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STRATEGIC TRADE IN POLLUTION PERMITS

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Strategic trade in pollution permits

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Abstract

Markets for pollution have become a popular regulatory instrument. Yet these markets are often highly concentrated, which may lead to strategic behavior by all participants. In this article we investigate the implications of strategic trade in pollution permits. The permit market is developed as a strategic market game, where all firms are allowed to behave strategically and their roles as buyers or sellers of permits are determined endogenously with price-mediated trade. In a second stage, firms transact on a product market and we allow for a variety of market structures. Our framework establishes the endogenous determination of equilibrium price, market structure, and levels of exchange in the permit market.

Key words: Pollution market, Market power, Strategic market game.

JEL classification: C72, D43, D51, L13, Q53.

1 Introduction

Markets for pollution permits have emerged as a mainstream regulatory instrument. Since the early adoption of the US Acid Rain Program numerous schemes have been established to control pollution.[1] Behind this spirited regulatory response lies the economic rationale of least-cost pollution control: aggregate control costs are minimized when players trade pollution permits. This least-cost result relies on the existence of low transactions costs as well as players acting competitively.[2] Yet players' strategic behavior in these markets—and the resulting social losses—are a real concern (Montero, 2009; Hintermann, 2015).[3] The actions of large and influential players in the market have

[1]Examples include the European Union Emissions Trading Scheme (EU ETS), Regional Greenhouse Gas Initiative (RGGI), Western Climate Initiative (WCI), and New Zealand Emissions Trading Scheme (NZ ETS). Markets are also commencing in South Korea, China, and India.

[2]In early schemes transaction costs appeared to be problematic, for example, in the Fox river (O'Neil et al., 1983) and RECLAIM (Foster and Hahn, 1995). Yet in most modern permit markets prohibitive transaction costs do not appear to be a significant problem. Aside from cost effectiveness, a whole host of explanations can be proposed for explaining inefficiency within schemes, such as the political economy aspects of regulation, compliance issues, and uncertainty.

[3]For example, Montero (2009) highlights these problems both within the U.S. sulfur permit market—where 43% of permits allocations were allocated to just four players—as well as an international carbon market, where strategic behavior may exist between countries. More recently, Hintermann (2015) provides evidence of price manipulation in the EU-ETS.

the potential to distort the equilibrium permit price, reduce the cost effectiveness of pollution control, and influence the product market equilibrium. Although the existence of market power and the associated losses may be significant, the fundamental aspects of this problem—the interactions between players in the permit market—are not well understood. In particular, very little is known about the formation of equilibria in the permit market when all players behave strategically.

To address this problem, we derive a strategic market game (Shapley and Shubik, 1977) that takes into consideration strategic behavior in the permit market. Our model comprises of two stages. In the first stage, traders participate in a strategic permit market game. Traders in the permit market participate by submitting either an offer (of permits) or bid (of money). A trading post then aggregates the offers and bids and determines the price of permits in a way that clears the market. Trade is thus price mediated: whether a trader wishes to buy or sell permits depends on their abatement technology and on their conjecture of the price in the market, which is determined by their beliefs about the market actions of other traders. In the second stage, once firms receive their final allocation of permits from the permit market, they transact on the product market. We provide alternative product market structures to assist in our investigation; namely, we begin by allowing regulated firms to act as perfect competitors in the product market then advance our analysis so that the firms act as independent monopolists, as well as considering an oligopoly market structure. Industries regulated by cap-and-trade markets are often highly concentrated (regionally segregated) markets, for example, this has been evident in the electricity (Wolfram, 1999; Borenstein et al., 2002; Bushnell et al., 2008) and cement (Ryan, 2012; Fowlie et al., 2016) industries. With such a framework, we are thus interested in the structure and formation of equilibria in both the permit and product markets when all firms act strategically. We focus our attention on the incentives for players to trade, the overall cost effectiveness of regulation, and the equilibrium structure of the markets.

In our price-mediated model of permit exchange we consider the existence of an equilibrium with trade in permits, and demonstrate that the market equilibrium is always cost inefficient. Indeed, we find that even in the presence of gains from trade autarky may be the only outcome if the gains from trade are not sufficiently large. Our framework also shows that strategic trade can alter the structure of the market, as the role of firms (buyers or sellers) and the equilibrium price are now endogenously determined: buyers (sellers) in a competitive market can switch their role in a market with strategic trade. When firms place bids or offers in the permit market, they also take into account future strategic product market behavior. A firm holding a permit not only has a *direct* effect on reducing their abatement cost they will also experience an *indirect* effect in which holding the permit will increase optimal production as well as the ability to raise rivals' costs.

The idea that firms attempt to manipulate the permit market price has long been recognized. A vast literature has followed the contribution by Hahn (1984).[4] In his

[4]See Montero (2009) and Reichenbach and Requate (2013) for comprehensive literature surveys on market power in pollution markets. Using frameworks that model exhaustible resources, market power in pollution markets has also been considered when pollution permits are storable (Liski and Montero, 2006, 2011).

study, Hahn developed a permit market model with a single large trader and a price-taking competitive fringe of small traders. This framework, however, is restrictive. These traditional models exogenously impose a behavioral restriction on some agents: not all agents are permitted to behave strategically and there is a requirement for an auctioneer. Thus, models that assume a monopolist and a competitive fringe framework have some peculiar features. In such models, for example, the competitive fringe 'soaks up' the excess demand from the monopolist and as the size of the competitive fringe reduces, trade ceases to take place. Aside from these peculiarities, the basic competitive fringe framework remains popular and has been extended in a number of directions. First, models have accounted for additional players that act as oligopolists competing against each other in the presence of a competitive fringe.[5] Yet precisely the same conclusion holds (even though the oligopolists could effectively trade with each other). Second, analysis has concentrated on establishing links between permit and product markets. The work by Misiolek and Elder (1989) was the first to present a model where a dominant firm can alter permit trades in order to manipulate rival firms' costs in the product market.[6] Although the model provides a link between both a permit and product market, the analysis suffers from the same weakness as other competitive fringe frameworks; namely, behavioral assumptions are still required to allow only one firm to manipulate the permit and product market. Thus, in general, these models are rather limited in describing how trade might take place in an economy populated by large firms in which the assumption of a substantial competitive fringe is not appropriate. In order to provide insight to this problem, we must investigate strategic behavior from a different perspective.

In this article we provide a new framework to investigate strategic behavior in a permit market. We do this by using a strategic market game and provide a unifying framework that incorporates strategic behavior for all firms in the permit market as well as considering the impact of market power in the product market. To provide a full equilibrium characterization, we follow a three-step approach that exploits the aggregative properties of the game played. In Step 1, we hypothesize a permit price and consider whether firms would be (potential) buyers or sellers of permits. In Step 2, we consider the behavior of each side of the market separately at the hypothesized permit price, deducing the aggregate supply of, and demand for, permits at that price. Finally, in Step 3, we check whether the hypothesized permit price is consistent with aggregate demand and supply. If so, then we have identified a Nash equilibrium. Once a permit market equilibrium is determined, firms participate in the product market. We start by providing a benchmark case of a perfectly competitive product market. We then develop our framework to allow each firm to be an independent monopolist within their product market as well as providing an oligopolistic product market structure. Allowing for such an approach provides a comprehensive, realistic, and tractable structure to analyze strategic trade in permit markets (and the associated product market).

[5] See, for example, Westskog (1996) and, more recently, Hagem (2013) that discusses the choice of strategic behavior.

[6] See Salop and Scheffman (1983, 1987), and Rogerson (1984) for the underlying framework.

1.1 Recent literature

To overcome the drawbacks of a competitive fringe framework, a small number of alternative mechanisms have been advocated. These alternatives use the supply function approach of Klemperer and Meyer (1989) to model trade in pollution permits (Malueg and Yates, 2009; Wirl, 2009; Lange, 2012).[7] Using this supply function approach identifies the losses associated with strategic behavior and shows that, although a bilateral oligopoly leads to the same equilibrium permit price as the competitive solution, trading volume is lower. In supply function frameworks, firms set up trade functions and specify the number of permits that are to be bought or sold, conditional on the equilibrium price. The market maker then collects these schedules and determines a market-clearing price. Although this approach does provide additional understanding of strategic permit trade, the main disadvantage of using a supply function approach is that price determination is a 'black box': the market maker determines the equilibrium price where aggregate net trades are zero without any attempt to focus on a price-mediated solution.[8] Establishing a price-mediated solution, therefore, may provide a richer (and more plausible) approach to modeling strategic trade in permits—something we consider in this article.

We provide a trading mechanism in a bilateral oligopoly framework that allows all traders to behave strategically and in which the sides of the market (i.e., the sets of buyers and sellers of permits) form endogenously, and is very much in the spirit of price-mediated trade via quantity competition à la Cournot. As such, the model does not take place in a 'black box' with a requirement for an auctioneer to clear the market, instead, we outline an explicit price-formation mechanism. Our mechanism incorporates a trading post that aggregates the bids and offers of all players and the equilibrium price is determined via the ratio of total amount of money bid to the total number of permits offered. Any exchanges are therefore determined subject to the bids and offers made as well as the resulting permit market price.

We are able to directly compare our framework with the supply function literature. In direct contrast to the key findings of this literature, we show the equilibrium permit price in our model of bilateral oligopoly will generically be different to the competitive equilibrium permit price. Under certain conditions, therefore, some firms may switch between selling permits (in the competitive equilibrium) to buying permits (in the bilateral oligopoly), and vice versa. Moreover, we show that for trade to take place it is necessary that there are 'sufficient' gains from trade, meaning autarky is the only equilibrium in some markets even though gains from trade may exist. By contrast, in supply function models, autarky is only an equilibrium when the initial permit allocation is efficient—something originally observed in Hahn (1984).

[7] For a further discussion see Godal (2011). For experimental findings of this approach see Schnier et al. (2014).

[8] As noted by Malueg and Yates (2009), Wirl (2009), and Lange (2012), the method of obtaining uniqueness in supply function equilibria requires additional assumptions over-and-above the requirement of unspecified price determination. For example, Malueg and Yates (2009) requires that firms have identical marginal abatement cost slopes as well as single parameter linear net-trade functions, whereas Lange (2012) requires that all strategies are consistent with small (stochastic) changes in the demand functions. As our approach focuses on an explicit trading mechanism, which produces a price-mediated solution, we do not require any of these additional assumptions.

In our general framework, we analyze firm behavior when the permit and product markets are linked. In particular, we combine the strategic permit market with alternative product market structures.[9] Earlier literature has also investigated the connectivity between the permit and product market, but this has been framed through a traditional competitive fringe framework (along with the subsequent weaknesses) (e.g., Sartzetakis, 1997; Hintermann, 2011).[10] Using our framework, we show that the introduction of independent monopolists in the product market unambiguously lowers the equilibrium permit price as the strategic supply (demand) of permits increases (decreases). When this product market structure is replaced by an oligopoly product market, counterbalancing strategic effects occur such that there may be an increase in the demand for permits and upward pressure on the associated permit price.

Our contribution is to provide a framework to model fully strategic trade in pollution permits, that is both realistic and tractable to allow for the full equilibrium characterization of the permit market. This, then, provides a basis for the evaluation of contemporary cap-and-trade markets when strategic behavior exists for all market participants. By combining our analysis with alternative product market structures, we also provide an encompassing model that incorporates many current regulatory market structures. Our approach can be used to nest previous attempts at strategic behavior in the product market (e.g., Misiolek and Elder, 1989) as well as complementing the recent literature on strategic permit markets that has yet to investigate the fundamental links between permit and product markets (Malueg and Yates, 2009; Wirl, 2009; Lange, 2012).

This article is structured as follows. Section 2 outlines the economic environment, determines the equilibrium characterization of a strategic market game and product market equilibrium. Section 3 provides a discussion of the permit market equilibrium. Section 4 extends the framework to include strategic behavior in the product market. We then conclude in Section 5.

2 The model

2.1 The economic environment

Consider an economic environment that is populated by an index set of firms $I = \{1, \ldots, N\}$, where firm $i \in I$ has an initial stock of money $m_i \geq 0$. Firms operate in a product market where the production of goods generates pollution. This pollution is regulated by a cap-and-trade scheme. Firms have the option to either hold a permit to cover emission liabilities, or reduce emissions by utilizing (costly) abatement technologies. Before undertaking production, firm i is allocated an initial endowment of permits $\omega_i > 0$ with the opportunity to engage in permit trade.[11] The regulator's pollution target is $\Omega = \sum_{i \in I} \omega_i$. We consider a two-stage environment: in the first stage permit

[9]Recently, Fowlie et al. (2016) investigated the adoption of market-based instruments (without market power) on a highly concentrated product market (a regionally segregated cement industry). Fowlie et al. (2016) finds that the establishment of a market-based instrument coupled with the market-power distortions in the product market generate losses over-and-above any benefits associated with emissions mitigation. See also Ryan (2012).

[10]For additional insights see De Feo et al. (2013).

[11]The analysis can also allow for traders who have no initial allocation of permits but that might want to transact in the permit market.

allocations are determined in the permit market which become common knowledge; in the second stage firms make production decisions in the product market.

Let $x_i \in \mathbb{R}$ be the number of permits that firm i is allocated by the market after trading: $x_i > 0$ for purchases of permits and $x_i < 0$ for sales. Let $\mathbf{x}$ denote the vector of final allocations for all firms, and $\mathbf{x}_{-i}$ the vector of all allocations excluding that of firm i. The final permit holdings of firm i are $\omega_i + x_i$ and we denote the price of permits by p. In the product market firm i's output is denoted by z_i, and the product price ϕ is determined by an inverse demand relationship $\Phi(Z)$, where $Z = \sum_{i \in I} z_i$ is the aggregate supply of the good. Production of the good generates pollution and the quantity of pollution emitted in producing z_i is given by $f_i(z_i)$. Any pollution that is not covered by a permit must be abated; accordingly, pollution abatement required by firm i is $a_i \equiv f_i(z_i) - (\omega_i + x_i)$. Firms undertaking production incur direct production costs and pollution abatement costs, so firm i's total cost of production is given by $C_i(z_i, a_i)$.

Assumption. *For each firm $i \in I$ the functions $f_i(\cdot)$ and $C_i(\cdot, \cdot)$ are twice continuously differentiable; $f_i', f_i'' \geq 0$; $C_i^z, C_i^a \geq 0$ with a strict inequality if $z_i > 0$; $C_i^{zz}, C_i^{aa} > 0$ and $C_i^{za} \geq 0$; $C_i^{zz} C_i^{aa} - (C_i^{za})^2 > 0$; and finally $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$.*

Firm i's payoff is comprised of any initial wealth m_i, revenue or costs associated with permit market activity $x_i p$, and, after accounting for all costs of production, the profit from productive activity:

$$V_i = m_i - x_i p + z_i \phi - C_i(z_i, f_i(z_i) - (\omega_i + x_i)).$$

Once initial permit endowments have been set (which are common knowledge), firms have the opportunity to trade permits and the market mechanism will determine the final allocation of permits. To capture firms' strategic behavior in the market for pollution permits, we turn to a model of bilateral oligopoly with a market mechanism in which market actions are quantity-based and trade is price mediated; no price-taking assumptions are imposed *ex ante* and the role of firms as buyers or sellers of permits is determined endogenously in the market. Such 'strategic market games' were introduced by Shapley and Shubik (1977) to model fully strategic behavior in general equilibrium settings, which we restrict to the case of two commodities—a good (permits) and money (see Dickson and Hartley, 2008). Trade takes place by way of an *explicit* trading mechanism: there is a 'trading post' to which firms submit an *offer* of permits to be exchanged for money or a *bid* of money to be exchanged for permits, depending on whether they want to sell or buy permits.[12] The trading post aggregates the offers and bids of all firms and determines the price of permits as the *ratio* of the total amount of money bid to the total number of permits offered. Exchanges are then determined according to the offers and bids made and the resulting market price. Trade is therefore price mediated, and each individual firm considers that their actions influence this price. Whether a firm wishes to buy or sell permits will depend on their abatement technology and their belief about the price in the market.

[12] This is in contrast to the existing literature on strategic trade in pollution permits (e.g., Hahn, 1984; Hintermann, 2011) that invariably assumes the presence of a 'competitive fringe' necessitating a 'black box' (auctioneer) approach to market clearing.

Formally, firm i can make an *offer* of permits $0 \leq q_i \leq \omega_i$ to be exchanged for money, or make a *bid* of money $0 \leq b_i \leq m_i$ to be exchanged for permits.[13] We assume firms only buy permits from their initial money holdings and we rule out firms making 'wash trades', i.e., contemporaneously buying *and* selling permits. The set of strategies available to firm $i \in I$ is therefore

$$\mathcal{S}_i = \{(b_i, q_i) : 0 \leq b_i \leq m_i, 0 \leq q_i \leq \omega_i, b_i \cdot q_i = 0\}.$$

The role of the trading post is to aggregate the offers and bids and determine trades. Let the aggregate offer and the aggregate bid be $Q = \sum_{i \in I} q_i$ and $B = \sum_{i \in I} b_i$, respectively. If either B or Q are zero then the trading post is deemed closed and any offers or bids that are made are returned. So long as $B, Q > 0$, the price of permits (denominated in units of money) is determined as $p = B/Q$, and the number of permits allocated to firm i (in addition to their initial holdings) is given by

$$x_i = \begin{cases} b_i/p & \text{if } b_i > 0, q_i = 0 \text{ or} \\ -q_i & \text{if } q_i > 0, b_i = 0. \end{cases} \tag{1}$$

The change in firm i's money holdings is thus

$$-x_i p = \begin{cases} -b_i & \text{if } b_i > 0, q_i = 0 \text{ or} \\ q_i p & \text{if } q_i > 0, b_i = 0. \end{cases}$$

An intuitive interpretation of this mechanism is as follows: the total supply of permits to the market from those that want to sell (Q) is shared among those traders that want to buy in proportion to their bids (b_i/B), for which a per-unit price of p is transferred to the sellers.

Once permit trading has taken place, permit allocations become common knowledge and firms engage in production decisions in the product market. In our baseline model we assume that firms behave as price-takers in the product market by modeling it as a perfectly competitive market. Later in the article, we explore the implications of firms having market power in the product market.

2.2 Product market decisions

Let ϕ denote the product market price that is set by the Walrasian auctioneer in a perfectly competitive product market. Then the profit of a typical firm $i \in I$ from their product market activity is

$$\tilde{\pi}_i(z_i; x_i) = z_i \phi - C_i(z_i, f_i(z_i) - (\omega_i + x_i)).$$

Once the permit market has cleared and firm i has a permit allocation x_i, the product market profit function $\tilde{\pi}_i(z_i; x_i)$ depends only on z_i. Firm $i \in I$ will seek to choose z_i to

[13] Throughout it is assumed that a sufficiently large penalty can be levied on firms for offering more permits than are in their possession, or making bids that exceed their money holdings, that this will never constitute equilibrium behavior. For example, this occurs with the non-compliance penalty in the EU-ETS, which was set at €100 per tonne of CO_2 in 2013 and increases with the Eurozone inflation rate: significantly higher than the equilibrium permit price.

maximize $\tilde{\pi}_i(z_i; x_i)$, where the first-order condition is

$$\frac{\mathrm{d}\tilde{\pi}_i(z_i;x_i)}{\mathrm{d}z_i} \leq 0 \Leftrightarrow C_i^z(z_i, f_i(z_i) - (\omega_i + x_i)) + f_i'(z_i)C_i^a(z_i, f_i(z_i) - (\omega_i + x_i)) \geq \phi, \tag{2}$$

with equality if $z_i > 0$.[14] Since we assume $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$ the solution will always be interior where the first-order condition holds with equality, and we denote the solution to (2) by $\tilde{z}_i(\phi; x_i) > 0$.

The competitive equilibrium product price determined by the Walrasian auctioneer must satisfy $\tilde{\phi} = \Phi(\sum_{i=1}^n \tilde{z}_i(\tilde{\phi}; x_i))$. This, of course, depends on the distribution of permit allocations, so where appropriate we will write $\tilde{\phi}(\mathbf{x})$ as the solution to this equation.[15] With a slight abuse of notation we write $\tilde{z}_i(x_i)$ for the supply of firm i to the product market in the competitive equilibrium, which is derived by firms equating their 'overall marginal cost'—comprised of the marginal cost of production and abatement—to the price of the good.

The relationship between a firm's behavior in the product market and their actions in the permit market is given by

$$\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}s_i} = \frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i}, \text{ for } s = \{b, q\}, \tag{3}$$

which follows by virtue of firm i's product market strategy depending only on its own allocation of permits. Implicit differentiation of (2) yields

$$\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i} = \frac{C_i^{za} + C_i^{aa} f_i'}{C_i^{zz} + 2C_i^{za} f_i' + C_i^{aa}(f_i')^2 + C_i^a f_i''} > 0 \tag{4}$$

under our assumptions. Intuitively, if a firm acquires more permits in the permit market then less abatement is required for a given level of output. This has two effects relevant for product market decisions: since $C_i^{aa} > 0$ the marginal cost of abatement falls; and since $C_i^{za} > 0$ the marginal cost of production falls. Both effects work to favor an increase in product market output when the firm is in possession of more permits.

To understand the effect of a change in the permit allocation on a firm's profitability in the product market let us, again with a slight abuse of notation, write the optimized profit function in the product market as

$$\tilde{\pi}_i(x_i) = \tilde{z}_i(x_i)\tilde{\phi} - C_i(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i)). \tag{5}$$

Since this is only influenced by x_i, we can write

$$\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}s_i} = \frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i}, \tag{6}$$

[14] This first-order condition is both necessary and sufficient under the assumptions stated in Subsection 2.1; the second-order condition is $-C_i^{zz} - C_i^{aa}(f_i')^2 - 2C_i^{za}f_i' - C_i^a f_i'' < 0$.

[15] Note that since firms are assumed to be price-takers in the product market no firm takes into consideration the effect of their permit allocation on the product price.

in which

$$\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} = \frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i}[\tilde{\phi} - C_i^z - C_i^a f_i'] + C_i^a \\ = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i)) \tag{7}$$

as the first-order condition (2) implies $\tilde{\phi} - C_i^z - C_i^a f_i' = 0$. Equations (6) and (7) show a direct link between the permit and product markets: this will be used to investigate the firm's actions within the permit market.

2.3 Permit market equilibrium

Foreseeing the consequences of permit market activity on actions in the product market, each firm $i \in I$ can be seen as solving the problem

$$\max_{(b_i,q_i)\in\mathcal{S}_i} m_i - x_i p + \tilde{\pi}_i(x_i),$$

where $x_i = b_i/p - q_i$, $p = B/Q$, and $\tilde{\pi}_i$ is defined in (5). This problem is concave in both b_i and q_i so the first-order conditions are both necessary and sufficient in identifying a best response.[16]

When engaging in permit market activity, a firm affects its product market profitability (according to (6)) and also its expenditure in the permit market. When choosing $s = \{b, q\} \in \mathcal{S}_i$ the firm will balance the marginal change in product market profitability with the marginal change in permit market expenditure, so that

$$\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} \leq \frac{\mathrm{d}x_i p}{\mathrm{d}s_i},\ s = \{b, q\},$$

where the inequality is replaced with an equality if $s_i > 0$.

For a buyer of permits for whom $s = b$, $x_i = b_i/p$, and so it follows that $\frac{\mathrm{d}x_i}{\mathrm{d}s_i} = (1 - b_i/B)p^{-1}$ and $\frac{\mathrm{d}x_i p}{\mathrm{d}s_i} = 1$. As such, the first-order condition for a buyer of permits is

$$\frac{\mathrm{d}\tilde{\pi}_i(b_i/p)}{\mathrm{d}x_i} \leq (1 - b_i/B)^{-1}p, \tag{8}$$

where the inequality is replaced with an equality if $b_i > 0$.

For a seller of permits for whom $s = q$, $x_i = -q_i$ and we have $\frac{\mathrm{d}x_i}{\mathrm{d}s_i} = -1$ and $\frac{\mathrm{d}x_i p}{\mathrm{d}s_i} = -(1 - q_i/Q)p$. Consequently, the first-order condition is

$$\frac{\mathrm{d}\tilde{\pi}_i(-q_i)}{\mathrm{d}x_i} \geq (1 - q_i/Q)p, \tag{9}$$

with equality if $q_i > 0$.

[16]This follows by noting that for $s = \{b, q\}$, the first derivative of the payoff function is $-\frac{\mathrm{d}x_i p}{\mathrm{d}s_i} + \frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i}$ and so the second derivative is $-\frac{\mathrm{d}^2 x_i p}{\mathrm{d}s_i^2} + \frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}\left(\frac{\mathrm{d}x_i}{\mathrm{d}s_i}\right)^2 + \frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}^2 x_i}{\mathrm{d}s_i^2}$. When $s = b$: $x_i = (b_i/B)Q$ so $\frac{\mathrm{d}x_i}{\mathrm{d}b_i} = \frac{B-b_i}{B^2}Q$ and $\frac{\mathrm{d}^2 x_i}{\mathrm{d}s_i^2} = -\frac{2(B-b_i)}{B^3}Q$; and $x_i p = b_i$ so $\frac{\mathrm{d}^2 x_i p}{\mathrm{d}b_i^2} = 0$. When $s = q$: $x_i = -q_i$ so $\frac{\mathrm{d}^2 x_i}{\mathrm{d}q_i^2} = 0$; and $x_i p = -(q_i/Q)B$ so $\frac{\mathrm{d}x_i p}{\mathrm{d}q_i} = -\frac{Q-q_i}{Q^2}B$ and $\frac{\mathrm{d}^2 x_i p}{\mathrm{d}q_i^2} = +\frac{2(Q-q_i)}{Q^3}B$. As noted, $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} > 0$ and we will subsequently show in Lemma 1 that $\frac{\mathrm{d}^2\tilde{\pi}_i}{\mathrm{d}x_i^2} < 0$, which establishes the claim.

Since firms are heterogeneous in their cost structure, pursuing a standard best-response analysis of this game would be fruitless as the dimensionality of the problem makes it intractable. Rather than imposing additional assumptions to instil tractability (e.g., restricting firms to be one of two types), we follow an approach—first presented in Dickson and Hartley (2008) and later extended to the case of 'interior endowments' by Dickson and Hartley (2013)—that exploits the fact that firms' payoffs depend only on their own action and the aggregation of other firms' actions in B and Q, which themselves influence the price p. Here we present the reasoning for permit exchange coupled with subsequent product market decisions. The method allows the construction of supply and demand functions in the permit market that account for strategic behavior and endogenous formation of the sides of the market, and can be used to identify a permit market equilibrium. The method proceeds as follows.

Step 1: Hypothesize a permit price p, and consider which firms would act on each side of the permit market if there was a Nash equilibrium with this price. We define

$$\tilde{p}_i^* \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) \tag{10}$$

as firm i's marginal abatement cost at its initial endowment and will show (in Proposition 2) that firm i will be a buyer of permits only if $\tilde{p}_i^* > p$ and a seller of permits only if $\tilde{p}_i^* < p$. When considering behavior consistent with a price p, this allows us to separate the set of firms into those that will potentially buy permits, and those that will potentially sell.

Step 2a: Hypothesize an aggregate supply of permits, Q, and consider the individual supplies of those firms that might sell permits at price p that are consistent with a Nash equilibrium with this Q and p. Then ask whether firms' individual supplies are consistent when aggregated, i.e., that individual supplies aggregate to Q. Let $\tilde{q}_i(p; Q)$ denote firm i's supply consistent with a Nash equilibrium in which the aggregate supply is Q and the price is p (which is given by the minimum of either the q_i that solves (9) or ω_i). Then we seek the value of Q such that $\sum_{\{i \in I : p_i^* < p\}} \tilde{q}_i(p; Q) = Q$, which is the aggregate supply consistent with a Nash equilibrium in which the price is p.

Step 2b: Hypothesize an aggregate bid B from those firms that might buy permits at price p, and deduce individual bids consistent with this aggregate bid, which we denote $\tilde{b}_i(p; B)$ (this is given by the minimum of either the b_i that solves (8) or m_i). Seek consistency of the aggregate bid, i.e., find the value of B such that $\sum_{\{i \in I : p_i^* > p\}} \tilde{b}_i(p; B) = B$.

Step 3: Seek a consistent price, i.e., a price such that the consistent aggregate offer from Step 2a and bid from Step 2b satisfy $B/Q = p$, which identifies a Nash equilibrium.

We begin by establishing Step 1. To do so, we first require the following lemma.

Lemma 1. *For each firm $i \in I$, $\frac{\mathrm{d}^2 \tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} < 0$.*

Proof. Recall from (7) that $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i))$. As such,

$$\begin{aligned}\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} &= C_i^{za}\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i} + C_i^{aa}\left(f_i'\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i} - 1\right)\\ &= \frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i}(C_i^{za} + C_i^{aa}f_i') - C_i^{aa}.\end{aligned}$$

In (4) we deduced that $\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}x_i} = \frac{C_i^{za}+C_i^{aa}f_i'}{C_i^{zz}+2C_i^{za}f_i'+C_i^{aa}(f_i')^2+C_i^a f_i''}$, implying

$$\begin{aligned}\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} &= \frac{(C_i^{za} + C_i^{aa}f_i')^2}{C_i^{zz} + 2C_i^{za}f_i' + C_i^{aa}(f_i')^2 + C_i^a f_i''} - C_i^{aa}\\ &= \frac{(C_i^{za})^2 - C_i^{zz}C_i^{aa} - C_i^a C_i^{aa} f_i''}{C_i^{zz} + 2C_i^{za}f_i' + C_i^{aa}(f_i')^2 + C_i^a f_i''},\end{aligned}$$

which is negative as a result of our assumptions on cost and pollution generation functions. □

We are now in a position to complete Step 1. The following proposition allows us to understand, once a permit price has been hypothesized, how firms are determined as either buyers or sellers of permits.[17]

Proposition 2. *If there is a Nash equilibrium with price p then firm $i \in I$ will be a buyer (seller) of permits only if $\tilde{p}_i^* > (<) p$.*

Proof of Proposition 2. Let $\tilde{p}_i^* > p$ and assume, by contradiction, that i sells permits. Then $q_i > 0$ and $x_i = -q_i < 0$, so Lemma 1 implies $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} > \left.\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}\right|_{x_i=0} \equiv \tilde{p}_i^*$ (see (10)). But from the first-order condition for sellers, $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} = (1 - q_i/Q)p < p$. As such, $\tilde{p}_i^* < \frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} < p$, yielding a contradiction. Thus, if $\tilde{p}_i^* > p$ for firm i then this firm will only buy permits in equilibrium. Demonstrating that if $\tilde{p}_i^* < p$ then firm i will only sell permits is similar and so omitted. □

Operationally, the consistent behavior of firms is represented using *share functions*. Take a typical firm i. If $p > \tilde{p}_i^*$ then we know that the firm will only be a seller of permits at such prices and we consider their behavior consistent with a Nash equilibrium in which the permit price is p and the aggregate supply of all 'potential sellers' (those firms $j \neq i \in I$ for whom $p > \tilde{p}_j^*$) is $Q > 0$. Let $\sigma_i = q_i/Q$ be firm i's share of the total supply; then using (9) we can deduce that firm i's optimal share of the total supply is given by its 'selling share function' $\tilde{s}_i^S(p; Q) = \min\{\sigma_i, \omega_i/Q\}$ where σ_i is the solution to

$$\tilde{l}_i^S(\sigma_i, Q, p) \equiv \frac{\mathrm{d}\tilde{\pi}_i(-\sigma_i Q)}{\mathrm{d}x_i} - (1 - \sigma_i)p \geq 0, \tag{11}$$

with equality if $\sigma_i > 0$.

It is useful to ascertain the properties of share functions. The share function $\tilde{s}_i^S(p; Q)$ is implicitly defined, and implicit differentiation of (11) reveals that it is decreasing in

[17] This is similar to Dickson and Hartley (2013, Lemma 1), but is included here for the case of permit exchange for a self-contained treatment.

Q and non-decreasing in p; in addition, study of (11) reveals that $\lim_{Q\to 0} \tilde{s}_i^S(p;Q) = 1 - \frac{\tilde{p}_i^*}{p}$.[18]

Consider now the case where $p < \tilde{p}_i^*$: firm i will only be a buyer of permits. The behavior of firm i consistent with a Nash equilibrium in which the price is p and the aggregate bid is $B > 0$ is represented by its 'buying share function' $\tilde{s}_i^B(p;B) = \min\{\sigma_i, m_i/B\}$ where, using (8), σ_i is the solution to

$$\tilde{l}_i^B(\sigma_i, B, p) \equiv \frac{\mathrm{d}\tilde{\pi}_i(\sigma_i B/p)}{\mathrm{d}x_i} - (1-\sigma_i)^{-1}p \leq 0, \tag{12}$$

with equality if $\sigma_i > 0$.

To deduce the properties of a buyer's share function, we note that if the aggregate bid is B and the price is p, the implied demand is B/p; thus, we write firm i's share function as $\tilde{s}_i^B(p, [B/p]p)$. Implicit differentiation of (12) reveals that the share function is strictly decreasing in $[B/p]$ for fixed p, strictly decreasing in p for fixed $[B/p]$, and has the property $\lim_{[B/p]\to 0} \tilde{s}_i^B(p;B) = 1 - \frac{p}{\tilde{p}_i^*}$.[19]

These share functions represent each firm's consistent behavior at a particular price, with particular aggregate bids or offers. We now seek consistency of these aggregates to complete Steps 2a and 2b above. Consistency of the aggregate offer at price p requires the sum of the individual offers of all firms that wish to sell at price p to be equal to the aggregate offer, or, dividing both sides of this equation by Q, for the sum of the share functions to be equal to one. Define $\tilde{S}^S(p;Q) \equiv \sum_{\{i\in I:\tilde{p}_i^* < p\}} \tilde{s}_i^S(p;Q)$. Then at price p we identify the *strategic supply*, denoted by $\tilde{\mathcal{Q}}(p)$, as that level of Q where

$$\tilde{S}^S(p;Q) = 1. \tag{13}$$

For a given p, all firms for whom $\tilde{p}_i^* < p$ will be included in $\tilde{S}^S(p;Q)$ and since each $\tilde{s}_i^S(p;Q)$ is continuous and decreasing in Q, $\tilde{S}^S(p;Q)$ will inherit this property implying $\tilde{\mathcal{Q}}(p)$, where defined, is a function. When p changes, the share functions of those firms who remain sellers change in a smooth way, and those firms who become sellers as the price rises (or drop out of the set of sellers as the price falls) again do so in a smooth way, implying that $\tilde{S}^S(p;Q)$ is continuous in p and consequently $\tilde{\mathcal{Q}}(p)$ varies continuously in p. Moreover, consideration of the equation implicitly defining $\tilde{\mathcal{Q}}(p)$ reveals it is non-decreasing in p.[20] The range of prices for which $\tilde{\mathcal{Q}}(p)$ is defined is $p > \tilde{P}^S$, where $\tilde{P}^S$ is

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[18]Recall from Lemma 1 that $\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} < 0$. As such, $\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial\sigma_i} = -Q\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} + p > 0$ so there is at most one solution to $l_i(\sigma_i, Q, p) = 0$: $\tilde{s}_i^S(p;Q)$ is a function. Moreover, implicit differentiation of (11) gives $\frac{\partial \tilde{s}_i^S(p;Q)}{\partial Q} = -\frac{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial Q}}{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial \sigma_i}} = -\frac{-\sigma_i\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}}{-Q\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}+p} < 0$ and $\frac{\partial \tilde{s}_i^S(p;Q)}{\partial p} = -\frac{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial p}}{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial \sigma_i}} = -\frac{-(1-\sigma_i)}{-Q\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}+p} > 0$.

[19]The fact that $\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} < 0$ (Lemma 1) is again important. With this in mind, note that $\frac{\partial \tilde{l}_i^B(\sigma_i,B,p)}{\partial\sigma_i} = B/p\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} - (1-\sigma_i)^{-2} < 0$, so we are ensured $\tilde{s}_i^B(p;B)$ is a function. Undertaking implicit differentiation, $\frac{\partial \tilde{s}_i^B(p;[B/p]p)}{\partial [B/p]} = -\frac{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial [B/p]}}{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial \sigma_i}} = -\frac{\sigma_i\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}}{B/p\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}-(1-\sigma_i)^{-2}} < 0$. In addition, $\frac{\partial \tilde{s}_i^B(p;[B/p]p)}{\partial p} = -\frac{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial p}}{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial \sigma_i}} = \frac{(1-\sigma_i)^{-1}}{B/p\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2}-(1-\sigma_i)^{-2}} < 0$. The limit is a consequence of taking limits in (12) as $[B/p] \to 0$.

[20]Although $\sum_{\{i\in I:\tilde{p}_i^* < p\}} \tilde{s}_i^S(p;Q)$ is continuous in p, it is not differentiable at values of p where new firms enter the set of sellers so implicit differentiation cannot be used. Rather, suppose by contradiction that for

uniquely defined by the equation

$$\sum_{\{i \in I: \tilde{p}_i^* < \tilde{P}^S\}} 1 - \frac{\tilde{p}_i^*}{\tilde{P}^S} = 1. \tag{14}$$

For $p \leq \tilde{P}^S$, the aggregate share function $\tilde{S}^S(p; Q)$ takes a value less than one when Q is close to zero and, since it is decreasing in Q, this is also true for higher values of Q; accordingly, it is never equal to one. Conversely, for $p > \tilde{P}^S$ it exceeds one when Q is small enough and since it is continuous and decreasing in Q it is equal to one at exactly one value of Q: the strategic supply.

On the buyers' side, we seek to find the consistent level of $[B/p]$, which is the aggregate demand for permits. This requires that individual bids when aggregated exactly equal the aggregate bid B, or that the sum of share functions equals one. Defining $\tilde{S}^B(p; [B/p]p) \equiv \sum_{\{i \in I: \tilde{p}_i^* > p\}} \tilde{s}_i^B(p; [B/p]p)$, the *strategic demand* for permits, denoted by $\tilde{\mathcal{D}}(p)$, is that level of $[B/p]$ which satisfies

$$\tilde{S}^B(p; [B/p]p) = 1. \tag{15}$$

Continuity of the strategic demand function follows by similar deductions to those made for strategic supply, and study of the condition implicitly defining strategic demand allows us to deduce that strategic demand is decreasing (strictly) in p.[21] The range of prices for which $\tilde{\mathcal{D}}(p)$ is defined is $p < \tilde{P}^B$, where $\tilde{P}^B$ is uniquely defined by the equation

$$\sum_{\{i \in I: \tilde{p}_i^* > \tilde{P}^B\}} 1 - \frac{\tilde{P}^B}{\tilde{p}_i^*} = 1. \tag{16}$$

For reasons that are similar to those elucidated for strategic supply, if $p \geq \tilde{P}^B$ then the aggregate share function is less than one for all values of $[B/p]$ so for these prices strategic demand is undefined whereas it takes positive values for $p < \tilde{P}^B$.

Turning finally to Step 3, a permit price p is consistent with a Nash equilibrium in which trade in permits takes place if and only if strategic supply and demand are equal at that price, for only then will the aggregate offer of permits and bid of money be consistent with the price. Since strategic demand is strictly decreasing in p and strategic supply is non-decreasing in p, if strategic supply and demand cross they do so only once, implying that there is at most one Nash equilibrium in which trade in permits takes place. This will be the case so long as $\tilde{P}^S < \tilde{P}^B$. Under such circumstances

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$p' > p$ we have $\tilde{\mathcal{Q}}(p') < \tilde{\mathcal{Q}}(p)$. Then the fact that share functions are decreasing in Q and non-decreasing in p implies

$$1 = \sum_{\{i \in I: \tilde{p}_i^* < p\}} \tilde{s}_i^S(p; \tilde{\mathcal{Q}}(p)) \leq \sum_{\{i \in I: \tilde{p}_i^* < p'\}} \tilde{s}_i^S(p; \tilde{\mathcal{Q}}(p)) < \sum_{\{i \in I: \tilde{p}_i^* < p'\}} \tilde{s}_i^S(p'; \tilde{\mathcal{Q}}(p')) = 1,$$

a contradiction.

[21] Suppose by contradiction that $p' > p$ and $\tilde{\mathcal{D}}(p') \geq \tilde{\mathcal{D}}(p)$. Then the facts previously deduced that the share function is *strictly* decreasing in p (and $[B/p]$) implies

$$1 = \sum_{\{i \in I: \tilde{p}_i^* > p\}} \tilde{s}_i^B(p; \tilde{\mathcal{D}}(p)p) \leq \sum_{\{i \in I: \tilde{p}_i^* > p'\}} \tilde{s}_i^B(p; \tilde{\mathcal{D}}(p)p) < \sum_{\{i \in I: \tilde{p}_i^* > p'\}} \tilde{s}_i^B(p'; \tilde{\mathcal{D}}(p')p') = 1,$$

a contradiction.

let $\hat{p}$ be the equilibrium price at which $\tilde{\mathcal{Q}}(\hat{p}) = \tilde{\mathcal{D}}(\hat{p})$ then the equilibrium aggregate supply of permits to the market is $\hat{Q} = \tilde{\mathcal{Q}}(\hat{p})$; the equilibrium aggregate bid of money is $\hat{B} = \hat{p}\hat{Q}$; the equilibrium supply of each firm for whom $\tilde{p}_i^* < \hat{p}$ is $\hat{q}_i = \hat{Q}\tilde{s}_i^S(\hat{p};\hat{Q})$ and the equilibrium bid of each firm for whom $\tilde{p}_i^* > \hat{p}$ is $\hat{b}_i = \hat{B}\tilde{s}_i^B(\hat{p};\hat{B})$. Equilibrium permit allocations are $\hat{x}_i = \hat{b}_i/\hat{p} - \hat{q}_i$. If $\tilde{P}^S \geq \tilde{P}^B$ then there is no Nash equilibrium in which trade in permits takes place; in such circumstances the only Nash equilibrium is autarky (which is always an equilibrium in bilateral oligopoly) and each firm's final allocation of permits is their initial endowment.[22]

3 Features of the permit market equilibrium

With our framework established in the previous section, it is pertinent to consider features of the permit market equilibrium and the consequences of strategic behavior. In particular within this section we will focus on the existence, structure, and cost effectiveness of the permit market equilibrium as well as the comparative statics of the model.

3.1 Existence of equilibrium

In bilateral oligopoly, as just noted, there is always an autarkic Nash equilibrium in which no trade takes place. An important question is whether it is the only equilibrium. The existence of a *non-autarkic* Nash equilibrium—and therefore whether any trade takes place—in the market for permits hinges on whether $\tilde{P}^S$ defined in (14) is less than $\tilde{P}^B$ defined in (16). To better understand the relationship between these two objects we next elucidate the details of their construction. Recall that $\tilde{p}_i^* \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i)$ is firm i's marginal abatement cost with its initial endowment of permits. Given an initial distribution of permit endowments we can, without loss of generality, re-order firms according to the magnitude of their marginal abatement cost: $\tilde{p}_1^* \leq \tilde{p}_2^* \leq \cdots \leq \tilde{p}_N^*$. Now we construct two functions that each depend on p. The first function, that identifies $\tilde{P}^S$, is

$$\sum_{\{i \in I: \tilde{p}_i^* < p\}} 1 - \frac{\tilde{p}_i^*}{p}, \tag{17}$$

which is increasing in p. For $p \leq \tilde{p}_1^*$ the function is undefined; for $\tilde{p}_1^* < p \leq \tilde{p}_2^*$ it takes the value $1 - \frac{\tilde{p}_1^*}{p}$; for $\tilde{p}_2^* < p \leq \tilde{p}_3^*$ it takes the value $2 - \frac{\tilde{p}_1^* + \tilde{p}_2^*}{p}$; for $\tilde{p}_n^* < p \leq \tilde{p}_{n+1}^*$ it takes the value $n - \frac{\sum_{i=1}^n \tilde{p}_i^*}{p}$. The second function, which will identify $\tilde{P}^B$, is

$$\sum_{\{i \in I: \tilde{p}_i^* > p\}} 1 - \frac{p}{\tilde{p}_i^*}, \tag{18}$$

which is decreasing in p and piecewise linear. Working from large values of p to smaller values, for $p \geq p_N^*$ the function is undefined; for $\tilde{p}_{N-1}^* \leq p < \tilde{p}_N^*$ it takes the value $1 - \frac{p}{\tilde{p}_N^*}$; for $\tilde{p}_{N-2}^* \leq p < \tilde{p}_{N-1}^*$ it takes the value $2 - \frac{p}{\tilde{p}_N^*} - \frac{p}{\tilde{p}_{N-1}^*}$; and for $\tilde{p}_{N-n}^* \leq p < \tilde{p}_{N-n+1}^*$ it takes the value $n - \sum_{i=N-n+1}^N \frac{p}{\tilde{p}_i^*}$.

[22] It is readily verified by inspection of payoffs that if the bids and offers of all other firms are zero then any positive bid or offer gives a lower payoff than being inactive, making autarky a Nash equilibrium.

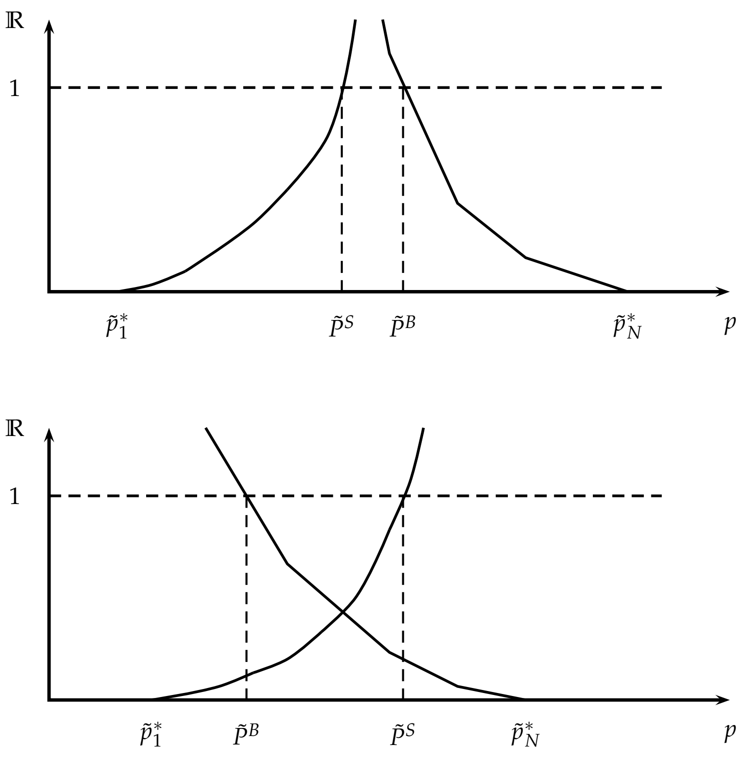

Figure 1: The construction of $\tilde{P}^S$ and $\tilde{P}^B$. The upward-sloping functions are $\sum_{\{i \in I: \tilde{p}_i^* < p\}} 1 - \frac{\tilde{p}_i^*}{p}$, which identifies $\tilde{P}^S$, and the downward-sloping functions, which identify $\tilde{P}^B$, are $\sum_{\{i \in I: \tilde{p}_i^* > p\}} 1 - \frac{p}{\tilde{p}_i^*}$.

$\tilde{P}^S$ is identified by the value of p where (17) is equal to one; $\tilde{P}^B$ is given by the value of p where (18) is equal to one. Figure 1 plots these functions for two different economies. In the upper panel the $\tilde{p}_i^*$s are widely dispersed and it is clear that in this case $\tilde{P}^S < \tilde{P}^B$ and therefore a non-autarkic Nash equilibrium in which trade in permits takes place exists in this economy. In the lower panel, however, the $\tilde{p}_i^*$s are less dispersed and in this case $\tilde{P}^S > \tilde{P}^B$, so the only equilibrium here involves no trade in permits.

The dispersion of the $\tilde{p}_i^*$s measures the gains from trading permits: if they are all equal there are no gains from trade and as they become more dispersed the gains from trade increase. As our illustration makes clear, the existence of gains from trade is not sufficient to ensure trade will take place: $\tilde{p}_1^* < \tilde{p}_N^*$ does not imply $\tilde{P}^S < \tilde{P}^B$. Rather, for a non-autarkic permit market equilibrium to exist there must be 'sufficient' gains from trading permits.

3.2 Cost efficiency of equilibrium

If a non-autarkic equilibrium does exist (i.e., the economy is such that $\tilde{P}^S < \tilde{P}^B$) will this equilibrium reduce pollution levels to Ω in a cost-effective way? If we were willing to assume that firms act as price-takers then the standard Walrasian equilibrium of the permit market would be used to describe equilibrium. Well-known results tell us that at the Walrasian equilibrium marginal abatement costs will be equalized; thus, whenever gains from trade in permits exist trade will take place, and emission reductions will be achieved in a cost-effective manner (Montgomery, 1972). In our model, consider two firms i and j that are active in a non-autarkic equilibrium with permit price $\hat{p}$, where i is a seller of permits ($\tilde{p}_i^* < \hat{p}$) and j is a buyer of permits ($\tilde{p}_j^* > \hat{p}$). Then it follows from (8) and (9) that

$$(1-\hat{\sigma}_i)^{-1}C_i^a(\tilde{z}_i(\hat{x}_i), f_i(\tilde{z}_i(\hat{x}_i)) - (\omega_i + \hat{x}_i)) = \hat{p} = (1-\hat{\sigma}_j)C_j^a(\tilde{z}_j(\hat{x}_j), f_j(\tilde{z}_j(\hat{x}_j)) - (\omega_j + \hat{x}_j)). \tag{19}$$

From (19), the following proposition is immediate.

Proposition 3. *In any permit market equilibrium in which trade takes place there exist $i, j \in I$ for whom*

$$C_i^a(\tilde{z}_i(\hat{x}_i), f_i(\tilde{z}_i(\hat{x}_i)) - (\omega_i + \hat{x}_i)) < C_j^a(\tilde{z}_j(\hat{x}_j), f_j(\tilde{z}_j(\hat{x}_j)) - (\omega_j + \hat{x}_j)),$$

so emissions reductions are not achieved in a cost-effective manner, unless all firms are negligible (so $\hat{\sigma}_i \approx 0$ for all $i \in I$).

This implies that between any buyer and seller (with non-negligible market share), further cost reductions are possible by transferring more permits from the seller to the buyer. All firms in bilateral oligopoly behave strategically; those that sell permits will restrict supply to try to increase the price, those that buy will restrict their bids to put downward pressure on the price. These strategic tensions combine to result in generic inefficiencies in the final allocation of permits.

3.3 Structure of the market

In the permit trading model developed in this article the sides of the market form endogenously: whether a firm becomes a seller or buyer of permits in equilibrium depends on their marginal abatement cost at their endowment in relation to the permit price, which depends on the actions of all firms. Since there is nothing in our model to suggest that the permit price will be the same with strategic behavior as with price-taking firms in a Walrasian model of permit exchange, *prima facie* it is unclear whether firms will take the same role as seller or buyer in these two market structures.

Proposition 4. *Suppose $\tilde{P}^S < \tilde{P}^B$ so there is a permit market equilibrium with trade. Let p^W be the price of permits in a competitive market, and suppose that in the permit market equilibrium $\hat{p} < [>] p^W$ and there is a firm i for whom $\hat{p} < \tilde{p}_i^* < p^W$ [$\hat{p} > p_i^* > p^W$]. Then in a competitive market firm i would be a seller [buyer], but when firms are modeled as behaving strategically the same firm, if active, is on the opposite side of the market.*

Proof. Let $\hat{p} < \tilde{p}_i^* < p^W$. If firm i was a buyer in a competitive market then $x_i > 0$ and $C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i - x_i)) = p^W$. But the fact that $\frac{\mathrm{d}^2\tilde{\pi}_i(x_i)}{\mathrm{d}x_i^2} < 0$ (Lemma 1) implies that $\tilde{p}_i^* \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) > C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i - x_i)) = p^W$, a contradiction. Thus, in a competitive market, firm i is a seller. In a strategic market, if firm i is also a seller then $x_i < 0$ and $C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i - x_i)) = (1-\hat{\sigma}_i)\hat{p}$. But then Lemma 1 again implies $C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) < C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i - x_i))$ so we have the inequality $\tilde{p}_i^* < (1-\hat{\sigma}_i)\hat{p} < \hat{p}$, a contradiction. The proof of the case $\hat{p} > \tilde{p}_i^* > p^W$ is similar and so omitted. □

Malueg and Yates (2009) present a competing model of fully strategic trade in permits that relies on the supply function approach of Klemperer and Meyer (1989). Although their focus is on the role of private information in permit markets, to ensure tractability of the model they must restrict supply functions to be linear. This has the consequence that, regardless of the distribution of market power, the equilibrium price will be equivalent to the competitive permit price (their Proposition 1). The equivalence of the equilibrium price between a strategic framework and a perfectly competitive framework, regardless of the distribution of market power, is a rather unrealistic feature of the supply function approach. In our bilateral oligopoly framework, the equilibrium price under strategic behavior is only equal to the competitive price if there is a perfect balance in strategic manipulation between both sides of the market, which, generically, will not be the case.

3.4 Comparative statics

As observed throughout this article, a number of fundamentals determine how firms trade permits: firms' endowments; their production (and abatement) technologies; as well as the demand in the goods market. We now consider the influence of these features on the permit market equilibrium.

Recall that the equilibrium in the permit market is identified by the intersection of the strategic supply and demand functions, the construction of which relies on aggregating firms' share functions defined in (11) and (12). A merit of the approach is that the properties of these share functions are relatively straightforward to deduce, allowing a comparative static analysis of equilibrium.

A firm's 'selling share function' is determined by the first-order condition $\frac{\mathrm{d}\tilde{\pi}_i(-\sigma_i Q)}{\mathrm{d}x_i} - (1-\sigma_i)p = 0$, the left-hand side of which is increasing in σ_i (by Lemma 1). As such, anything that increases [decreases] $\frac{\mathrm{d}\tilde{\pi}_i(-\sigma_i Q)}{\mathrm{d}x_i}$ will decrease [increase] the share function. Also note that strategic supply is determined by $\sum_{\{i \in I: \tilde{p}_i^* < p\}} s_i^S(p; Q) = 1$, the left-hand side of which is decreasing in Q. Consequently, if a firm's selling share function decreases [increases] then, other things equal, strategic supply will decrease [increase], for the range of prices where this firm would be a seller.

A similar rationale can be made for buyers' share functions. A firm's 'buying share function' is determined by $\frac{\mathrm{d}\tilde{\pi}_i(\sigma_i B/p)}{\mathrm{d}x_i} - (1-\sigma)^{-1}p = 0$, the left-hand side of which is decreasing in σ_i. Thus anything that increases [decreases] $\frac{\mathrm{d}\tilde{\pi}_i(\sigma_i B/p)}{\mathrm{d}x_i}$ will increase [decrease] the share function. Again recall that strategic demand is determined by $\sum_{\{i \in I: \tilde{p}_i^* > p\}} s_i^B(p, [B/p]p) = 1$, the left-hand side of which is decreasing in $[B/p]$. It follows that if a firm's buying share function increases [decreases] then strategic demand will increase [decrease], over the range of prices where this firm would be a buyer.

Now, from (7) we know that $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i))$. Our assumptions on firms' cost functions then implies that $\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}x_i}$ will increase [decrease] if (a) there is an increase [decrease] in demand in the product market that results in $\tilde{z}_i(x_i)$ increasing [decreasing] for all x_i; (b) the pollution generated from a given level of production increases [decreases], where a reduction may be due to, for example, improvements in abatement technology; and (c) the firm's permit allocation decreases [increases].

Consider a situation, then, where demand increases in the product market, which influences all firms. Each firm's selling share function will decrease, which decreases the strategic supply of permits, and each firm's buying share function will increase, which increases the strategic demand for permits (recall that strategic supply is an increasing function of p, and strategic demand is strictly decreasing in p). Consequently, an increase in demand in the product market increases the equilibrium price of permits. The effect on the equilibrium volume of permits traded is unclear since, while supply has contracted, the permit price has increased.

Consider next a situation where abatement technologies become more efficient so less pollution is generated from the production of goods and suppose this influences all firms equally. Then selling share functions will decrease, which will result in an increase in the strategic supply of permits, and buying share functions will decrease resulting in a reduction in strategic demand for permits. The effect of more efficient abatement technologies is to reduce the equilibrium price of permits, but the effect on the quantity of permits traded is unclear.

If the regulator wishes to reduce total emissions Ω, which it implements by reducing the endowment of all firms, then the effect is to decrease all firms' selling share functions which reduces strategic supply, and increase their buying share functions which increases strategic demand. The consequence will be upward pressure on the equilibrium price of permits. Note, however, that changes in permit endowments are often not undertaken in a uniform way. For example, we may consider a situation where a regulator changes policy from an equitable distribution of permits to a distribution where more highly polluting firms receive more permits. Suppose that with an equitable distribution of permits the equilibrium price is $\hat{p}$ and suppose further that the regulator increases the endowment of permits to those who are buyers (i.e., for whom $\tilde{p}_i^* > \hat{p}$) and reduces the endowment of permits to sellers (i.e., those firms for whom $\tilde{p}_i^* < \hat{p}$). For those firms that received a greater [smaller] endowment, their buying share function reduces [increases] and their selling share function increases [reduces], with the necessary implication that for all $p \geq \hat{p}$ strategic demand is lowered and, likewise, for all $p \leq \hat{p}$ strategic supply is also lowered. Consequently, the equilibrium quantity of permits traded will decline under the new regulation. In fact it is even possible that under an equitable distribution of permits where $\tilde{P}^S < \tilde{P}^B$, a change to the initial endowment towards a 'grandfathered' distribution of permits contracts both the strategic supply and demand enough to make $\tilde{P}^S \geq \tilde{P}^B$, so no trade in permits takes place: referring back to Figure 1, grandfathering may shift the economy from a situation depicted in the upper panel, to that depicted in the lower panel.[23] The effect on the equilibrium permit price when there remains a non-autarkic equilibrium is unclear, and even if the aggregate endowment of permits declines it does not necessarily follow that the permit price will increase.

[23]Note that if a firm's $\tilde{p}_i^*$ under an egalitarian distribution of permits is low then it will increase under grandfathering, whereas if $\tilde{p}_i^*$ is high it will decrease under grandfathering, thus reducing the gains from trade.

4 Market power in the product market

We now turn to consider non-competitive product market structures and the effect on the permit market equilibrium. In the previous framework it was assumed that firms were price-takers in the product market; yet it is possible that some element of market power may exist. This is, in fact, quite likely as many industries regulated by a cap-and-trade scheme are highly concentrated, such as the electricity market (Wolfram, 1999; Borenstein et al., 2002; Bushnell et al., 2008) and the cement industry (Ryan, 2012; Fowlie et al., 2016). The manifestation of market power in a product market is the restriction of supply to increase the price. We deduced in our comparative statics exercise that there is a positive relationship between firms' supply in the product market and their net demand for permits, and therefore with the equilibrium permit price. As such, if firms have market power in the product market and the supply of goods to the market reduces, the 'market-power effect' will put downward pressure on permit prices relative to the situation where firms are assumed to be price takers. If firms are independent monopolists in the product market—which would be the case if the output of their production process was sufficiently differentiated, or firms served regional markets—then the market-power effect is the only additional consideration, the details of which we elucidate in the next subsection. After this we consider the issues associated with imperfect competition in a product market, where the strategic importance of a firm's cost function (in relation its competitors) provides a richer link between the product and permit markets.

4.1 Independent monopolists

Consider a situation in which firms serve independent monopolies following the conclusion of the permit market: firms have market power in the product market, but there is no strategic interaction. This structure may occur, for example, when electricity companies participate in a permit market and are subsequent natural monopolists for their electricity supply (e.g., Ellerman et al., 2000).

To begin, let $\bar{\Phi}_i(z_i)$ be the inverse demand function in the market of firm i, then each firm's payoff function takes the form

$$\bar{V}_i = m_i - x_i p + \bar{\pi}_i(z_i, x_i) \text{ where}$$
$$\bar{\pi}_i(z_i, x_i) = z_i \bar{\Phi}_i(z_i) - C_i(z_i, f_i(z_i) - (\omega_i + x_i)).$$

To ensure that we can compare behavior in an independent monopoly market structure with that in a competitive product market we require some equivalence between the markets. Thus, we assume that if a firm supplies an identical quantity either as a monopolist or in a competitive market, then the price it will receive will be the same. With the functions we have defined, this requires that for any vector of permit allocations $\mathbf{x}$, $\bar{\Phi}_i$ is such that

$$\bar{\Phi}_i(\tilde{z}_i(x_i)) \equiv \tilde{\phi}(x_i, \mathbf{x}_{-i}).$$

We also assume $\bar{\Phi}_i' < 0$ and $2\bar{\Phi}_i' + z_i \bar{\Phi}_i'' < 0$, which are the standard monotonicity and decreasing marginal revenue assumptions that ensure concavity of firms' product market profit functions.

For a given permit market allocation, the first-order condition governing optimal behavior in the product market is $\frac{\mathrm{d}\bar{\pi}_i(z_i,x_i)}{\mathrm{d}z_i} \leq 0$ with equality if $z_i > 0$, implying

$$C_i^z(z_i, f_i(z_i) - (\omega_i + x_i)) + f'(z_i)C_i^a(z_i, f_i(z_i) - (\omega_i + x_i)) \geq \bar{\Phi}_i(z_i) + z_i\bar{\Phi}_i'(z_i).$$

We denote the solution by $\bar{z}_i(x_i)$ which is an interior solution since we assume $C_i^z + f_i'C_i^a = 0$ when $z_i = 0$.

With a given permit market allocation, a firm will supply less if it is an independent monopolist compared to a perfectly competitive firm since it understands its influence on the price. With a reduction in product supply and our given assumptions governing market equivalence this implies $\bar{z}_i(x_i) < \tilde{z}_i(x_i)$ for all x_i.

The reduced-form profit function (slightly abusing notation) is

$$\bar{\pi}_i(x_i) = \bar{z}_i(x_i)\bar{\Phi}_i(\bar{z}_i(x_i)) - C_i(\bar{z}_i(x_i), f_i(\bar{z}_i(x_i)) - (\omega_i + x_i)).$$

The value of a change in permit market strategy on product market profitability is, therefore,

$$\begin{aligned}
\frac{\mathrm{d}\bar{\pi}_i(x_i)}{\mathrm{d}s_i} &= \frac{\mathrm{d}\bar{\pi}_i(x_i)}{\mathrm{d}x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} \\
&= \frac{\mathrm{d}x_i}{\mathrm{d}s_i}\left[\frac{\mathrm{d}\bar{z}_i(x_i)}{\mathrm{d}x_i}[\bar{\Phi}_i + \bar{z}_i\bar{\Phi}_i' - (C_i^z + f'C_i^a)] + C_i^a\right] \\
&= \frac{\mathrm{d}x_i}{\mathrm{d}s_i}C_i^a(\bar{z}_i(x_i), f_i(\bar{z}_i(x_i)) - (\omega_i + x_i)).
\end{aligned}$$

As such, the permit market optimality condition has the same basic form as the case of a competitive product market. However, as noted, an independent monopolist will supply less to the market so the arguments in the function are different; as the marginal cost of abatement increases with output ($\frac{\mathrm{d}C_i^a}{\mathrm{d}z_i} = C_i^{za} + f'C_i^{aa} > 0$) this implies that an independent monopolist will value permits less than a firm in a competitive market. For a given permit market price this will reduce the demand from those firms that still wish to buy permits, increase the supply of those firms that wish to sell permits, and means that firms that previously wanted to acquire permits may now want to switch to the supply side. The following proposition is derived.

Proposition 5. *When firms are independent monopolists in the product market, strategic demand (supply) for permits is always smaller (larger) compared to price-taking firms in the product market. Consequently, the effect of market power in the product market is to lower the equilibrium permit price.*

Proof. First, we confirm that a firm supplies less as an independent monopolist than it does as a price taker: $\bar{z}_i(x_i) < \tilde{z}_i(x_i)$ for all x_i. With an output of $\tilde{z}_i(x_i)$,

$$\Phi_i(\tilde{z}_i(x_i)) + \tilde{z}_i(x_i)\bar{\Phi}_i'(\tilde{z}_i(x_i)) = \tilde{\phi}(x_i, \mathbf{x}_{-i}) + \tilde{z}_i(x_i)\bar{\Phi}_i'(\tilde{z}_i(x_i)) < \tilde{\phi}(x_i, \mathbf{x}_{-i}).$$

Since $\tilde{\phi}(x_i, \mathbf{x}_{-i}) = C_i^z(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i)) + f'(\tilde{z}_i(x_i))C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i))$, this implies that $\bar{\Phi}_i(\tilde{z}_i(x_i)) + \tilde{z}_i(x_i)\bar{\Phi}_i'(\tilde{z}_i(x_i)) < C_i^z(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i)) +$

$f'(\tilde{z}_i(x_i))C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i))$ and consequently the monotonicity of the left- and right-hand sides of the first-order condition imply that $\bar{z}_i(x_i) < \tilde{z}_i(x_i)$.

Since $C_i^{za} + f_i' C_i^{aa} > 0$ this implies that

$$C_i^a(\bar{z}_i(x_i), f_i(\bar{z}_i(x_i)) - (\omega_i + x_i)) < C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - (\omega_i + x_i))\, \forall x_i. \tag{20}$$

This allows us first to establish that the price at which a firm switches between being a potential seller and potential buyer falls when they are an independent monopolist:

$$\bar{p}_i^* \equiv C_i^a(\bar{z}_i(0), f_i(\bar{z}_i(0)) - \omega_i)) < C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i)) \equiv \tilde{p}_i^*,$$

implying that at a given permit price p, firms are either sellers in both market structures; buyers in both market structures; or switch from being a buyer to a seller, but not vice versa.

Following the method of analysis in our discussion of comparative statics, the inequality in (20) implies that the selling share function in the permit market of an independent monopolist will exceed that of a price taker, and the buying share function in the permit market of an independent monopolist will be less than that of a price taker, implying that if all firms are independent monopolists, the strategic demand for permits will be lower, and the strategic supply of permits higher, than if all firms are price takers, and consequently the equilibrium price of permits will fall. □

4.2 Oligopolistic product market

When firms compete in both the permit and product market the interaction between markets is much richer. The effect of trading permits changes the firm's cost function for the product market and, importantly, influences the marginal cost of production. If the market is perfectly competitive (or if firms serve independent monopolies), this 'direct effect' of permit market activity is the only effect that influences firms' optimal output. If strategic behavior is considered in the product market, however, the outcome from engaging in Cournot competition hinges crucially on the firm's marginal cost in relation to those of its competitors. This raises two additional effects of permit market activity: an 'indirect effect' that results from the change in the product market equilibrium attributable to a change in the firm's own marginal cost; and, since the total number of permits is fixed, a 'changing rivals' costs' effect that results from a change in the product market equilibrium attributable to the change in other firms' marginal costs. These effects provide an additional incentive to acquire permits, thereby (at least) partially mitigating the suppressed net demand for permits that occurs due to the existence of product market power (and therefore restricting output).

To consider the effect of strategic interaction in the product market, suppose that the firms participating in the permit market then go on to supply the same product market in which they compete à la Cournot. The price in the goods market will be determined as $\Phi(Z)$, which depends on the aggregate supply of all firms $Z = \sum_{i=1}^n z_i$. Consider a product market subgame in which the vector of permit allocations is $\mathbf{x} = \{x_i\}_{i=1}^n$. In this subgame, we want to deduce the Cournot equilibrium. The payoff function of firm i in this subgame takes the form

$$V_i = m_i - x_i p + \pi_i(z_i, Z, x_i) \text{ where}$$
$$\pi_i(z_i, Z, x_i) = z_i \Phi(Z) - C_i(z_i, f(z_i) - (\omega_i + x_i)).$$

When engaging in Cournot competition firms can be seen as maximizing their payoff with respect to z_i, taking the actions of other traders as given, which implies

$$C_i^z + f^{'} C_i^a \geq \Phi(Z) + z_i \Phi^{'}(Z),$$

with equality if $z_i > 0$. Since we assume $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$ each firm will be active in a Cournot equilibrium, and we denote by $\hat{z}_i(Z; x_i)$ the output of firm i consistent with a Nash equilibrium in which the aggregate supply of all firms is Z, which satisfies the above first-order condition with equality. A Nash equilibrium in the subgame requires that these consistent individual supplies are also consistent with the aggregate supply. As such, the aggregate supply at the Cournot equilibrium in the subgame in which the vector of permit allocations is $\mathbf{x}$ is given by $\hat{Z}(\mathbf{x})$, defined by:

$$\hat{Z}(\mathbf{x}) = \{Z : \sum_{i=1}^{n} \hat{z}_i(Z; x_i) - Z = 0\}. \tag{21}$$

Notice that this depends on the entire vector of permit allocations. The equilibrium supply of firm i is then written $\hat{z}_i(\hat{Z}(\mathbf{x}); x_i)$. Our assumptions on demand and cost functions imply that individual 'replacement functions' $\hat{z}_i(Z; x_i)$ are decreasing in Z and therefore that $\sum_{i=1}^{n} \hat{z}_i(Z; x_i)$ is decreasing in Z so there is a unique fixed point and so a unique Cournot equilibrium.[24]

Returning now to first-stage decisions in the permit market, the reduced-form payoff function for firm i is

$$V_i = m_i - x_i p + \pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i) \text{ where}$$
$$\pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i) = \hat{z}_i(\hat{Z}(\mathbf{x}); x_i)\phi(\hat{Z}(\mathbf{x})) - C_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), f(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i)) - (\omega_i + x_i)).$$

When considering its optimal action in the permit market, a firm needs to consider the marginal effect on its allocation of permits and the benefits (or costs) that this brings in terms of product market profitability. With $s = \{b, q\}$, the first-order condition governing optimal behavior in the permit market requires

$$\frac{\mathrm{d}\pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i)}{\mathrm{d}s_i} \leq \frac{\mathrm{d}x_i p}{\mathrm{d}s_i}.$$

The right-hand side of this first-order condition is the same as when we assumed the firm is a price-taker in the product market. The left-hand side, however, is somewhat different as it accounts not only for the direct effect of permit market activity on product market profitability, but also the indirect and changing rivals' cost effects.

Decomposing the effect of permit market activity on product market profitability, we

[24]This method was first used in the analysis of Cournot equilibrium by Novshek (1985).

find

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}s_i} = \frac{\partial \pi_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} + \frac{\partial \pi_i}{\partial z_i}\frac{\mathrm{d}\hat{z}_i}{\mathrm{d}s_i} + \frac{\partial \pi_i}{\partial Z}\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}. \tag{22}$$

Now, in the second term,

$$\frac{\mathrm{d}\hat{z}_i}{\mathrm{d}s_i} = \frac{\partial \hat{z}_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} + \frac{\partial \hat{z}_i}{\partial Z}\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}. \tag{23}$$

In both (22) and (23) the effect on the equilibrium aggregate output $\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}$ can be decomposed into the direct effect from firm i's permit market strategy, and the indirect effect that comes through firm i's strategy influencing the permit holdings of other firms:

$$\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i} = \frac{\partial \hat{Z}}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} + \sum_{j\neq i}\frac{\partial Z}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}s_i}. \tag{24}$$

Inserting (24) and (23) into the initial decomposition (22) and re-arranging yields

$$\begin{aligned}\frac{\mathrm{d}\pi_i}{\mathrm{d}s_i} =& \frac{\partial \pi_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} + \frac{\mathrm{d}\pi_i}{\mathrm{d}z_i}\frac{\partial \hat{z}_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} \\ &+ \left(\frac{\partial \pi_i}{\partial z_i}\frac{\partial \hat{z}_i}{\partial Z} + \frac{\partial \pi_i}{\partial Z}\right)\frac{\partial \hat{Z}}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}s_i} \\ &+ \left(\frac{\partial \pi_i}{\partial z_i}\frac{\partial \hat{z}_i}{\partial Z} + \frac{\partial \pi_i}{\partial Z}\right)\sum_{j\neq i}\frac{\partial \hat{Z}}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}s_i}.\end{aligned} \tag{25}$$

The first line of (25) captures the direct effect of permit market activity on profit that comes about from a change in optimal supply; the second line captures the indirect effect of permit market activity that comes from the change in firm i's permit holdings influencing the equilibrium in the product market; and the third line captures the changing rivals' cost effect that changes the product market equilibrium indirectly through the effect of firm i's actions on the permit holdings of others. Note that in the final term $\frac{\mathrm{d}x_j}{\mathrm{d}s_i} = 0$ for those traders $j \neq i$ that are sellers of permits, since their permit allocation is unilaterally decided by $x_j = -q_j$, so the changing rivals' cost effect only materializes for firms on the demand side of the permit market.

This expression presents two complications for a general model of trade in permits. First, it depends on the whole vector of permit allocations implying optimal permit market actions cannot be written only as a function of aggregations of others' strategies; consequently, this makes the analysis of permit market equilibrium that relied on the aggregative properties of the game more complicated. In Dickson and MacKenzie (2016) we show that this is tractable for conventional cost and demand functional forms used in the environmental economics literature. Second, the sign of $\frac{\mathrm{d}\pi_i}{\mathrm{d}s_i}$ is ambiguous. Consider the decision of a permit buyer that engages in a strategic product market compared to the buyer in a competitive product market. Their output in a strategic market will be less than in a competitive market which will serve to reduce their demand for permits. However, by acquiring permits the firm lowers its marginal cost relative to others: purchasing permits reduces its own marginal cost and simultaneously increases the marginal costs of other permit buyers since any permits acquired by the firm in question cannot be acquired by other firms. These strategic considerations serve to increase the demand for permits. Which of these effects dominates depends on a multitude of factors, not least the competitiveness of the product market and the firm's market power in that market.

5 Conclusion

The purpose of this article is to investigate the implications of strategic trade in pollution markets. By establishing a strategic market game where firms' roles as buyers or sellers are determined endogenously, we create a two-stage framework, where in the first stage firms participate in a price-mediated permit market and, in the second stage, firms select their level of production.

In the permit market, we use a strategic market game to identify firms' roles as buyers or sellers of permits and allow for price-mediated trade. We show that the equilibrium is generically inefficient even if trade in permits takes place, and indeed that the only equilibrium may involve no trade if there are insufficient gains from trade. Our framework also shows that strategic trade can alter the structure of the market, as the role of firms (buyers or sellers) and the equilibrium price are now endogenously determined: buyers (sellers) in a competitive market can switch their role in a market with strategic trade. Thus we show the use of strategic trade via a price-mediated strategic market game has fundamental consequences for the cost efficiency, level of exchange, equilibrium permit price, and structure of the market.

As cap-and-trade markets are now frequently implemented to control major pollution problems, it is important to identify how, in the presence of non-competitive behavior, the market equilibrium is established, and, of course, the associated cost inefficiencies. Our approach, by focusing on endogenous market formation and a price-mediated solution, has identified the fundamental links between strategic behavior, cost inefficiency, market formation, and the nature of the equilibrium.

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