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CONSUMPTION EXPENDITURES IN ECONOMIC IMPACT STUDIES: AN APPLICATION TO UNIVERSITY STUDENTS

By

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**Consumption Expenditures in Economic Impact Studies:
An Application to University Students**

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Abstract

This paper examines how appropriately to attribute economic impact to consumption expenditures. Consumption expenditures are often treated as either wholly endogenous or wholly exogenous, following a distinction from Input-Output analysis. For many applications, such as those focusing on the impacts of tourism or benefits systems, such binomial assumptions are not satisfactory. We argue that consumption is neither wholly endogenous nor wholly exogenous but that the degree of this distinction is rather an empirical matter. We set out a general model for the treatment of consumption expenditures and illustrate its application through the case of university students. We examine individual student groups and how the impacts of students at particular institutions. Furthermore we take into account the binding budget constraint of public expenditures (as is the case for devolved regions in the UK) and examine how this affects the impact attributed to students' consumption expenditures.

Keywords: Input-Output, Impact, Higher Education, Students, Expenditures

JEL-Codes: I23, I25, R12, R15

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1 Introduction

There is some ambiguity in applied demand-driven economic impact analysis as how to treat consumption expenditures. For Input-Output analysis this is clear-cut in principle. Household consumption is either treated as wholly endogenous (Type-I) or wholly exogenous (Type-II). In many practical applications, however, this either-or distinction is not fit for purpose. This is manifested, for example, when determining the impact of tourism spending where in some cases not all of it is wholly exogenous (see for example Allan et al (2007) for the case of sports tourism in Glasgow) or when analysing the consumption spending of social groups whose consumption is partially supported by state transfers (Dunlop, 2001). These problems tend to be solved by researchers on a case by case basis, but have (to our knowledge at least) received limited formal attention.

We seek to address this apparent lacuna by setting out a model, which treats the degree of exogeneity of consumption expenditures essentially as an empirical matter, determining it via a simple accounting approach, drawing on available survey data. The case is illustrated through an application to the case of students at Higher Education Institutions (HEIs) in Scotland. This case has three particular merits: Firstly, it has been studied somewhat extensively by applied economists in the past; secondly, different types of students draw on different sources of income, therefore raising interesting challenges as how to treat the impact of these; and the availability of sufficiently good data, in particular recent surveys of students incomes and expenditures (Warhurst et al., 2009).

A number of studies have examined the impact of student's consumption (e.g. Love and McNicoll 1988, Steinacker 2005), often in the context of the host institution's expenditure impacts (e.g. Armstrong 1993, Bleaney et al 1992, Brownrigg 1973, Harris 1997, Hermansson et al 2013ab, Love & McNicoll 1990). There have been two alternative treatments of student expenditures in past impact studies: one incorporates only the expenditures of in-coming students (e.g. Kelly *et al*, 2004), the other includes all student expenditures, irrespective of their origin (e.g. Harris, 1997).

We argue that each of these past treatments represents an approximation to an input-output accounting approach in which the crucial distinction is between the exogenous and endogenous components of student expenditures. External students' expenditures (net of direct imports) can be regarded as exogenous to the host region. However, home students' expenditures cannot legitimately be treated as either wholly endogenous, which would validate the first approach, nor wholly exogenous, which would validate the second. We apply our model based on a recent survey of sources of student incomes in Scotland (Warhurst *et al*, 2009), to allow a more nuanced, and more informed, distinction between the exogenous and endogenous components of student expenditures.

A part of student's consumption is funded by public grants. Devolved regions like Scotland are subject to a public expenditure constraint determined by the Barnett formula. This implies that

public funds allocated to students cannot be used to finance other types of public expenditure. Similarly, public funding of students at HEIs in Scotland has an opportunity cost for the Scottish Government (Hermannsson et al, 2013ab). We explore the impact of accommodating this funding constraint in order to isolate that part of the overall impact of student expenditures that is attributable to students *per se*, rather than to the public funding that they receive.

The next section discusses the conventional treatment of consumption in Input-Output models, focussing on the example of higher education students. The third section presents a general model for consumption impacts. This is a skeletal model set out to illustrate the approach. A more detailed theoretical discussion is provided in Appendix. The fourth section analyses the expenditure impacts of students' consumption expenditures and shows how results based on empirically determined exogeneity parameters differ from those derived under conventional 'rule of thumb' assumptions applied in the literature. Furthermore, the model is used to accommodate the binding budget constraint of devolved public expenditures. The fifth section analyses the expenditure impact of students at individual institutions, in order to explore the heterogeneity of impacts and its causes. Again, the balanced expenditure impacts of students' consumption expenditures are also examined. Brief conclusions are presented in Section 6.

2 The treatment consumption in IO models and established conventions for student expenditures

In Input-Output analysis household consumption is either treated as wholly exogenous or entirely endogenous. These are referred to as Type-I and Type-II cases (for a general discussion see Leontief 1986, Miller 1998, Miller & Blair 2009). In the first case households' consumption expenditures provide an exogenous injection to the economy under analysis and the multiplier captures the internal feedbacks this drives. In the second case the model is closed with regards to wages and household expenditures. In this setting the remaining exogenous elements (typically government expenditures, investments and exports) drive endogenous feedbacks, i.e. intermediate trade and household consumption, which respond passively to responses in external stimuli.

Elaborations on the type-II principle include Type-III and Type-IV multipliers. Whereas Type-II multipliers assume a linear relationship between total income and total consumption spending for all consumers, Type-III multipliers (Myernik, 1967) adjust these for the spending patterns of different income groups. Type-IV multipliers (Batey & Madden 1983) make a distinction between those households where average consumption coefficients apply (such as in the Type-II case) and those where marginal consumption coefficients are more relevant, such as in-migrants or those that are re-employed out of unemployment. Although these are useful refinements of the Type-II multiplier they relate to the pattern of expenditures generated by household consumption rather than the determination of endogeneity or exogeneity of the income driving those expenditures. Therefore, the basic dichotomy between all in or nothing remains.

This dichotomous distinction can be overly simplistic for applied work. In many cases household consumption is partially endogenous and partially exogenous. Capturing that nuance can be important for accurately assessing the economic impact of the subject being studied. An example is the case of football tourists in Glasgow as illustrated by Allan *et al* (2007). Glasgow's football fixtures attract a large number of spectators, from within the city, other parts of Scotland and further afield. Some of the associated consumption expenditures are net injection to the city economy (exogenous), while the expenditures of local spectators displace other consumption expenditures and are therefore endogenous. Other examples include the impact of transfer payments (Dunlop, 2001) and students (Hermannsson, 2013ab), which we shall focus on in this paper.

Two conflicting approaches have been applied to measure the impact of students' consumption expenditures. The first is typically motivated, in effect, by an assumption that all students would have studied elsewhere in the absence of local HEIs, as in Battu *et al* (1998) and Harris (1997). On this assumption the whole of all students' expenditures in the host region are directly attributed to that region's HEI(s). In the remainder of the paper we shall refer to this as the 'All Included' (AI) approach. The alternative treatment takes the expenditures of students who move into the region to study to be the only genuinely additional part of student expenditures, as in Kelly *et al* (2004). For convenience we label this as the 'External Only' (EO) approach. This view is often motivated in terms of the notion that indigenous students are likely to remain in the region even in the absence of HEIs. For small host regions with a single HEI, which is the case Battu *et al* (1998) and Harris (1997) were considering, the former (AI) view may appear to be more convincing since many of the students are likely to be in-migrants anyway, and it may be easy for any indigenous students to travel to study at an alternative HEI. However, in the context of a study of the expenditure impacts of students at individual HEIs in a much larger region like Scotland, with 20 HEIs, the second (EO) treatment is often favoured.

In this context, attributing regional economic activity to the whole of students' expenditures is tantamount to assuming that all of these expenditures are exogenous to the region. In contrast, attributing regional economic activity only to in-migrant students effectively assumes that all of the expenditures of home students are endogenous, since no activity is attributed to their spending. Neither of these limiting assumptions is likely to be satisfied in practice, rather some parts of student expenditures are likely to be exogenous and some parts are likely to be endogenous.

In order to clarify this it is helpful to consider the different groups of students that will comprise the typical HEI's student body: overseas students; students from other regions of the UK, and home students. For students who are attracted into the region from outwith the UK, it is reasonable to assume that the whole of their expenditure is exogenous, since it is entirely funded by activity in the rest of the World (whether by students' own saving, family or governmental transfers). However, for students who come to study from another region of the UK matters are less clear cut. Local employment is much more likely than it is for overseas students, given the shared language, culture and absence of legal barriers to employment. This

element of student funding is clearly endogenous to the host region. However, expenditure financed by parents in RUK, from borrowing, by other regional governments, or by the national government, is appropriately treated as exogenous. Finally, it seems very unlikely that local or home students' expenditures can legitimately be treated as either wholly exogenous or wholly endogenous. Many students work part-time and many receive support out of parental incomes, and so at least a part of their expenditures are very likely to be endogenous. However, the presence of some Scottish government funding and credit financing suggest that at least a part of Scottish students' expenditures should be regarded as exogenous.

In contrast, we develop an alternative approach, the Exogenous-Endogenous Attribution (EEA) approach. The key focus is on the distinction between those elements of student expenditures that are exogenous to the host economy and those that are endogenous. Endogenous economic activity is then attributed to each of the exogenous expenditures using a multiplier model.

3 A model of student's consumption expenditure impact

In practice accounting for the implicitly linked (exogenous) students' expenditure involves 3 steps: determining the level of student spending; judging the extent to which this is exogenous to the host economy; and identifying how student expenditures are distributed among sectors. Then an Input-Output model can be used to derive the knock-on impacts of these expenditures.

In the Leontief model, gross output in the economy (\mathbf{q}) is determined as the product of the Leontief inverse $(\mathbf{1} - \mathbf{A})^{-1}$ and a vector of final demands (\mathbf{f}):

$$\mathbf{q} = (\mathbf{1} - \mathbf{A})^{-1}\mathbf{f} \quad (1.a)$$

In Input-Output parlance, what we need to do is to estimate the share of final demand attributable to students (\mathbf{f}^s). This is determined as:

$$\mathbf{f}_n^s = \mathbf{v}_c^s c_n^s x_n (1 - \delta) \quad (2.a)$$

where \mathbf{v}_c^s is a vector that reveals the sectoral breakdown of students' consumption expenditures¹, c_n^s is the average gross consumption expenditures of student type n, x_n is the share of gross consumption expenditures of student group n that is exogenous and δ is the direct import share, which is equal across all student groups and fixed at 32.0% (equal to that of households in the Scottish IO-tables).

¹ This is obtained from a survey carried out by Kelly et al (2004). We assume this same consumption pattern holds across all student groups.

The columns of the Leontief inverse can be summed to obtain output multipliers. From this point of view a sector's output (q) can be determined as the product of the final demand for the sector's outputs f and the relevant multiplier (m):

$$q = f m \quad (1.b)$$

An output multiplier for students- consumption expenditures (m^c) can be obtained by post multiplying the Leontief inverse with the sectoral breakdown of students' consumption (v_c^s):

$$m^c = (\mathbf{1} - \mathbf{A})^{-1} v_c^s \quad (3)$$

This multiplier is effectively a weighted average of the output multipliers of all the local sectors students purchase from². Furthermore, using the scalar notation introduced in equations 1.b and 3, equation 2.a can be modified so that the final consumption demand of a student representative of type/group n can be expressed as:

$$f_n^s = c_n^s x_n (1 - \delta) \quad (2.b)$$

where c_n^s is the average gross consumption expenditures of student type n , x_n is the share of gross consumption expenditures of student group n that is exogenous and δ is the direct import share, which is equal across all student groups and fixed at 32.0% (equal to that of households in the Scottish IO-tables). Furthermore, Inserting equations 2.b and 3 into equation 1.b the output impact of a student's consumption expenditures can be represented as:

$$q^s = m^c c_n^s x_n (1 - \delta) \quad (4)$$

The earlier approaches can be seen, in effect, as special cases, which resolve the difficulty simply by assuming that either all, or only external, students' expenditures can be regarded as exogenous.

4 IO-impacts under different assumptions about the exogeneity of students' consumption expenditures

In the case of *external* students the identification of exogenous expenditures is straightforward. The whole of external students' expenditures are unambiguously exogenous as their incomes are derived from an external location, i.e. $x_n = 1$. The treatment of their expenditure is similar to that of tourists. For *local* students, however, the distinction between their endogenous and exogenous consumption is less clear cut. To a large extent their income, and hence consumption, is endogenous to the local economy in that it comes from wages earned from local industries and transfers from within local households. However, under the EEA approach local students' expenditures contain some exogenous elements. We take these

² That is: $m^c = \sum \alpha_i^s m_i$ where $\alpha_i^s = f_i^s / \sum f_i^s$.

to be expenditures that are financed from commercial credit taken out during their years of study, student loans and education-related grants and bursaries.

For details of Scottish students' income and expenditures this study draws on the comprehensive survey by Warhurst *et al* (2009). We begin by identifying the scale of Scottish students' expenditures and how they are funded. The full details of how student expenditures are determined are reported in the Appendix of Hermannsson *et al* (2013a) but the results are summarised in Table 1. To illustrate how the results are obtained, it is useful to run through each column of the table. For Scottish students the starting point is the average expenditure of Scottish domiciled undergraduate students as reported in Warhurst *et al* (2009) £6,230. From this a number of deductions are made for income sources that are endogenous to the Scottish economy. These are all derived directly or indirectly from results reported by Warhurst *et al* (2009). Furthermore an estimate for expenditures supported by students taking out additional commercial credit during their studies is added back. This results in an estimate of the exogenous element of the expenditures of the average student, which is equivalent to $c_n^s x_n$ in the expenditure model. Then the direct import component of these expenditures is deducted based on the direct import rate of Scottish households ($\delta=32\%$). This results in the exogenous expenditures of the average student of a Scottish domicile on local sectors (£1,724), or $c_n^s x_n (1 - \delta)$ in the model. Once this per student impact has been determined it is straightforward to scale this up to the impact of the entire student group by drawing on FTE student numbers³.

Table 1 Derivation of exogenous student spending by place of domicile

Location of domicile		Scotland	Rest of the UK	Rest of the World
Gross average student spending £	+	6,230	7187	7,187
Income from employment £	-	1,945	1,945	
Within household transfers £	-	453		
Other income £	-	570		
Dissaving £	-	1,073		
Spending attributable to new commercial credit £	+	346		
Exogenous average per student spending £ ($c_n^s x_n$)	=	2,535	5,242	7,187
Direct imports £ ($\delta=32\%$)	-	811	1,677	2,300
Final demand for output of local sectors per student £ ($c_n^s x_n (1-\delta)$)	=	1,724	3,565	4,887
Number of students FTE's	x	114,262	22,052	24,555
Net contribution to final demand for output of local sectors £m (f_n^s)	=	197.0	78.6	120.0

The same process is applied to incoming students. However, the starting point is different. In the absence of survey evidence of the expenditure levels of incoming students we use the average expenditures of Scottish students living independently (£7,187) as a proxy of the living expenditures of incoming students. For incoming students these expenditures are exogenous to the Scottish economy, except that we assume students from the rest of the UK to participate in the labour market at the same rate as Scottish students. The same procedure is applied as before to deduct the direct import content of the exogenous expenditures.

³ These are obtained from the Higher Education Statistics Agency (HESA).

There are a number of points worth noting from Table 1. First the estimate of exogenous expenditure per local student ($c_n^S x_n$) is £2,535, which is less than half of the estimated £5,242⁴ of exogenous expenditures by the typical student from RUK studying in Scotland, and only 35% of the corresponding estimate for an ROW student. Accordingly, our treatment implies that it would be inappropriate to regard the whole of Scottish students' expenditures as exogenous, since elements are clearly endogenous (including those related to income from employment). Equally, while the endogenous expenditures of local students constitute 41% of their total expenditure, this is well below the 100% often assumed.

Per student contribution to local final demand is obtained by deducting direct imports from exogenous student expenditures. This is much higher for RUK and ROW students than for home students. To obtain total impacts on final demand this is multiplied by the number of students in each category, as reported in the final row of Table 1. These estimates imply that local student expenditures contribute £197.0 million to local final demand, about one and a half times more than the amount contributed by ROW students and over two and a half times that of RUK students.

Once students' contribution to final demand has been determined the next step is to estimate the knock-on impacts of their consumption spending. The Type-II output multiplier for student consumption spending (m^C) derived from the IO tables is 1.8. Hence, a direct injection of £395.6 million (the sum of the elements in the bottom row of Table 1) generates £710.6 million of output in the Scottish economy or approximately 0.4% of total output. Despite the relatively modest impact *per student*, Scottish students make up approximately two thirds of the student population and therefore drive a significant portion of total student final demand and account for approximately half of the total student consumption impact. The consumption spending of students from the rest of the world is a little less, accounting for 30% of the total consumption impact, and the remaining 20% is attributable to the expenditure of students from the rest of the UK.

4.1 Comparison of students' consumption impacts under different approaches

Table 1 demonstrates how the exogenous expenditures of different student groups add up to reveal differences in final demand expenditures by student groups. Furthermore, equation 4 shows how, the differences in final demand expenditures by student groups can be accommodated in an impact model. For each of these student groups the consumption multiplier is the same but the differences in impacts are driven by their gross expenditures c_n^S and the extent to which these expenditures are exogenous x_n .

Multipliers offer the benefit of a scale-independent metric, which is convenient for comparison. Therefore, it is useful to derive an expenditure multiplier for each student group

⁴ Incoming students from the UK are assumed to participate in the Scottish labour market whereas students from the rest of the World are assumed not to.

to facilitate comparison of their impacts. To obtain a multiplier we can divide through with gross consumption c_n^s , so that⁵:

$$m_n^s = \frac{q_n^s}{c_n^s} = m^c x_n (1 - \delta) \quad (5)$$

The estimates for exogenous and endogenous consumption expenditures of different student types can easily be summarised in terms of the parameters of the student impact model. A summary of these parameters is presented in Table 2 below. These individual impacts can be scaled up to reflect the impact of whole student populations, but it is also straightforward to aggregate group impacts to show combinations of different student groups for entire regions, such as Scotland, or individual HEIs there within.

Table 2 Comparison of parameters for the student spending model and estimated per student impacts, by student origin.

		Scotland	Rest of the UK	Rest of the World
Gross consumption	c_n^s	6,230	7,187	7,187
Consumption multiplier	m^c	1.80	1.80	1.80
Exogenous share of expenditures	x_n	41%	73%	100%
Share of direct imports in expenditures	δ	32%	32%	32%
Students' gross-expenditure/output multiplier	m_n^s	0.50	0.89	1.22
Output impact of consumption expenditures	q_n^s	3,097	6,403	8,779
Number of students FTEs	v_n	114,262	22,052	24,555
Impact of student group/type n (£m)	\bar{q}^s	354	141	216

The impact of a single student representative of group/type n can be multiplied with the number of FTE students belonging to that category (s_n). Using "bar" to represent a whole group of students of type n, then the output impact of their consumption expenditures can be represented as:

$$\bar{q}_n^s = m_n^s c_n^s s_n = m^c x_n (1 - \delta) c_n^s s_n \quad (5)$$

Furthermore, impact of different student groups n can be aggregated to provide the consumption impact of the entire student population:

$$\bar{q}^s = \sum_n m_n^s c_n^s s_n = m^c \sum_n x_n (1 - \delta) c_n^s s_n \quad (6.a)$$

However, we may want to obtain a scale independent metric of the impacts of the group of students attributable to particular regions or institutions (who are in turn made up of combinations of student groups with different expenditure characteristics). The obvious

⁵ However, this multiplier is not directly comparable with conventional IO output multipliers, as it does not relate final demand to output impact, but has been modified to show the link between gross student expenditures and the output impact of students' consumption expenditures.

solution would be to divide through the impact estimate with the final demand. However, since in this case the variation between different student types comes through the final demand but not the expenditure structure as such, this would only reveal the consumption multiplier m^c , which is the same for all students. However, dividing through equation 6.a with the number of students (s_{ni}) reveals the output impact of the consumption expenditures of the average student. Using a "hat" to denote the average impact of students at institution i , this can be expressed as

$$\hat{q}_i^s = \frac{\bar{q}_i^s}{\sum_n s_{ni}} \quad (7)$$

where $\sum_n s_{ni}$ represents the total of all students (sum over all n types) at HEI i .

As we saw in Table 2 there are two elements that drive the difference in impacts between students of different origins. First, students from different origins differ in their gross consumption expenditures. Secondly, how these gross expenditures translate into economy-wide impacts varies as the share of exogenous expenditures differs between different student types. This, in turn, affects the gross-expenditure/output multiplier m_n^s . However, in this study the share of exogenous expenditures x_n is determined empirically, whereas in previous literature this parameter is based on simplifying assumptions

For the External Only (EO) approach the assumption is that $x=0$ for local students but that $x=100\%$ for incoming students. In the case of Scotland this would mean that $X_{SCO}=0$ and $X_{RUK}=X_{ROW}=1$. For the All Included (AI) approach we are effectively assuming that $X=1$ for all students, so that in the case of Scotland this would mean that $X_{SCO}=X_{RUK}=X_{ROW}=1$. Therefore, we can see that the EO approach underestimates the impact of local students and overestimates the impact of RUK students, but is an accurate assumption for ROW students. The AI approach, however, overestimates both the impacts of Scottish and RUK students, while being accurate for the case of those coming in from the rest of the World.

Table 3 summarises the treatment of students' consumption expenditures under each of the three approaches and how this results in significantly different impacts. The left hand panel presents the exogeneity shares x_n , whilst the right hand panel presents corresponding per student impacts. The rightmost column shows the average across all students at Scottish HEIs.

Table 3: Exogeneity shares (x_n) and impacts per student (q^s) under different assumptions about the exogeneity of students' consumption expenditures.

Approach	Exogenous expenditures as a share of gross expenditures			Impact of average student £			
	x_{SCO}	x_{RUK}	x_{ROW}	\hat{q}_{SCO}^s	\hat{q}_{RUK}^s	\hat{q}_{ROW}^s	\hat{q}_{All}^s
Endogenous							
Exogenous Attribution (EEA)	0.41	0.73	1	3,097	6,403	8,779	4,417
External Only (EO)	0	1	1	0	8,779	8,779	2,544
All Included (AI)	1	1	1	7,610	8,779	8,779	7,949

The three different approaches result in significantly different results. The impact of the average student is £4,417 under the EEA approach. However, the commonly applied EO approach results in an average impact of 2,544, which is just under 60% of the baseline approach. Alternatively, the AI approach results in an average per student impact amounting to £7,949. That is nearly twice that of the EEA approach and more than three times that of the EO approach. As is expected, these variations in x_n impact student groups in different ways. The biggest impact is on the estimate for local students, with impacts per Scottish student ranging from £0 under the EO approach to £7,610 under the AI approach, and £3,097 under the EEA approach.

4.2 Re-examining the exogeneity of devolved public expenditures

Part of Scottish students' expenditures are funded by grants provided by the Scottish Government. In the previous section we treated these expenditures as exogenous, as in Input-Output analysis government expenditures are typically treated as exogenous. In this subsection we revise this assumption and explore what implications it has for the impacts of students' consumption expenditures. This is important as the devolved Scottish Government effectively has a binding budget constraint, so that the Scottish Government's expenditure on HEI students displaces other public expenditure in Scotland. This is because the Scottish Government is financed through a block grant from the UK Government using the population-based Barnett formula (Christie and Swales, 2009)⁶; has no borrowing powers and only a limited ability to shift expenditure between accounting periods.

Given available information on the extent that students' consumption expenditures are contingent upon Scottish Government funding, adjusting the attribution of students' expenditures so that these are treated as endogenous is straightforward. Warhurst et al (2009, Table 2.4, p. 24) report that 'Education Related Grants and Bursaries' constitute £759 of

⁶ The Scottish Parliament does have the power to vary the standard rate of income tax by up to 3p in the pound. We abstract from this possibility here since all of the Scottish political parties are committed to not using this power. Lecca *et al.* (2010) give an analysis of the consequences of this tax-raising power being exercised by the Scottish Parliament.

students income. This amounts to 12.2% of Scottish students' average expenditures and is to a significant extent funded by the Scottish block grant⁷.

Table 4 reveals how this acknowledgement of the binding budget constraint of the Scottish Government impacts the exogeneity share of students' expenditures and the output impact of the average student. The degree of Scottish students' expenditures is significantly reduced, from 41% to 29%, or approximately by a third. Incoming students are not affected by this as they are not funded by the Scottish Government. As we can see from the right hand column of the table, the impact of the average Scottish student is reduced by £928, whereas the impact of the average student over the entire student population is reduced by £659 or just under 15%.

Table 4 Exogeneity shares (x_n) and impacts per student (q^s) under the EEA-approach given different assumptions about the exogeneity of Scottish Government expenditures.

Approach	Exogenous expenditures as a share of gross expenditures			Impact of average student £			
	x_{SCO}	x_{RUK}	x_{ROW}	\hat{q}_{SCO}^s	\hat{q}_{RUK}^s	\hat{q}_{ROW}^s	\hat{q}_{All}^s
EEA (ScotGov = Exogenous)	41%	73%	100%	3,097	6,403	8,779	4,417
EEA (ScotGov = Endogenous)	29%	73%	100%	2,169	6,403	8,779	3,759

5 Impact of students at individual institutions

It is clear that traditional simplifying assumptions systematically over- or underestimate the consumption impact of students at the aggregate level. However, HEIs are heterogeneous (Hermannsson et al, 2013b) and serve widely different student populations. Therefore, it is instructive to know to what extent the biases of the two traditional approaches impact individual institutions differently. The impact of the student population at an individual HEI will be driven by two factors: the scale of the student population and its composition. Going back to equation 6.b, the output impact driven by the consumption expenditures of students at institution i can be represented as:

$$\bar{q}_i^s = \sum_{ni} m_{ni}^s c_{ni}^s s_{ni} = m^c \sum_{ni} x_{ni} (1 - \delta) c_{ni}^s s_{ni} \quad (6.b)$$

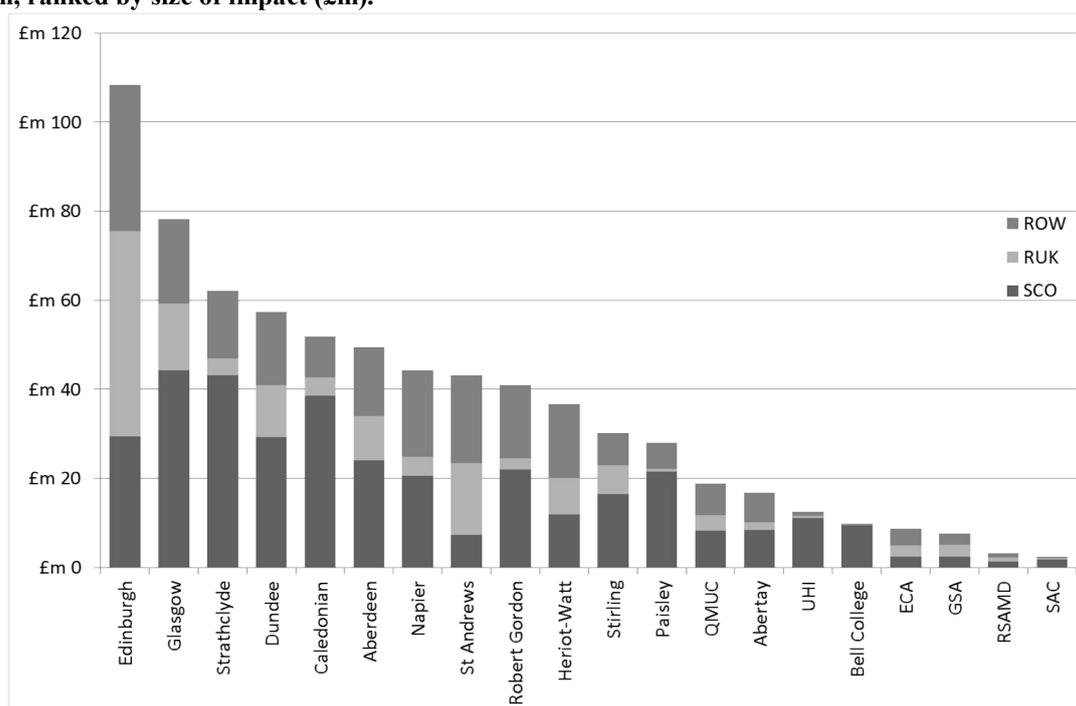
Table 5 below presents the Type-II output impact of students at different institutions. The first set of columns reveal the absolute size (as measured in FTEs) and composition (%) of the student population by origin, then we present estimates of students' exogenous consumption expenditures under the EEA approach and the next set of columns reveal the output impact of these exogenous injections. All these figures are broken down by student origin. Finally, we present the impact of the average student at each institution. A further graphic summary of the impacts of students' consumption expenditures at each institution is provided in Figure 1.

⁷ The category also includes support from private charities. Here the conservative stance is adopted that the charities are funded from Scottish contributions and therefore represent a re-distribution within the Scottish economy rather than an additional injection.

Table 5 Students' consumption impacts by institution

Formal name	Abbreviated name (used in the remainder of the paper)	Student numbers FTE				Exogenous student spending £m				Output impact of student spending, £m				Output impact as % of Scottish output
		SCO	RUK	ROW	Total	SCO	RUK	ROW	Total	SCO	RUK	ROW	Total	
The University of Aberdeen	Aberdeen	7,749	1,557	1,774	11,079	13.4	5.5	8.7	28	24.00	10.0	15.6	49.5	0.03%
University of Abertay Dundee	Abertay	2,704	278	749	3,731	4.7	1.0	3.7	9	8.37	1.8	6.6	16.7	0.01%
Bell College	Bell College	3,067	19	4	3,091	5.3	0.1	0.0	5	9.50	0.1	0.0	9.7	0.01%
The University of Dundee	Dundee	9,462	1,810	1,868	13,140	16.3	6.5	9.1	32	29.30	11.6	16.4	57.3	0.03%
Edinburgh College of Art	ECA	799	379	442	1,620	1.4	1.4	2.2	5	2.48	2.4	3.9	8.8	0.00%
The University of Edinburgh	Edinburgh	9,495	7,201	3,745	20,440	16.4	25.7	18.3	60	29.40	46.1	32.9	108.4	0.06%
Glasgow Caledonian University	Caledonian	12,466	629	1,054	14,149	21.5	2.2	5.2	29	38.60	4.0	9.3	51.9	0.03%
Glasgow School of Art	GSA	789	423	289	1,501	1.4	1.5	1.4	4	2.44	2.7	2.5	7.7	0.00%
The University of Glasgow	Glasgow	14,267	2,360	2,145	18,773	24.6	8.4	10.5	43	44.18	15.1	18.8	78.1	0.04%
Heriot-Watt University	Heriot-Watt	3,859	1,276	1,892	7,027	6.7	4.5	9.2	20	11.95	8.2	16.6	36.7	0.02%
Napier University	Napier	6,627	675	2,220	9,522	11.4	2.4	10.8	25	20.52	4.3	19.5	44.3	0.02%
The University of Paisley	Paisley	6,940	114	661	7,716	12.0	0.4	3.2	16	21.49	0.7	5.8	28.0	0.02%
Queen Margaret University College, Edinburgh	QMUC	2,648	549	817	4,013	4.6	2.0	4.0	11	8.20	3.5	7.2	18.9	0.01%
The Robert Gordon University	Robert Gordon	7,121	395	1,867	9,383	12.3	1.4	9.1	23	22.05	2.5	16.4	41.0	0.02%
The Royal Scottish Academy of Music and Drama	RSAMD	439	135	105	678	0.8	0.5	0.5	2	1.36	0.9	0.9	3.1	0.00%
The University of St Andrews	St Andrews	2,370	2,512	2,245	7,128	4.1	9.0	11.0	24	7.34	16.1	19.7	43.1	0.02%
Scottish Agricultural College	SAC	603	46	26	675	1.0	0.2	0.1	1	1.87	0.3	0.2	2.4	0.00%
The University of Stirling	Stirling	5,344	1,011	811	7,165	9.2	3.6	4.0	17	16.55	6.5	7.1	30.1	0.02%
The University of Strathclyde	Strathclyde	13,913	611	1,729	16,253	24.0	2.2	8.4	35	43.08	3.9	15.2	62.2	0.03%
University of Highlands and Islands	UHI	3,599	72	114	3,785	6.2	0.3	0.6	7	11.14	0.5	1.0	12.6	0.01%
		114,262	22,052	24,555	160,870	197	79	120	396	354	141.2	215.6	710.6	0.40%

Figure 1: The Type-II output impact of students' consumption expenditures by institution and student origin, ranked by size of impact (£m).



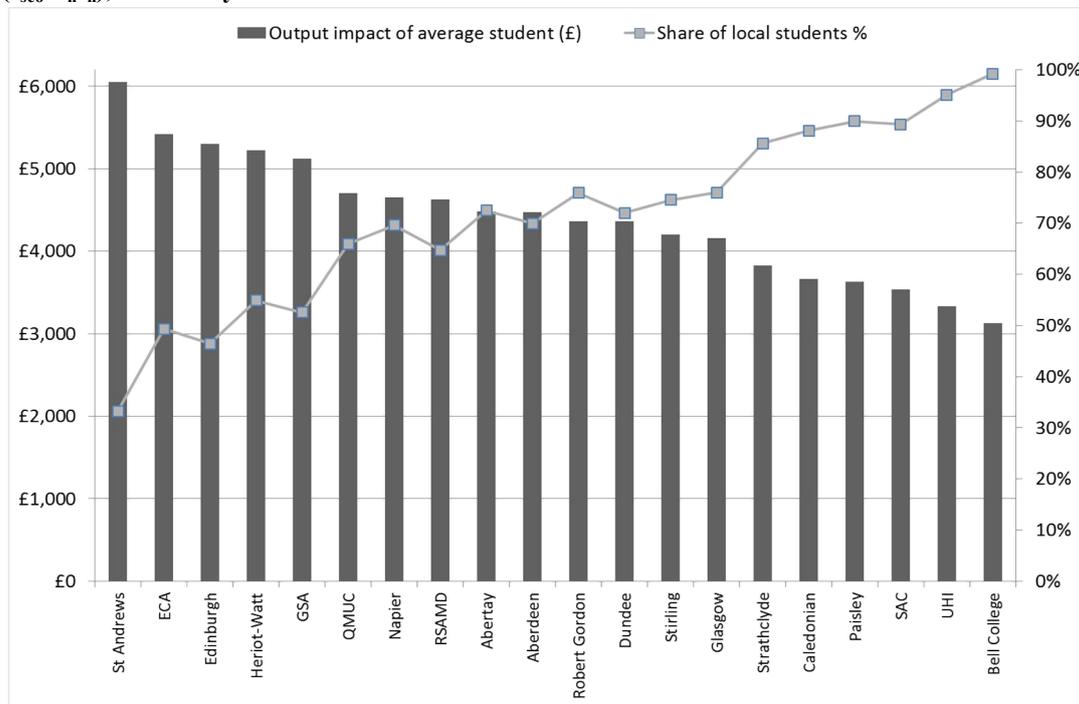
As is evident from Figure 1 Scottish HEIs vary significantly both in terms of the scale of the impact of their students' consumption expenditures and the extent to which this impact is being driven by local or incoming students. This again reflects the scale and composition of the respective student populations. The largest impact is driven by students at the University of Edinburgh (£108), this is 45 times larger than the smallest impact, that of the students at the Scottish Agricultural College, who support 2.4m of output in the Scottish economy. The composition of this impact also differs significantly. Edinburgh, St Andrews, Heriot-Watt, the Edinburgh College of Arts and the Glasgow School of Arts stand out when it comes to attracting external students (this is more readily visible in Figure 2). Furthermore, several HEIs have quite mixed student populations, such as Glasgow, Strathclyde, Dundee, Aberdeen, Napier and Stirling, while some are almost entirely attended by local students (UHI, Bell College, SAC).

These institutions are of course vastly different in terms of the size of their student populations with the largest student population (Edinburgh) being 30 times that of the smallest one (SAC). Therefore, in order to abstract from scale, it is useful to look at the impact of the average student at each of these institutions. This is revealed in the final column of Table 5, but to clarify the presentation this is also presented in a diagrammatical format, ranked by scale, in Figure 2 below, which also presents the share of local students in the student population.

As we can see from the diagram, the average student at St Andrews drives the largest impact at £6,052, almost double the impact of a representative student at Bell College £3,126. It is clear that the scale of the average impact is driven by the composition of the student population and is negatively associated with the share of local students in the student population. For the University of St Andrews only one third of all students are Scottish, whereas at Bell College almost all are. The

impact of the average student at Bell College is very close to that of the representative student of Scottish origin (\hat{q}_{SCO}^S) £3,097.

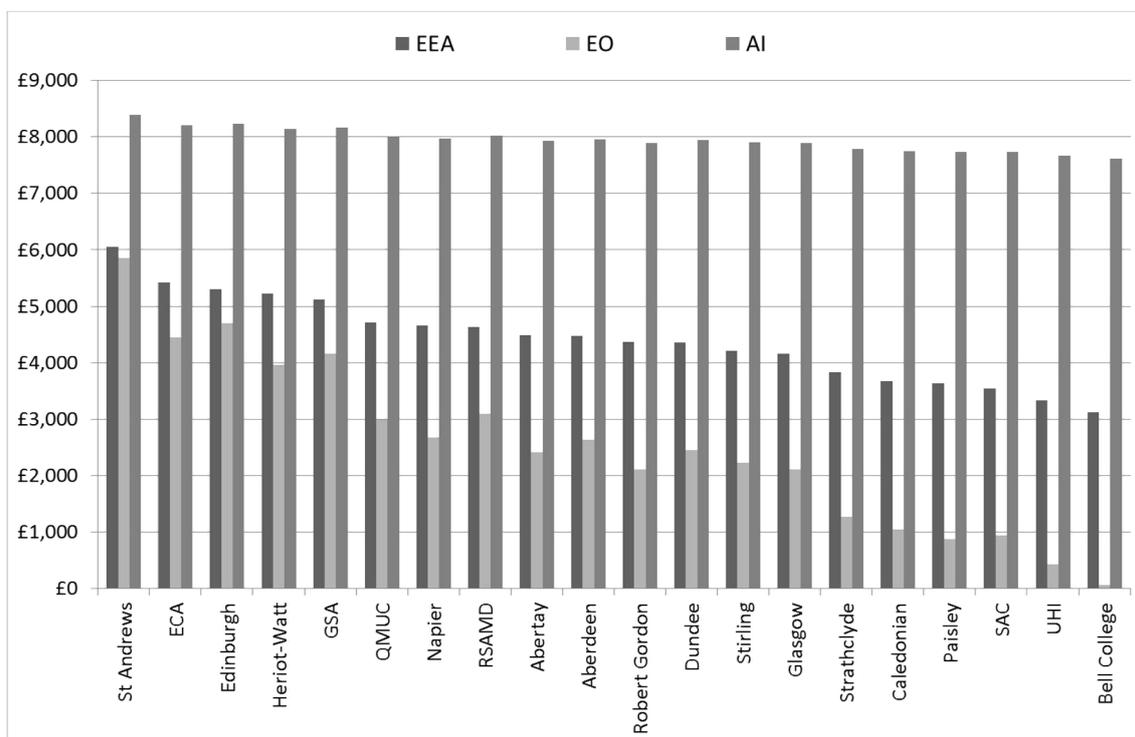
Figure 2 The output impact of the average student (\hat{q}_{All}^S) (£) and the share of local students (%) at each HEI ($s_{SCO}/\sum_n s_n$), ranked by size.



So far we have analysed the impacts of student populations attending individual institutions only in terms of the EEA approach. However, it is of interest to analyse the biases induced by the External Only (EO) and All Included (AI) assumptions, and how these affect individual institutions differently. Drawing on Table 3 on the impact of the average student of each type under each of the 3 assumptions, it is straightforward to multiply these with the shares of different student types at each HEI, presented in Table 5 above, in order to derive the expenditure impact of the average student at each institution under each of the three assumptions. The results are presented in Figure 3 below.

The AI assumption overstates impacts significantly vis-à-vis the EEA approach in all cases. It performs least bad in the case of the institutions with the lowest share of local students. The External Only assumption provides a reasonable approximation when analysing the impact of student populations where local students are a relatively small share. In the best case, that of St Andrews, this assumption only underestimates impacts of the average student by £192 or 3%. However, for institutions where the student population is mostly local this assumption is entirely inappropriate.

Figure 3 Output impact of the average student at HEI i under each of the three assumptions (EEA, EO and AI), ranked by the size of the impact estimated under the EEA approach (£).

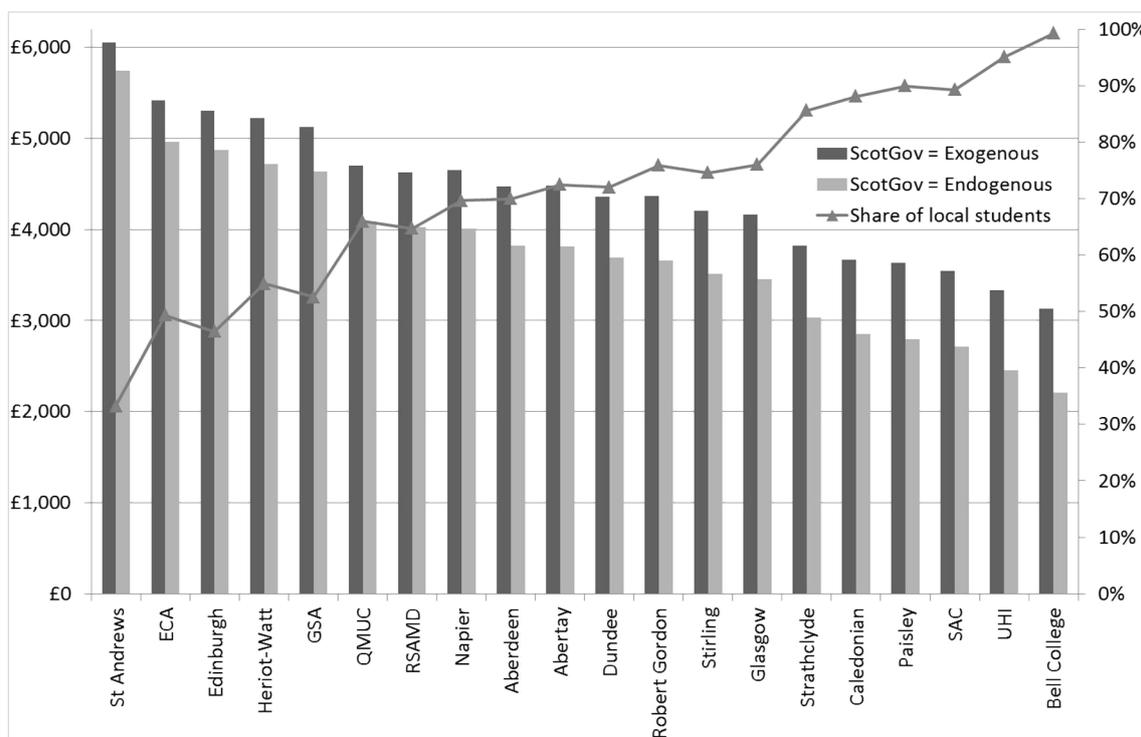


5.1 *Balanced expenditure impacts of students at individual HEIs*

Section 4.2 revealed that treating Scottish Government funding of students as endogenous reduces the impact attributed to Scottish students by 30% and the impact of the average student in Scotland by 15%. However, the composition of the student populations at different HEIs is heterogeneous and therefore, a priori, we expect this change to further increase the heterogeneity of the impacts of students at individual institutions. Figure 4 shows the average impact per student as estimated using EEA approach under the conventional assumption of treating public expenditures as exogenous (dark bars) and treating the expenditures of the Scottish Government as endogenous (light grey bars). Furthermore, we show how this is influenced by the composition of the student body as proxied by the share of Scottish students (right hand scale).

Those HEIs who already demonstrate the strongest per student consumption impacts are least sensitive to the acknowledgement of the budget constraint of the Scottish Government. The rationale is straightforward, the larger the share of incoming students the less is the impact of reducing the exogeneity of Scottish students' consumption expenditures from $X_{SCO} = 41\%$ to $X_{SCO} = 29\%$. Whereas, institutions that cater more for local students have student bodies that demonstrate a smaller consumption impact but this impact is more significantly affected from the change in X_{SCO} as their share of Scottish students is higher.

Figure 4 Comparison of output impacts under the EEA-approach treating Scottish Government expenditures as exogenous (dark bars) or as endogenous (light bars) and the share of local students (%) at each HEI, ranked by scale of impact.



6 Conclusions

This paper sets out a model to formally address a recurring problem in applied impact studies, that is how to distinguish between, and account for, endogenous and exogenous consumption expenditures. In standard IO analysis these are typically treated as either fully exogenous or wholly endogenous. Such as is manifested in the Type-I and Type-II assumptions. However, in many instances consumption expenditures are neither wholly endogenous nor exogenous. This is for example found in the cases of tourists, students and benefit claimants. To address this we set out a general model where the degree of exogeneity is treated as an empirical matter and demonstrate its application to the case of university students in Scotland. Students are heterogeneous in terms of their expenditures. Some are 100% exogenous (foreign students), but for local students much is endogenous. Previous attempts at analysing student impacts have applied simplifying assumptions to deal with this. We find these to be inaccurate, in particular with regard to the treatment of local students. Using conventional approaches their impacts are either overstated (100% exogenous) or understated (100% endogenous). However, we demonstrate the choosing between such limiting cases is not necessary given the availability of survey evidence. The biases involved are particularly distorting for HEIs that largely serve local students. Further influence on the exogeneity of the consumption expenditures is the public sector budget constraint, which is arguably binding for devolved government expenditures in the UK context. We find that acknowledging this budget constraint reduces the exogeneity (and hence the impact) of local students' consumption expenditures by about a third.

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Appendix: Theoretical treatment of students' consumption expenditures

We wish to identify the impact of student consumption using a modified Type II Input-Output (IO) approach. We take student consumption expenditure to be exogenous (as an investment in human capital). However, the funding of this expenditure potentially generates offsetting negative expenditures either taken as fully exogenous or endogenous. The demand for commodity i is given as the sum of intermediate and final demands:

$$x_i = \sum a_{i,j} x_j + \lambda_i h + \theta_i b_x + \sigma_i \bar{s} + \iota_i \bar{i} + \phi_i \bar{f} \quad (\text{A1})$$

where: x_i is the demand for the output of sector i , $a_{i,j}$ is the intermediate demand for sector i for a unit output of sector j ; h is the aggregate non-student household consumption expenditure; b_x is the total non-student expenditure of the Scottish Government; \bar{s} , \bar{i} and \bar{f} are the exogenous total investment, export and student consumption expenditures; and λ_i , θ_i , ϕ_i , ι_i , and σ_i are the corresponding expenditure coefficients.

In the IO demand driven model, wage income is determined as:

$$w = \sum a_{w,j} x_j \quad (\text{A2})$$

where w is total wage income, and $a_{w,j}$ is the wage input per unit of sector j . Non-student household income is determined as a linear function of total wage income, minus the share of student consumption financed by household transfers and student wage income, γ_h and γ_w respectively:

$$h = \alpha w - (\gamma_h + \gamma_w) \bar{s} \quad (\text{A3})$$

where α is calibrated on the initial IO table.

The Barnett non-student expenditure is determined by the exogenous total Barnett budget determined by Westminster, \bar{b} , minus the Scottish Government's expenditure on funding student consumption:

$$\bar{b} = b_x + \gamma_b \bar{s} \quad (\text{A4})$$

where γ_b is the share of student consumption expenditure financed by the Scottish Government.

Equations (1) to (4) can be expressed in an extended Input-Output manner as:

$$Mv + Jz = v \quad (\text{A5})$$

The elements of equation (5) are as follows. M is an $((n+2) \times (n+2))$ matrix of the form:

$$M = \begin{bmatrix} A & 0 & \lambda \\ a_w & 0 & 0 \\ 0 & \alpha & 0 \end{bmatrix} \quad (\text{A6})$$

with A an $(n \times n)$ matrix of technical coefficients, a_w is an $(1 \times n)$ vector of wage input coefficients, λ is an $(n \times 1)$ vector of household consumption coefficients, and α is the scalar relationship between total wage income and total household income, and wage income. v is an $((n+2) \times 1)$ vector of endogenous variables with the elements:

$$v = \begin{bmatrix} x \\ w \\ h \end{bmatrix} \quad (\text{A7})$$

where x is an $(n \times 1)$ vector of industry outputs, w and h are the total wage and non-student household income.

J is an $((n+2) \times 4)$ matrix that converts the aggregate exogenous final demand expenditures to exogenous shocks to direct sectoral demand or wage and household income. It takes the form:

$$J = \begin{bmatrix} \theta & \sigma - \gamma_b \theta & \iota & \phi \\ 0 & 0 & 0 & 0 \\ 0 & -\gamma_w - \gamma_h & 0 & 0 \end{bmatrix} \quad (\text{A8})$$

where θ , σ , ι and ϕ are $(n \times 1)$ vectors of government, student, investment and export coefficients and z is a (4×1) vector of aggregate exogenous final demands with element:

$$z = \begin{bmatrix} \bar{b} \\ \bar{s} \\ \bar{i} \\ \bar{f} \end{bmatrix} \quad (\text{A9})$$

Making the familiar rearrangement to equation (5) gives:

$$Jz = (1 - M)v \quad (\text{A10})$$

Pre-multiplying both sides of equation (10) by $(1-M)^{-1}$ produces:

$$(1 - M)^{-1} Jz = v \quad (\text{A11})$$

The $(1-M)^{-1}$ term is an extended version of the Type II Input-Output multiplier that can be partitioned as:

$$(1 - M)^{-1} = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix} \quad (\text{A12})$$

where the partitioned elements are interpreted as follows. The $(n \times n)$ $M_{1,1}$ matrix is the standard Type II inverse, so that the element $\mu_{i,j}^{1,1}$ is the increase in activity in sector i that results from a unit increase in final demand in sector j . Similarly the $(1 \times n)$ vectors $M_{2,1}$ and $M_{3,1}$ are the Type II wage and household multiplier values, so that i th element of these vectors are the increase in wage and household income, respectively, generated by a unit increase in final demand for the output of sector i . The $M_{1,2}$ and $M_{1,3}$ partitions are $(n \times 1)$ vectors. The i th elements of these vectors give the increase in activity in sector i that results from an exogenous increase in wage income or household income respectively. Finally the partitions $M_{2,2}$, $M_{3,2}$, $M_{2,3}$ and $M_{3,3}$ are all (1×1) scalars. $M_{2,2}$ and $M_{3,2}$ are the increases in wage and household income that result from a unit increase in exogenous

wage income and $M_{2,3}$ and $M_{3,3}$ are the increases in household income that result from an exogenous increase in household income.

The exogenous disturbance to the Type II Input Output model is an $((n+2) \times 1)$ vector generated as the product of the $((n+2) \times 4)$ J matrix and the (4×1) z vector. The z vector simply identifies the values for the four types of exogenous final demand. The J matrix converts the aggregate exogenous expenditures into disaggregated industry, wage and household demand shocks.

In a conventional Type II approach, all the columns of the J matrix would take a form similar to that in the government, investment and export columns (columns 1, 3 and 4) as expressed in equation (8). In these columns, the first n elements are expenditure coefficients, identifying the industrial composition of the aggregate expenditures. The entries for the final two elements are zero. That is to say, these expenditures include no direct exogenous wage or household expenditure injection⁸.

There are two aspects of the student's expenditure column in the J matrix that are distinctive. These all relate to the fact that student expenditure is not generally fully exogenous. In particular, the funding of student consumption expenditure typically implies a degree of expenditure switching, with reductions in expenditures elsewhere. First, there is an offsetting element to the direct industry expenditure coefficients. This reflects the fall in public sector expenditure that occurs as a result of the proportion, γ_b , of student consumption expenditure being funded by grants from the Scottish Government. Given the budget constraint imposed by the Barnett funding formula, this means a corresponding reduction in government expenditure.

A second set of offsetting expenditure reductions come through the changes in household income generated by exogenous student consumption. As this is a Type II model household consumption is treated as endogenous by definition. Further we adopt the conventional approach of linking household income to local wage payments (Miller and Blair, 2009) but treat student consumption expenditure separately from other household expenditure (equation (3)). Essentially we assume that (non-student) household consumption expenditure is endogenous, but that student consumption expenditure is exogenous. This means that there are two negative coefficients that enter as the last element in the expenditure column of matrix J. These identify the reduction in total household consumption expenditure as a result of a proportion, γ_h , of student consumption being financed as an intra-household transfer and a proportion, γ_w , from students' own wages. These are both treated as negative shocks to exogenous household income, with corresponding reductions in economic activity generated through the consumption multiplier⁹.

From equation (11), the multiplier is expressed as $(1-M)^{-1}J$. That is to say, if we read down the elements of one of the i th columns of this matrix, this will give the appropriate increase in industry,

⁸ Clearly in a wider SAM approach such transfer injections would in general occur. However, our treatment of student expenditure is rather different.

⁹ An alternate way of treating student income from wages would be to enter the term $-\gamma_w$ in the second (wage income) row in the J matrix. This would be the appropriate treatment in a full SAM model. We favour our present approach because of the looseness of the link between wage income and household income in the standard type II IO model.)

wage or household income that would be generated by a unit increase in the i th category of exogenous final demand, that is Scottish Government student, investment or export expenditure. As stated earlier, the $((n+2) \times (n+2))$ matrix $(I-M)^{-1}$ is a standard Type-II Leontief inverse with two additional rows and columns for wage and household income. Variation in the output multiplier values for any of the four exogenous aggregate expenditures is derived from $((n+2) \times 4)$ J matrix, and in particular the composition of the columns. For the government, investment and export expenditures, the differences relate only to the corresponding $(n \times 1)$ vectors of expenditure coefficients: θ , ι and ϕ . However, for the student consumption expenditure there are not only the direct coefficients, given by the $(n \times 1)$ vector σ , but also the offsetting government expenditures, $\gamma_b\theta$, and the negative shock to household income, γ_h and γ_w .

These exogenous expenditure elements are then multiplied by the appropriate multiplier values to generate the impact on the aggregate economy. For the elements of student consumption expenditure and the offset reduction in government expenditure on other services, these are the corresponding Type II sectoral output multipliers. For the offset reduction in household expenditure, this is the Type II consumption multiplier.

Expressed more formally, for a unit increase in student consumption expenditure, the change in the vector of endogenous variables, Δv^s , is given by:

$$\Delta v^s = (I - M)^{-1} \begin{pmatrix} \sigma - \gamma_b \theta \\ 0 \\ -\gamma_h - \gamma_w \end{pmatrix} \quad (A13)$$

and the conventional Type II output multiplier is the sum of the first n elements of the Δv vector.