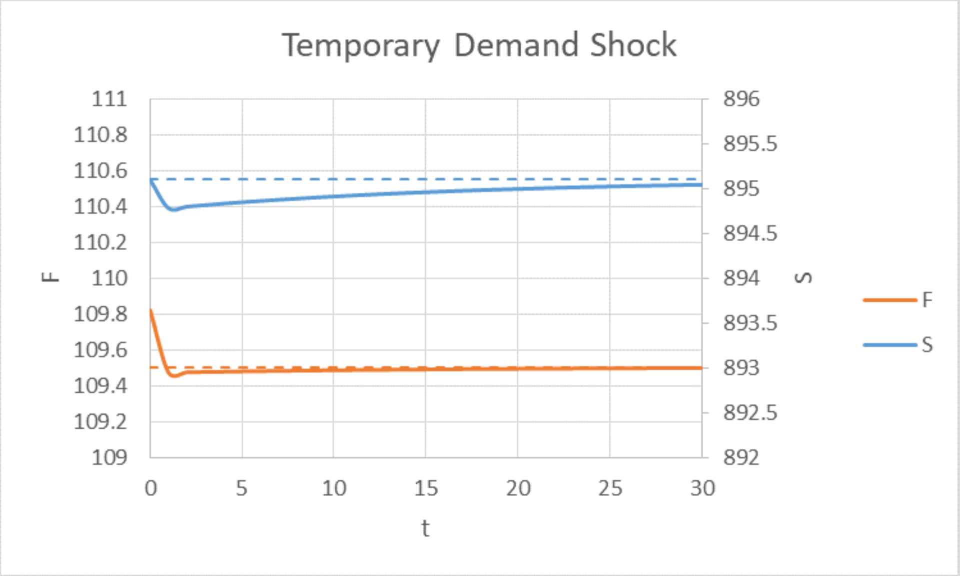
This subsection briefly describes the additional calibration procedure needed to calibrate the environmentally augmented CGE (over and above the calibration procedure normally followed for the standard CGE).

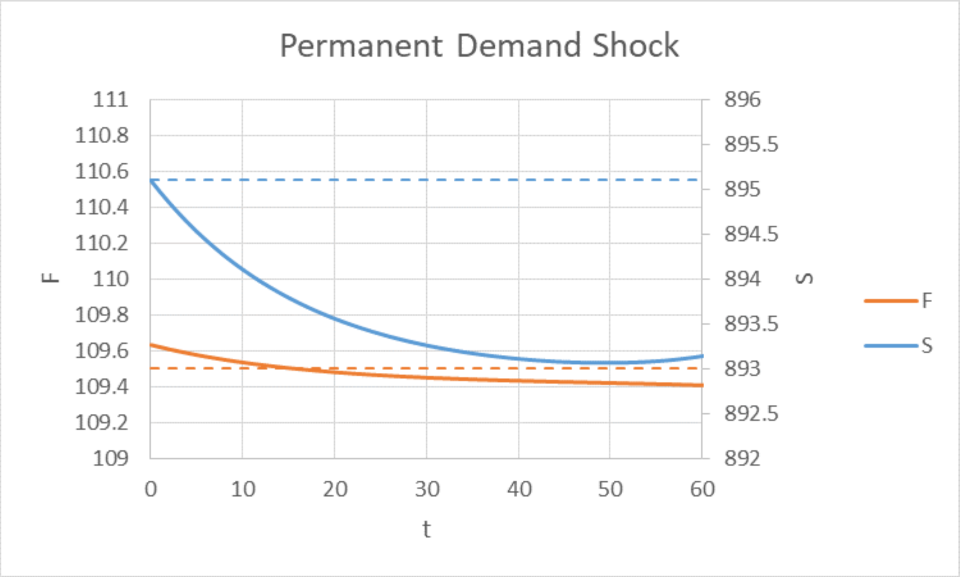
- We have pF from data (as noted above).
- In steady state, all prices are 1. Therefore we have F^* directly from the data since we assume the economy is initially in a steady state.
- Profit maximisation in the agricultural sector, Eqn (12) (as well as equivalent for other inputs) allows us to determine the share of the ecosystem service in value added, δ.
- Steady state relationships for physical stocks and flows, Eqns (9) and (10) imply a relationship between the unexploited stock value $\bar{S}$ and the regeneration rate g. Other data, or expert opinion, should be obtained to disentangle these. Once this is done (e.g. estimate from the dynamics of crop rotations and length of time fields left fallow?) then we have g, $\bar{S}$, and S^*, and the model is fully calibrated.
- To project the model forward, we use the policy function for ecosystem service supply, Eqn (8); the logistic equation of motion for the stock, Eqn (7); the marginal product pricing formula for the ecosystem service, Eqn (13); and the production function for agricultural output, Eqn (14).

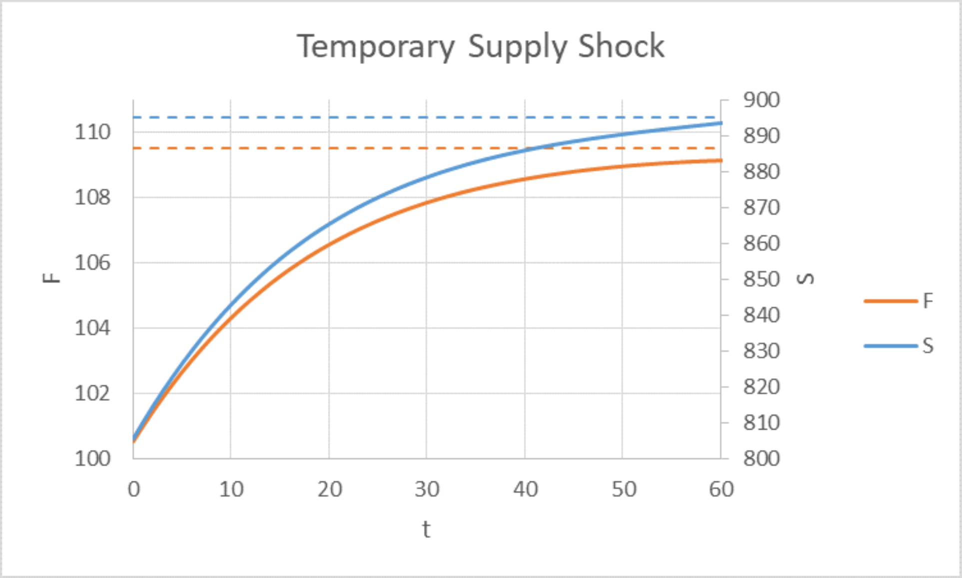
Results

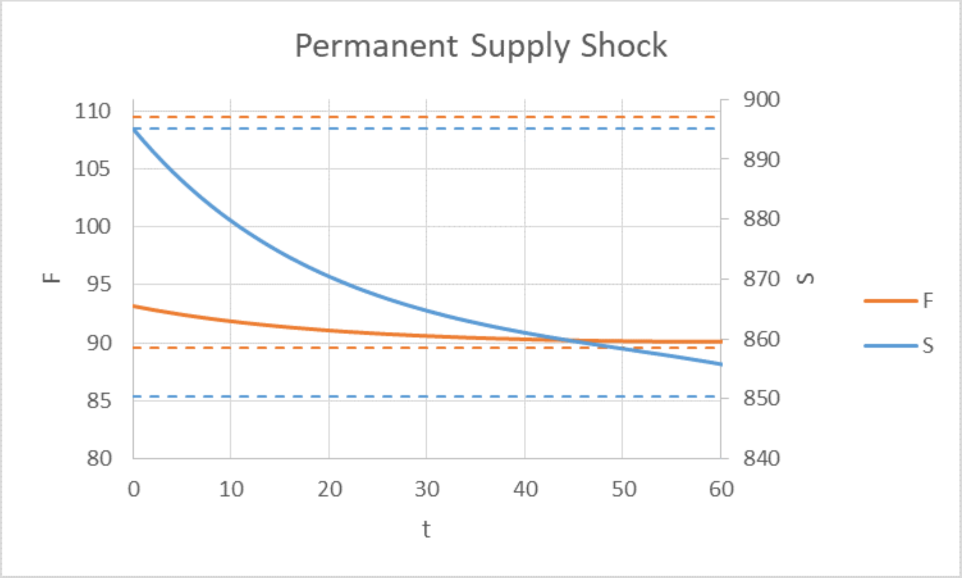
Here we show illustrative results, generated using a partial equilibrium model, in which labour, capital, and intermediates are assumed to be fixed factors, so that there is only a price response from these; and we have an endogenous price and quantity response from natural capital. We show the impacts of temporary and permanent demand and supply shocks:

- Demand shocks: 1% rise in nominal expenditure on agricultural output
- Temporary supply shock: -10% change in real value of natural capital stock
- Permanent supply shock: fall in value of the regeneration rate, g









These graphs show the model’s responses to these shocks as the endogenous variables move towards their new steady states (which will be the same as the old steady states in the case of temporary shocks and demand shocks). The first graph shows the response of the supply of ecosystem service inputs, and the corresponding behaviour of the natural capital stock to a temporary demand shock. In the first period, there is a 1% rise in nominal expenditure on agricultural output. With fixed factors there would be no change in output, and this would simply lead to a 1% rise in prices. Here we have endogenous natural capital though. The rise in prices means that a higher level of ecosystem services are used in the first period, boosting agricultural output. This depletes the natural capital stock however below its steady state value. In future periods expenditure returns to the steady state level, but the below steady state level of natural capital stocks means that supply of ecosystem service inputs are also below the corresponding steady state value while the stock regenerates. Agricultural output is therefore below steady state, and all stocks and flows approach steady state as time passes.

The second graph shows a permanent 1% rise in expenditure on agricultural output, and the dynamics of the model’s response are similar to the one period expenditure shock except much more protracted. For a demand shock, the final steady state is again equal to the initial steady state (though nominal prices differ by 1%).

The third graph shows an environmental shock: the real value of the stock of natural capital is lowered below its steady state value. The optimal response is to reduce the supply of ecosystem services to agricultural production until the stock recovers. The natural capital stock, the ecosystem service flow supplied, and the level of agricultural output, all approach their steady state values from below. Note that with fixed expenditure on agricultural output, prices will be above their steady state level and approach their steady states from above.

Finally, the fourth graph shows the impact of a fall in the regeneration rate of natural capital: another form of environmental shock with permanent effects. The stock starts at the old steady state level which is above the new steady state. So the optimum supply of ecosystem service inputs to agricultural production is above the new steady state level. So both stock and the flow (and agricultural output) approach the new steady state values from above. Note however, that the new steady state has lower values for the stock, the flow and for levels of production than the initial steady state.

Next Steps: A Model Of Scotland (AMOS)

The Fraser of Allander Institute at the University of Strathclyde has developed the AMOS model. This is a Computable General Equilibrium (CGE) model, a type of general equilibrium macroeconomic model that is sufficiently complex (with a multi-sectoral production structure) as to require numerical simulation (rather than exhibiting properties which can be investigated analytically). This is the model we ultimately seek to augment with natural capital.

This model has been applied to a host of analyses not only of the Scottish economy but also to analyse the economic consequences of policies focused at energy technologies/use and environmental measures (e.g. carbon taxes).

The next step of this project is to implement the above agricultural biomass natural capital (sub-) model as well as modelling carbon emissions, into AMOS.

5. Conclusions

This paper has explored the challenges involved in integrating natural capital and ecosystem services within computable general equilibrium models. The general approach has been illustrated through the specific example of agricultural biomass ecosystem service flows as an input into agricultural production, a focus that precludes any impact on value-added that would, for example, arise through the incorporation of ecosystem flows that are currently not valued within measures of GDP. Issues of appropriate model specification are addressed through comparison with the current treatment of the agricultural sector within FAI’s computable general equilibrium model, AMOS, which does not separately identify natural capital stocks and their associated service flows. The data on natural capital stocks and ecosystem flows are discussed and are used, together with key parameter values, to calibrate an illustrative partial equilibrium model of the agricultural sector which augments the AMOS specification of that sector to incorporate a natural capital stock and its associated flow of ecosystem services. The properties of this model are then established through numerical simulation and appear plausible.

The next step is to fully incorporate the extended specification into a variant of the AMOS model that already incorporates a disaggregated agricultural sector and models carbon emissions. This will create a natural-capital-augmented CGE model of Scotland that incorporates necessary detail on the agriculture sector. The augmented model will allow us to capture the fully general equilibrium interdependencies of the economic and environmental subsystems. Accordingly the natural-capital-augmented CGE significantly extends the capabilities of the modelling framework in a number of important directions, including allowing us to:

- track how this natural capital stock interacts with economic activity, contributing to the modelling of (a key element of) Genuine Savings in the economy;
- implement environmental shocks and see how the agricultural sector and the whole economy (including aggregate and sectoral value added and employment effects) reacts, and how this reaction spills over through the production structure of the whole economy;
- explore the economic and environmental impact of a range of demand and supply-side policy interventions in agriculture, including export stimulation;
- conduct micro-to- macro analyses of the system-wide effects of, for example, rolling out various farm-level policies to stimulate (economic and/or environmental) productivity across the agriculture sector.

These analyses will also be useful for the consideration of environmental policies. A good example of where this analysis could be especially relevant in the current policy environment is in considering payment schemes which spend the budget previously spent under the EU’s Common Agricultural Policy. These can be targeted at environmental improvement and the provision of ecosystem services. Such policies could be looked at in a comprehensive model that jointly features agricultural profitability, ecosystem services, and agriculture’s links into the wider economy. Analysis in such a framework could be invaluable when considering analysis associated with Brexit and the environment.

In the longer-term we aim to incorporate further natural capital stocks and ecosystem service flows into CGE models, and this work is ongoing. Ultimately we would aspire to a comprehensive treatment of natural capital within our CGE models, which would allow us automatically to track the transmission mechanisms and impact of any disturbance, including policy interventions, simultaneously on green GDP and genuine savings as well as aggregate and sectoral economic activity. At present data restrictions, as well as limitations to our understanding of a number of policy transmission mechanisms, inhibit the speed with which we are likely to achieve that ambition, however the current Scottish Government research project is in the process of relaxing these inhibitors to further progress.

References

Anger, A., Shmelev, S., Morris, J., Zenghelis, D., & Di Maria, C. (2014) "*UK National Ecosystem Assessment Follow-on. Work Package Report 2: Macroeconomic implications of ecosystem service change and management: A scoping study*", UNEP-WCMC, LWEC, UK.

Comerford, D. (2017) "*Economic analysis of the agriculture and food sectors in Scotland using extended Input-Output analysis*", RESAS1.4.2ciii D1

Hanley, N., Dupuy, L., & McLaughlin, E. (2015) "*Genuine Savings and sustainability*", Journal of Economic Surveys, doi: 10.1111/joes.12120

Hartwick, J.M. (1977) "*Intergenerational Equity and the Investment of Rents from Exhaustible Resources*", American Economic Review, 67, December, pp. 972-74

Hartwick, J.M. (1990) "*Natural resources, national accounting and economic depreciation*", Journal of Public Economics 43: 291–304

Lecca, P., Swales, K., and Turner, K. (2011), "*An investigation of issues relating to where energy should enter the production function*", Economic Modelling 28 (2011) 2832-2841

Millennium Ecosystem Assessment (2005), "Ecosystems and Human Well-Being: Synthesis", Island Press, Washington

ONS (2016) "*UK natural capital: monetary estimates, 2016*", available online at https://www.ons.gov.uk/economy/environmentalaccounts/bulletins/uknaturalcapital/monetaryestimates2016/pdf

ONS (2016b) "*Changes to National Accounts: Imputed Rental*", available online at https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts/articles/changestonationalaccounts/imputedrental/pdf

Scottish Government (2016) "*Greenhouse Gas Estimation*", available online at http://www.gov.scot/Topics/Statistics/Browse/Economy/Input-Output/CarbonAssessment

Simon, H.A. (1956) "*Rational Choice and the Structure of the Environment*", Psychological Review. 63 (2): 129–138. doi:10.1037/h0042769. PMID 13310708.

1 Appendix

- Assume stock S_t at time t, with a logistic equation of motion

  $$S_{t+1} = S_t + gS_t \left(\frac{\bar{S} - S_t}{\bar{S}} \right) - F_t$$

  where the units of $\bar{S}$ is the quantity of the stock that would exist in the limit under no exploitation, S_t is the current stock, F_t is the flow of ecosystem services used in the economy, and g is the regeneration rate of the stock.

  - NB This formulation for the real value of the stock, in the same units as the real value of the flows under a logistic equation of motion, precludes defining the value of the stock as the present value of future flows. To see this, conside the steady state with $\bar{S} = 1$. It must be the case that $F^* = gS^*(1 - S^*)$, and this does not depend on the interest rate. Imposing $S^* = \frac{\beta}{1-\beta} F^*$ implies a specific expression for S^* in terms of g and β, which is not the same as the expression implied by the steady state under optimal behaviour. Therefore we can only use one of the natural capital or ecosystem services values from the ONS since they are related via $S^* = \frac{\beta}{1-\beta} F^*$

  At start of period t, the environmental sector sees p_t, chooses F_t and receives $p_t F_t$. Also $\beta_t(s)$ is known for all s, but need to use $E_t[\beta_\tau(s)]$ for all $\tau > t$.

- In principle, the environmental sector will plan to supply a sequence of real ecosystem service flows, $F_s, (s \geq t)$, in order to maximise the PV of profits, taking its expectation (at time t) of prices as given

  $$\max_{\{F_s\}} \Pi_t = \max_{\{F_s\}} \sum_{s=t}^{\infty} \beta_t(s) E_t[p_s] F_s$$

  where $\beta_t(s)$ is the discount factor applying over the period (t, s)

- Let $V(S_t) = \max_{\{F_s\}} \Pi_t$ be the value function for this optimisation problem and denote $\beta_t(t+1)$ as β_t, $\beta_{t+1}(t+2)$ as β_{t+1}, etc. The assumption is that we have already solved the maximisation problem i.e. we have optimally chosen $F_s, \forall s \geq t$. The value function is the value of the objective function evaluated at the optimal policy. Because we have already chosen the controls to maximise the objective function, the value function is no longer a function of the controls, it is a function of the states i.e.

  $$\begin{aligned} V(S_t) &= p_t F_t + \beta_t V(S_{t+1}) \\ &= p_t F_t + \beta_t V \left(S_t + gS_t \left(\frac{\bar{S} - S_t}{\bar{S}} \right) - F_t \right) \end{aligned}$$

The first order conditions are implicitly part of the optimisation problem that we have assumed to have already solved i.e.:

$$\frac{\partial \Pi}{\partial F_t} = 0 = \frac{\partial}{\partial F_t}\left[p_t F_t + \beta_t V\left(S_{t+1}\right)\right] = p_t - \beta_t \frac{\partial V\left(S_{t+1}\right)}{\partial S_{t+1}}$$

i.e.

$$p_t = \beta_t \frac{\partial V\left(S_{t+1}\right)}{\partial S_{t+1}}$$

The envelope theorem essentially says that the total derivative of $V\left(S_t\right)$ can be calculated by taking the partial derivative i.e.

$$\begin{aligned} \frac{\partial V\left(S_t\right)}{\partial S_t} &= \beta_t \frac{\partial V\left(S_{t+1}\right)}{\partial S_{t+1}} \frac{\partial S_{t+1}}{\partial S_t} \\ &= \beta_t \frac{\partial V\left(S_{t+1}\right)}{\partial S_{t+1}} \left(1 + g - 2g\frac{S_t}{\bar{S}}\right) \end{aligned}$$

combining

$$\begin{aligned} \frac{\partial V\left(S_{t+1}\right)}{\partial S_{t+1}} &= \beta_{t+1} \frac{\partial V\left(S_{t+2}\right)}{\partial S_{t+2}} \left(1 + g - 2g\frac{S_{t+1}}{\bar{S}}\right) \\ \frac{p_t}{\beta_t} &= E_t\left[p_{t+1}\right]\left(1 + g - 2g\frac{S_{t+1}}{\bar{S}}\right) \\ &= E_t\left[p_{t+1}\right]\left(1 + g - \frac{2g}{\bar{S}}\left(S_t + gS_t\left(\frac{\bar{S} - S_t}{\bar{S}}\right) - F_t\right)\right) \end{aligned}$$

i.e.

$$F_t = S_t + gS_t\left(\frac{\bar{S} - S_t}{\bar{S}}\right) - \frac{\bar{S}}{2g}\left[1 + g - \frac{p_t}{\beta_t E_t\left[p_{t+1}\right]}\right]$$

which is our general policy rule for he choice of $F_t = F\left(S_t\right)$. NB β_{t+1} is the discount factor which will apply over the period $(t+1, t+2)$.

- We can then derive steady state expressions using

  $$\begin{aligned} F^* &= S^* + gS^*\left(\frac{\bar{S} - S^*}{\bar{S}}\right) - \frac{\bar{S}}{2g}\left[1 + g - \frac{1}{\beta}\right] \text{ from policy rule} \\ F^* &= gS^*\left(\frac{\bar{S} - S^*}{\bar{S}}\right) \text{ from steady state condition for stock} \end{aligned}$$

  i.e.

  $$S^* = \frac{\bar{S}}{2g}\left[1 + g - \frac{1}{\beta}\right]$$

  NB need regeneration rate greater than the interest rate to aviod exhausting the stock.

- The nominal value of the ecosystem service flow at t, is $p_t F_t$. p_t is determined by the marginal product of F_t in production.

- The ecosystem service flows are used in agricultural production. In the model as it currently stands, agricultural output is given by:

$$\begin{aligned}
X &= B\left(\gamma\left[KL\right]^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left[VV\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= B\left(\gamma\left[A\left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left[VV\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= B\left(\gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left[VV\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

  where the elasticity of substitution between inputs is set in the SAM at $\sigma = 0.3$, and the share parameters γ and α and efficiency parameters A and B, are calibrated, given the base year input values for capital K, labour L and intermediate inputs $[VV]$.

- We modify this to include the agricultural biomass ecosystem service input (as part of value added):

$$X = B\begin{pmatrix} \gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}\delta F^{\frac{\sigma-1}{\sigma}} \\ +\gamma A^{\frac{\sigma-1}{\sigma}}(1-\alpha-\delta)L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left[VV\right]^{\frac{\sigma-1}{\sigma}} \end{pmatrix}^{\frac{\sigma}{\sigma-1}}$$

- The agricultural sector will maximise profits taking prices as given

$$\Pi_{Ag} = p_{Ag}X - rK - wL - E\left[p\right]F - p_i\left[VV\right]$$

$$0 = p_{Ag}\frac{\partial X}{\partial F} - E\left[p\right]$$

  i.e.

$$F = \left(\frac{p_{Ag}}{E\left[p\right]}\gamma\delta\right)^{\sigma}(AB)^{\sigma-1}X$$

- Marginal product of F

$$\begin{aligned}
p &= \frac{\partial}{\partial F}\left[p_{Ag}X\right] = p_{Ag}\frac{\partial X}{\partial F} \\
&= p_{Ag}\gamma\delta(AB)^{\frac{\sigma-1}{\sigma}}\left(\frac{X}{F}\right)^{\frac{1}{\sigma}}
\end{aligned}$$

- Other FOCs:

$$B^{-\frac{\sigma-1}{\sigma}}X^{\frac{\sigma-1}{\sigma}} = \begin{pmatrix} \gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}}\delta F^{\frac{\sigma-1}{\sigma}} \\ +\gamma A^{\frac{\sigma-1}{\sigma}}(1-\alpha-\delta)L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left[VV\right]^{\frac{\sigma-1}{\sigma}} \end{pmatrix}$$

$$\begin{aligned}
\frac{\partial \Pi}{\partial K} &= p_{Ag}B\frac{\sigma}{\sigma-1}\left(\right)^{\frac{\sigma}{\sigma-1}-1}\gamma A^{\frac{\sigma-1}{\sigma}}\alpha\frac{\sigma-1}{\sigma}K^{\frac{\sigma-1}{\sigma}-1} - r \\
&= p_{Ag}B\left(\right)^{\frac{1}{\sigma-1}}\gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{-\frac{1}{\sigma}} - r \\
&= p_{Ag}B^{\frac{\sigma-1}{\sigma}}X^{\frac{1}{\sigma}}\gamma A^{\frac{\sigma-1}{\sigma}}\alpha K^{-\frac{1}{\sigma}} - r = 0
\end{aligned}$$

$$K = \left(\frac{p_{Ag}\gamma\alpha}{r}\right)^{\sigma} (AB)^{\sigma-1} X$$

$$L = \left(\frac{p_{Ag}\gamma(1-\alpha-\delta)}{w}\right)^{\sigma} (AB)^{\sigma-1} X$$

$$X = B\left(\begin{array}{c} \gamma A^{\frac{\sigma-1}{\sigma}} \alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}} \delta F^{\frac{\sigma-1}{\sigma}} \\ + \gamma A^{\frac{\sigma-1}{\sigma}} (1-\alpha-\delta) L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)[VV]^{\frac{\sigma-1}{\sigma}} \end{array}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial [VV]} &= p_{Ag} B \frac{\sigma}{\sigma-1} ()^{\frac{\sigma}{\sigma-1}-1} (1-\gamma) \frac{\sigma-1}{\sigma} [VV]^{\frac{\sigma-1}{\sigma}-1} - p_i \\ &= p_{Ag} B^{\frac{\sigma-1}{\sigma}} X^{\frac{1}{\sigma}} (1-\gamma) [VV]^{-\frac{1}{\sigma}} - p_i = 0 \end{aligned}$$

$$[VV] = \left(\frac{p_{Ag}(1-\gamma)}{p_i}\right)^{\sigma} B^{\sigma-1} X$$

- $$X = B\left(\begin{array}{c} \gamma A^{\frac{\sigma-1}{\sigma}} \alpha K^{\frac{\sigma-1}{\sigma}} + \gamma A^{\frac{\sigma-1}{\sigma}} \delta F^{\frac{\sigma-1}{\sigma}} \\ + \gamma A^{\frac{\sigma-1}{\sigma}} (1-\alpha-\delta) L^{\frac{\sigma-1}{\sigma}} + (1-\gamma)[VV]^{\frac{\sigma-1}{\sigma}} \end{array}\right)^{\frac{\sigma}{\sigma-1}}$$

$$F = \left(\frac{p_{Ag}}{p}\gamma\delta\right)^{\sigma} (AB)^{\sigma-1} X$$

$$K = \left(\frac{p_{Ag}\gamma\alpha}{r}\right)^{\sigma} (AB)^{\sigma-1} X$$

$$L = \left(\frac{p_{Ag}\gamma(1-\alpha-\delta)}{w}\right)^{\sigma} (AB)^{\sigma-1} X$$

$$[VV] = \left(\frac{p_{Ag}(1-\gamma)}{p_i}\right)^{\sigma} B^{\sigma-1} X$$

$$\delta = \frac{\alpha p}{r}\left(\frac{F}{K}\right)^{\frac{1}{\sigma}}$$

$$\begin{aligned} \frac{F}{L} &= \left(\frac{\delta}{p}\frac{w}{1-\alpha-\delta}\right)^{\sigma} \\ &= \left(\frac{\frac{\alpha p}{r}\left(\frac{F}{K}\right)^{\frac{1}{\sigma}}}{p} \frac{w}{1-\alpha-\frac{\alpha p}{r}\left(\frac{F}{K}\right)^{\frac{1}{\sigma}}}\right)^{\sigma} \end{aligned}$$

$$\alpha = \frac{\left(\frac{F}{L}\right)^{\frac{1}{\sigma}} \frac{p}{w}}{\left(\frac{F}{L}\right)^{\frac{1}{\sigma}} \frac{p}{w} + \frac{p}{r}\left(\frac{F}{K}\right)^{\frac{1}{\sigma}} \left(1 + \left(\frac{F}{L}\right)^{\frac{1}{\sigma}} \frac{p}{w}\right)}$$

$$\frac{K}{L} = \frac{\left(\frac{\alpha}{r}\right)^{\sigma}}{\left(\frac{(1-\alpha-\delta)}{w}\right)^{\sigma}}$$

$$A = \left(\frac{F}{[VV]}\right)^{\frac{1}{\sigma-1}} \left(\frac{p}{p_i\delta}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}}$$