Strathclyde
Discussion Papers
in Economics

Climate Policy with Gains from Scale: Is China the Optimal Policymaker?

David Comerford, Slawomir Dzido

No. 26 – 1

February 2026

Department of Economics
University of Strathclyde, Glasgow

Climate Policy with Gains from Scale: Is China the Optimal Policymaker?

David Comerford* Slawomir Dzido†

Abstract

We build a Climate-economy IAM incorporating a monopolistically competitive supply side (Krugman, 1980) resulting in a model featuring increasing returns to scale. Optimal policy is both a carbon tax to shift composition towards low carbon sectors, and an investment subsidy to boost the size of the capital stock. Our main theoretical result is that the optimal carbon tax is decreasing in the the degree of increasing returns to scale in the economy. We use this model to look at Chinese overcapacity in manufactures, explaining the combination of cheap finance enabled investments in the capacity to create vast scale in EV and solar panel manufactures.

Keywords: Climate policy; increasing returns to scale; integrated assessment models; carbon taxation; industrial policy; investment subsidies; green transition; abundance.

JEL Codes: Q54; Q58; L13; O44.

*Strathclyde Business School, Department of Economics

†Strathclyde Business School, Department of Economics

1 Introduction

The global challenge of climate change requires not only a reduction in carbon emissions but also a rapid transformation of industrial capacity towards low-carbon technologies. Traditionally, climate change economics (since e.g. Nordhaus, 1991) has emphasised that policy should price carbon emissions equal to the marginal damages they cause. This approach to climate policy, implemented as instruments such as carbon taxes and tradable emissions permits, aims to correct the climate change market failure by changing relative prices and shifting economic activity towards less polluting sectors. However, this carbon pricing approach to climate policy has made only limited inroads in to the real world of climate policy. Recent experience, such as the Biden era Inflation Reduction Act in the US, has emphasised instead subsidies. In China in particular, low carbon industries have benefited from massive subsidies and other state support. This suggests another dimension of policy may be equally important in driving the green transition: intentionally fostering large-scale productive capacity.

In the Chinese case, a combination of state-backed finance, infrastructure expansion, and industrial subsidies has enabled a vast manufacturing capacity. This includes machinery or electronic equipment, but also solar photovoltaics, electric vehicles, and related green products. This “overcapacity” or subsidised abundance is criticised for provoking “involution” - that is, a profusion of competitors which would be loss making absent the subsidies, and which may be a highly inefficient allocation of resources. However, these policies have nonetheless delivered global diffusion of green technologies. It may be the case that China’s approach to climate policy, even if only a byproduct of an overall industrial strategy, is more effective than that advocated for by traditional climate change economics?

China’s policy, however, only makes sense if large parts of modern industry are characterised by increasing returns to scale. Across sectors such as solar photovoltaics, batteries, and electric vehicles, Chinese policymakers have deliberately pushed investment far beyond immediate domestic demand, sustaining firms through years of thin or negative margins while expanding productive capacity at extraordinary speed. What outsiders often criticise as “overcapacity” or “dumping” can instead be read as a coordinated attempt to drive industries down their cost curves—an intentional creation of abundance through scale. By underwriting losses and providing cheap finance, China effectively socialises the fixed costs of expansion so that private firms can reach the scale at which average costs collapse and global competitiveness follows. The resulting pattern: huge investment, then cost dominance and market leadership; suggests that key sectors of the modern economy operate under increasing returns. Perhaps climate policy needs to be cognisant of this?

This paper develops a climate-economy framework to analyse this case. Building on Krugman (1980), which exhibits increasing returns to scale (IRS), we introduce (in a single-region setting) a model in which both carbon taxes and investment subsidies play a role in decentralising optimal policy. Carbon taxes direct the energy sector towards cleaner means of production, while subsidies to investment expand capital stock (and hence size of the overall economy) beyond what exists in laissez-faire.

We are not the first to consider increasing returns and subsidy policy in the context of climate change (see e.g. Koomey et al. (2023)), but we believe we are the first to specify an analytic (in the tradition of Golosov et al., 2014) Integrated Assessment Model that features IRS. We therefore contribute to a very limited literature which considers IRS in the setting of climate change policy (discussed further in Section 2). We find that, in the presence of increasing returns to scale, an investment subsidy supplements a carbon tax as optimal policy instruments. This model shows formally that a deliberate "overcapacity/abundance" policy may constitute optimal climate policy. In this way, we go beyond the discussion typically oriented on (only) green investment subsidies (e.g. Jin et al. (2024)).

Furthermore, this paper adds to the discussion criticising any trade-off between climate change mitigation and growth, Stern and Stiglitz (2023)), as it suggests boosting investment (and hence growth) at the same time as pricing emissions. We also contribute to debates over the role of industrial policy in the climate transition and the design of policies that can achieve both rapid decarbonisation and sustained economic growth. To the best of our knowledge, we present the first formal conceptualisation of the pro-abundance policy's role in the climate context.

The paper proceeds as follows: Section 2 describes relevant literature; Section 3 outlines the model; Section 4 presents our policy scenarios and results; and Section 5 then concludes the paper.

2 Background and literature

China's recent industrial trajectory combines concentrated policy intervention with exceptionally rapid diffusion and cost declines in many sectors. Central and provincial authorities have long deployed instruments such as cash subsidies, tax benefits, subsidised credit, and subsidised land (Rotunno and Ruta, 2024) to accelerate domestic capacity and boost trade surplus. Policy has been generous across many sectors, but there has been some focus towards sectors pivotal to the so called Green Transition (Organisation for

Economic Co-operation and Development, 2025+, Mathiesen et al., 2023) such as photovoltaics, batteries, and electric vehicles. This has resulted in China becoming the global leader in manufacturing in these sectors; and the rollout of manufacturing in these sectors has contributed to massive cost declines e.g. by more than 99% in the case of solar photovoltaic panel manufacture (Our World in Data (Global Change Data Lab), 2025).

The positive impact of China's industrial policy is likely via many interacting mechanisms. These include the commonly considered endogenous growth and learning-by-doing effects, with their associated cost declines (Goulder and Mathai, 2000, Grübler and Gritsevskyi, 1999, Acemoglu et al., 2012), or clustering effects (Krugman, 1991). In this paper, we microfound our approach using the role of scale in enabling high fixed cost firms to operate profitably, Krugman (1980), but following Arkolakis et al. (2012), the model we derive is isomorphic to any New Trade Theory model in the "gravity model" class (such as Melitz, 2003, Eaton and Kortum, 2002). We simply want to analyse policy in the case that there are significant returns to scale, without probing too deeply in to the source of these scale economies.

The existence of IRS in China is consistent with Ding et al. (2016) who confirm the existence of IRS in the majority of Chinese sectors or Anguo et al. (2011) who do so for all examined manufacturing sectors, and the story appears similar for energy-intensive and energy-producing sectors Yang (2019). This is also consistent with previous non-Chinese work (such as Caballero and Lyons, 1992, Basu and Fernald, 1997, Baxter and King, 1991, Syverson, 2011).

However, the magnitude of such aggregate returns to scale may be disputed. Because our framework embeds a trade model, the degree of scale economies in our policy scenarios is governed by the trade elasticity parameters standard in quantitative trade models. In particular, we follow Simonovska and Waugh (2014) in calibrating the elasticity of substitution across varieties, which implies a relatively strong love-of-variety channel and, under free entry, a high degree of effective scale economies at the aggregate level. When mapped into a reduced-form aggregate production relationship, this corresponds to an elasticity of output with respect to aggregate inputs that is substantially above unity. Such magnitudes are larger than those typically inferred in the macro production-function literature, where empirical estimates of aggregate returns to scale are generally close to constant once utilisation and mark-ups are accounted for (e.g. Basu and Fernald, 1997, Caballero and Lyons, 1992). However, two papers by the same authors provide support for this calibration: (1) Cuñat and Zymek (2021) show that incorporating trade-mediated scale effects (consistent with a low trade elasticity and hence a high degree of IRS) can substantially reduce the unexplained residual in cross-country income comparisons; and

(2) Cuñat and Zymek (2025) similarly suggest that scale economies in the traded goods sector may lead to strongly adverse outcomes for trade deficit countries, explaining support for trade restrictions from e.g. the Trump administration in the US. This work then suggests that strong effective scale economies are a plausible feature of the aggregate economy rather than simply a modelling artefact.

To the extent that gains from trade and increasing returns to scale have been considered in the context of climate change, these discussions have typically focused on the benefits that scale economies may provide in lowering costs for renewable energies as this sector expands (Nordhaus, 2002, Stern et al., 2022*a*). Somewhat similarly, Dhami and Zeppini (2025) define IRS as a process during which an increased adoption of a technology reduces costs or delivers higher benefits. Focusing on the role of uncertainty, they show that – due to fiscal constraints – Pigouvian pollution taxes and targeted green subsidies are not always straightforward complements in driving the transition to clean technology adoption. Their "guardrail approach", however, makes societal impacts of the policies difficult to estimate. Although absent direct climate considerations, Kunce (2023) examines the impact of rising IRS in a game-theoretical model where symmetric regions decide between capital subsidies and environmental protection to find a negative association with the latter. Catalano et al. (2020) assume production IRS in an OLG model and assess various climate adaptation strategies (where climate change impacts capital depreciation). The authors observe higher GDP growth when early preventive action is taken, but the influence of the increasing returns on optimality of the fiscal policies, however, appears unclear. Yang (2019) examines the additional complexities and potential costs incurred as fossil fuel-reliant, energy-intensive sectors which feature IRS, adjust when faced with climate policies. Conte et al. (2025) build an IAM featuring IRS to look at how climate policy affects the spatial distribution of economic activity, and show that a unilateral carbon tax may be optimal for the EU by stimulating economic activity in high-productivity sectors. Farrokhi and Lashkaripour (2025) combine a dynamic climate–emissions environment with a sequence of static, multi-country general-equilibrium trade models. Trade, production, and policy choices are determined period by period within a quantitative new-trade-theory framework, while emissions accumulate over time and generate future climate damages. However, unlike standard IAMs (and unlike our paper), the model does not feature intertemporal optimisation of capital or policy paths.

Overall the literature is consistent with the claim in Stern et al. (2022*b*), who argue that the existing wave of IAMs is not suited to dealing with issues like IRS. This leaves the integration of the gains from scale with the economics of climate change largely untouched. This is precisely the gap we cover in this paper.

3 Model

In this section, we introduce our model, which is an adaptation of the seminal Krugman (1980) trade model, combined with a stripped back version of the analytic IAM of van der Ploeg and Rezai (2021) (which is in the class of IAMs described by Golosov et al., 2014). The key characteristics of the baseline framework are product differentiation and associated "love of variety" by consumers, and monopolistic competition between symmetric firms, subject to fixed costs and free entry, which leads to the existence of aggregate IRS.

A general summary of our model is as follows. Household consumption preferences define demand for varieties of goods. Firms pay fixed costs and produce the goods using capital, labour, and energy. Energy is a generated via labour, devoted either to a carbon-intensive (but more productive) technology or a less productive but zero emission technology. The relative use of energy determines the level of emissions which in turn damage the total factor productivity. We have a single region (so there is no trade) but the basic logic of trade models with IRS holds: as the market size grows, overall productivity rises.

Below we describe the model in more detail (with full details available in the Appendix).

3.1 Households

Within period, price-taking households derive utility from a CES aggregate of differentiated varieties. For given consumption levels q_i,

$$Q = \left[\int_0^n q_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

where θ denotes the elasticity of substitution across varieties. With p_i being a nominal price of variety i, P represents the price index, defined so that total expenditure on differentiated goods satisfies

$$PQ = \int_0^n p_i q_i di \tag{2}$$

3.2 Firms

The model features infinitesimal, differentiated, monopolistically competitive firms that take demand for their product as given and choose prices to maximise profits. The market is characterised by free entry, and so they make zero profits.

Firms pay fixed costs, f, and produce quantity, q_i, using capital, K_i, direct labour L_{iF}, and indirect labour in fossil energy production, L_{if}, and renewable energy production, L_{ir}.

The fossil and renewable energy are imperfect substitutes with $-1 < \rho < 1$ describing the degree of substitutability. Production is therefore given by

$$q_i = \phi K_i^{\alpha} \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho}\right)^{\frac{\nu}{\rho}} \left(L_{iF}\right)^{1-\alpha-\nu} - f \tag{3}$$

where ϕ denotes firm-level total factor productivity. κ characterises the productivity advantage related to fossil sources. α and ν represent shares of capital and energy in production respectively.

Firms take consumer demand and input prices as given, and choose input quantities to maximise profits.

3.3 Static Equilibrium

All markets clear i.e. we have:

$$K = nK_i \tag{4}$$

$$L_f = nL_{if} \tag{5}$$

$$L_r = nL_{ir} \tag{6}$$

$$L = nL_{iF} + L_f + L_r \tag{7}$$

We normalise the price level $P = 1$. The result is that firms choose prices and quantities for their output of:

$$p_i = \frac{\theta}{\theta - 1} \frac{r}{\alpha W} \tag{8}$$

$$q_{if} = f(\theta - 1) \tag{9}$$

where the composite input good price, W, and the capital price, r, are given by:

$$W = \frac{nf\theta}{K} \tag{10}$$

$$r = \frac{\alpha Q}{K} \tag{11}$$

with the measure of firms, n, and real output, Q, given by:

$$n = p_i^{\theta-1} \tag{12}$$

$$Q = f(\theta - 1) \left(\frac{W}{f\theta}\right)^{\frac{\theta}{\theta-1}} K^{\frac{\theta}{\theta-1}} \tag{13}$$

$$= f(\theta - 1) \left(\frac{\phi}{f\theta}\right)^{\frac{\theta}{\theta-1}} \left(K^{\alpha} \left(\kappa L_f^{\rho} + L_r^{\rho}\right)^{\frac{\nu}{\rho}} \left(L - L_f - L_r\right)^{1-\alpha-\nu}\right)^{\frac{\theta}{\theta-1}} \tag{14}$$

Note that real output exhibits IRS since $\theta/(\theta-1)>1$.

3.4 Dynamics

The population of this economy, N, is constant, while productivity grows at exogenous rate ω, so that the effective labour size, $L_t = N\omega^t$. The representative household makes consumption-saving decisions by maximising lifetime utility

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} N \ln\left(\frac{C_{t+s}}{N}\right) \tag{15}$$

subject to their budget constraint:

$$K_{t+1} = p_t^K K_t + \max\left\{w_{Ft}, w_{ft}, w_{rt}\right\} N\omega^t + T_t - C_t \tag{16}$$

where β is a discount factor, $p_t^K K_t$ defines total capital income, $\max\left\{w_{Ft}, w_{ft}, w_{rt}\right\} N\omega^t$ refers to labour income, and T_t refers to lump sum tax/transfer income. There is full depreciation of capital.

Following the optimisation procedure described in Appendix A.3, we find that labour is optimally allocated to where it earns the maximum wage (which means that all three sectors: direct labour inputs, fossil energy production, and renewable energy production; must pay the same wage for a non-degenerate outcome); and that the optimal consumption/savings choice is governed by the Euler Equation:

$$C_{t+1} = \beta p_{t+1}^K C_t \tag{17}$$

In the absence of policy, we have $p_t^K = r_t = \alpha Q_t / K_t$, and it can easily be shown that this implies:

$$K_{t+1} = \alpha\beta Q_t \tag{18}$$

$$C_t = (1-\alpha\beta)\, Q_t \tag{19}$$

3.5 Social planner's solution

The social planner needs to account for the aggregate resource constraint, as well as all externalities. These are defined by the system of two dynamic equations:

$$K_{t+1} = Q_t - C_t \tag{20}$$

$$E_{t+1} = E_t + L_{ft} \tag{21}$$

where:

$$Q_t = Ae^{-\chi E_t}\left(K_t^{\alpha}\left(\kappa L_{ft}^{\rho} + L_{rt}^{\rho}\right)^{\frac{\nu}{\rho}}\left(N\omega^t - L_{ft} - L_{rt}\right)^{1-\alpha-\nu}\right)^{\frac{\theta}{\theta-1}} \tag{22}$$

Comparing with Equation (14), we see that:

$$A = f\left(\theta - 1\right)\left(f\theta\right)^{-\frac{\theta}{\theta-1}} \tag{23}$$

$$e^{-\chi E_t} = \phi_t^{\frac{\theta}{\theta-1}} \tag{24}$$

The social planner maximises social welfare as described in Appendix A.4. The result is that the marginal product of labour in the direct labour supply to firms, is set equal to the marginal product of labour in renewable energy production; however, consistently with Golosov et al. (2014), the marginal product of labour in fossil fuel production is set higher than in these other sectors by a wedge equal to:

$$\frac{\beta\chi}{1-\beta}Q_t \tag{25}$$

which reduces labour supply to the fossil sector relative to *laissez faire*. And finally, the social planner also chooses next period capital stock equal to:

$$K_{t+1} = \alpha\beta\frac{\theta}{\theta-1}Q_t \tag{26}$$

i.e. capital accumulation is optimally larger than in *laissez faire*, the stronger is the degree of IRS.

3.6 The Policymaker

The economy will clear such that wages in each sector are equal at the rate:

$$w_t = w_F\left(t\right) = \left(1-\alpha-\nu\right)\frac{Q_t}{L_{Ft}} \tag{27}$$

$$= w_r\left(t\right) = \frac{\nu L_{rt}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}}Q_t \tag{28}$$

$$= w_f\left(t\right) = \frac{\nu\kappa L_{ft}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}}Q_t \tag{29}$$

i.e. at what firms perceive to be their marginal product of labour (which is less than the actual aggregate marginal product of labour in this economy by a factor of $\theta/(\theta-1)$, i.e. the degree of IRS).

We then imagine a policymaker who can intervene in the economy by setting an investment subsidy, $\varepsilon_1 \geq 1$, so that:

$$p_t^K = \varepsilon_1 r_t = \varepsilon_1 \frac{\alpha Q_t}{K_t} \tag{30}$$

and by charging a carbon tax, $\tau_t \geq 0$, that must raise gross wages paid by firms in the fossil energy sector, $w_f\left(t\right) = w_t + \tau_t$, so that net wages after paying the carbon tax are equalised to those in other sectors. The value of this carbon tax is:

$$\tau_t = \frac{\beta\chi}{\varepsilon_2\left(1-\beta\right)} Q_t \tag{31}$$

The policymaker funds the investment subsidy and distributes the carbon tax revenues via a lump sum tax or transfer to households:

$$T_t = \tau_t L_{ft} - \alpha(\varepsilon_1 - 1) Q_t \tag{32}$$

If $\varepsilon_1 = \varepsilon_2 = \theta/(\theta - 1)$ then the policymaker implements the Social Planner's Solution. Note that the optimal investment subsidy is increasing in the degree of IRS, whereas the optimal carbon tax is decreasing in the degree of IRS.

4 Policy Scenarios

In this section, we describe the implications of this model using simulations.

4.1 Calibration

The model is calibrated as follows:

Of major importance is the elasticity of substitution over varieties, which determines the degree of IRS. We use a central value of $\theta = 4$, following Simonovska and Waugh (2014).

We follow van der Ploeg and Rezai (2021) in choosing a fairly standard $\alpha = 0.3$ and $\nu = 0.04$ for the income shares of capital and energy respectively. We arbitrarily choose the fixed costs paid by firms as $f = 1$. Model periods are a decade, and we assume that the discount rate is $2\% p.a.$, giving a discount factor, $\beta = 0.82$. The substitution between fossil and renewable energy parameter, $\rho = 0.5$, is chosen consistently with the literature[1]. And then the relative productivity advantage of fossil energy, κ, is chosen so

[1] Acemoglu et al. (2012) use 0.67 and 0.9, Golosov et al. (2014) use 0, Hart (2019) uses 0.75, van der Ploeg and Rezai (2021) use -0.058, and Campiglio et al. (2024) use 0.67. Empirical estimates range between -1 and 0.67 (Stern, 2012, Papageorgiou et al., 2017).

that the model with no climate policy will match the approximate 80:20 fossil:low carbon energy mix seen in the data (Our World in Data – Global Change Data Lab, 2026):

$$\kappa = \left(\frac{80}{20}\right)^{\frac{1-\rho}{2-\rho}}$$

We initially assume that the model is in a no growth, no policy, no climate damages steady state i.e. $K_1 = \alpha\beta Q_0 = K_0$, which implies a value for K_0 and a value for L_{f0}. We take this as describing the state of the world in 2020, at which time we are 1^oC above pre-industrial temperatures. Consistently with Ritchie (2023), we take our "Carbon Budget" for reaching 2^oC to be 2.5 times (i.e. 25 years' worth) this value for L_{f0}. This then allows us to calibrate the climate damages parameter χ: we assume productivity growth of $1\% p.a.$ i.e. $\omega = 1.105$, and set χ such that optimal policy exhausts the carbon budget, and so reaches 2^oC, by 2100. The model is then fully calibrated for its central case.

Given this calibration, we can report the resulting optimal carbon tax or Social Cost of Carbon. Assuming that 'in the real world' in 2020, the implemented carbon price is approximately zero, then the optimal carbon price would raise revenues as a share of GDP of:

$$\frac{\chi\beta(\theta-1)}{\theta(1-\beta)} L_{f0} = 3.78\%$$

Where the L_{f0} used in the above assumes no climate policy has been implemented. Then, given global GDP in 2020 of $\approx \$100tr$ and global CO_2 (equivalent) emissions of $\approx 50Gt$ (World Bank, 2026), we derive an optimal carbon price of $\$76/t$ - well in keeping with the literature (see e.g. Tol, 2023, 2024). Of course, in the optimal policy simulation, when this price is actually imposed, firms respond by using less fossil energy inputs and so L_{f0} falls, and carbon pricing revenues as a share of GDP fall to 1.19% of GDP. But we know that this is because the carbon price of $\$76/t$ has been imposed, and we now have our mapping from modelled quantities in to real world prices.

We have one interesting result with respect to calibrating the model: it can be shown, see Appendix B, that changing θ and then recalibrating the model to the same targets (in particular recalibrating χ to continue to match optimal policy to meeting the 2^oC target at 2100) leaves the carbon price unchanged, i.e. in the carbon price formula, Equation (31), the change in $\varepsilon_2 = \theta/(\theta-1)$ is exactly offset by the change in χ. So in this sense, the optimal carbon price is independent of the degree of IRS. This is not what we mean, however, by the dependence of the optimal carbon tax upon the degree of scale economies in the aggregate economy. We are simply using this calibration method to calibrate our damages parameter χ. Once χ is calibrated, the optimal carbon tax does indeed depend upon the degree of scale economies in the aggregate economy.

4.2 No issues of *second best*

Before proceeding to our main results, we present one preliminary result: the optimal carbon tax in the presence of a suboptimal investment subsidy is still the optimal carbon tax; and the optimal investment subsidy in the presence of a suboptimal carbon tax is still the optimal investment subsidy, i.e. there are no issues of second best (Lipsey and Lancaster, 1956). We "prove" this computationally. Let:

$$\varepsilon_1 = 1 + \left(\frac{\theta}{\theta - 1} - 1\right) m \tag{33}$$

$$\tau_t = n\frac{(\theta - 1)\beta\chi}{\theta(1 - \beta)} Q_t \tag{34}$$

So that the parameters m and n tell us the fraction of optimal policy imposed for the investment subsidy and the carbon tax respectively. Then we run the model, from the same starting point, for many different values of m and n and look at social welfare across these runs. As we see in Figure 1, when there are no carbon taxes ($n = 0$) the optimal investment subsidy level is still characterised by $m = 1$, and likewise in Figure 2, when there is no investment subsidy ($m = 0$) the optimal carbon tax is still characterised by $n = 1$.

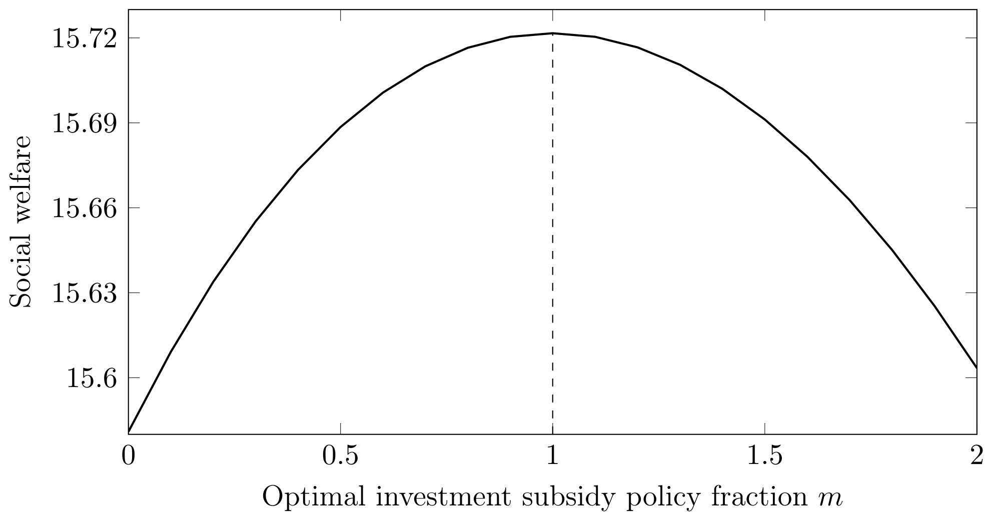


Figure 1: Social welfare as a function of the optimal investment subsidy policy fraction m, with the carbon tax policy fraction fixed at $n = 0$. Welfare is maximised at $m = 1$.

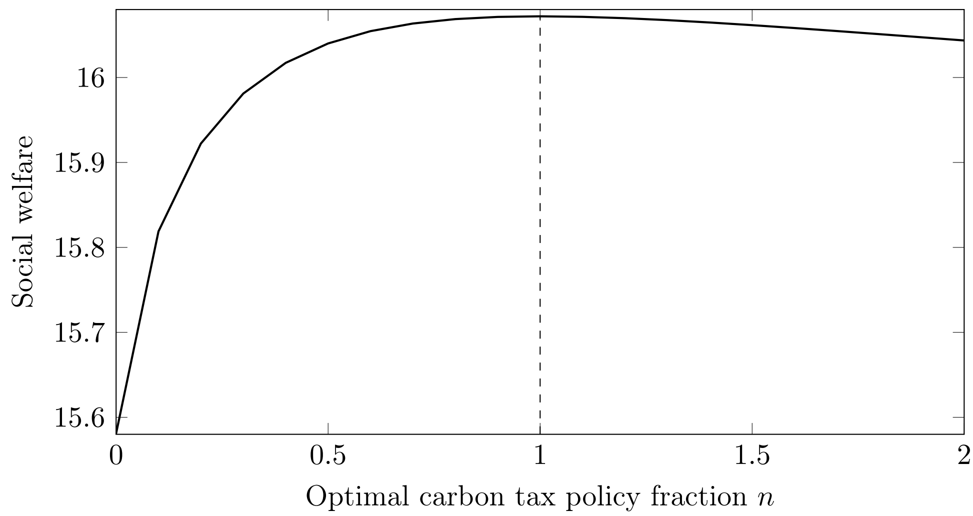


Figure 2: Social welfare as a function of the optimal carbon tax policy fraction n, with the investment subsidy policy fraction fixed at $m = 0$. Welfare is maximised at $n = 1$.

4.3 The dependence of Carbon Prices on the degree of IRS

Our main exercise is to compare the level of carbon taxes as a function of the degree of IRS, both in actuality and in policymaker beliefs. There are two ways of doing this.

Firstly, perhaps most reasonably, but almost trivially, we can compare the actions of three policymakers: one who behaves optimally (i.e. knows the correct degree of scale economies in the macroeconomy), another who believes $\theta = 8$ (i.e. believes that scale economies are weaker than they actually are), and the other who believes that the economy exhibits constant returns to scale. Clearly, the optimal policymaker implements both an investment subsidy and a carbon tax of \$76/t; the "Weak IRS policymaker" implements a smaller investment subsidy, and a larger carbon tax of \$89/t; and the "CRS policymaker" implements no investment subsidy, and a carbon tax of \$101/t. And, while the economy operating under the policies of the optimal policymaker reaches 2^oC above pre-industrial temperatures in 2100, the economy operating under the policies of the WIRS policymaker sees a temperature rise of 1.85^oC, and the that of the CRS policymaker sees a temperature rise of only 1.73^oC. The economy of the WIRS policymaker is 90% of the size of the optimal policymaker in 2100, while that of the CRS policymaker economy is only 83% of the size of the economy in 2100 under optimal policy.

Full details of the simulated results for the economy under each of these three policymakers are shown in Figures 3 and 4.

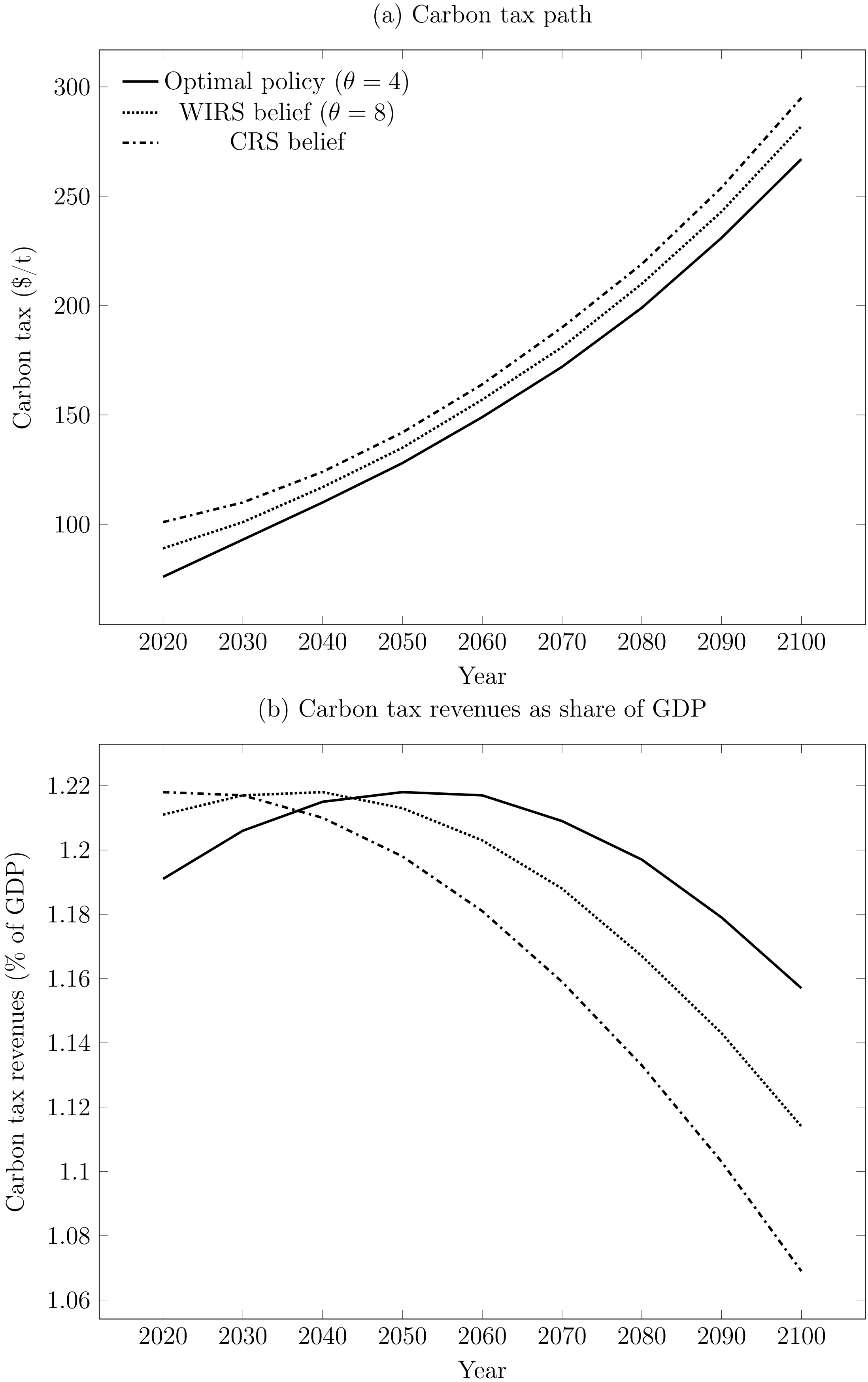


Figure 3: Carbon taxes under alternative beliefs about scale economies. The optimal policymaker correctly accounts for increasing returns to scale ($\theta = 4$), while alternative policymakers assume weaker increasing returns ($\theta = 8$), WIRS, or constant returns, CRS.

(a) Output

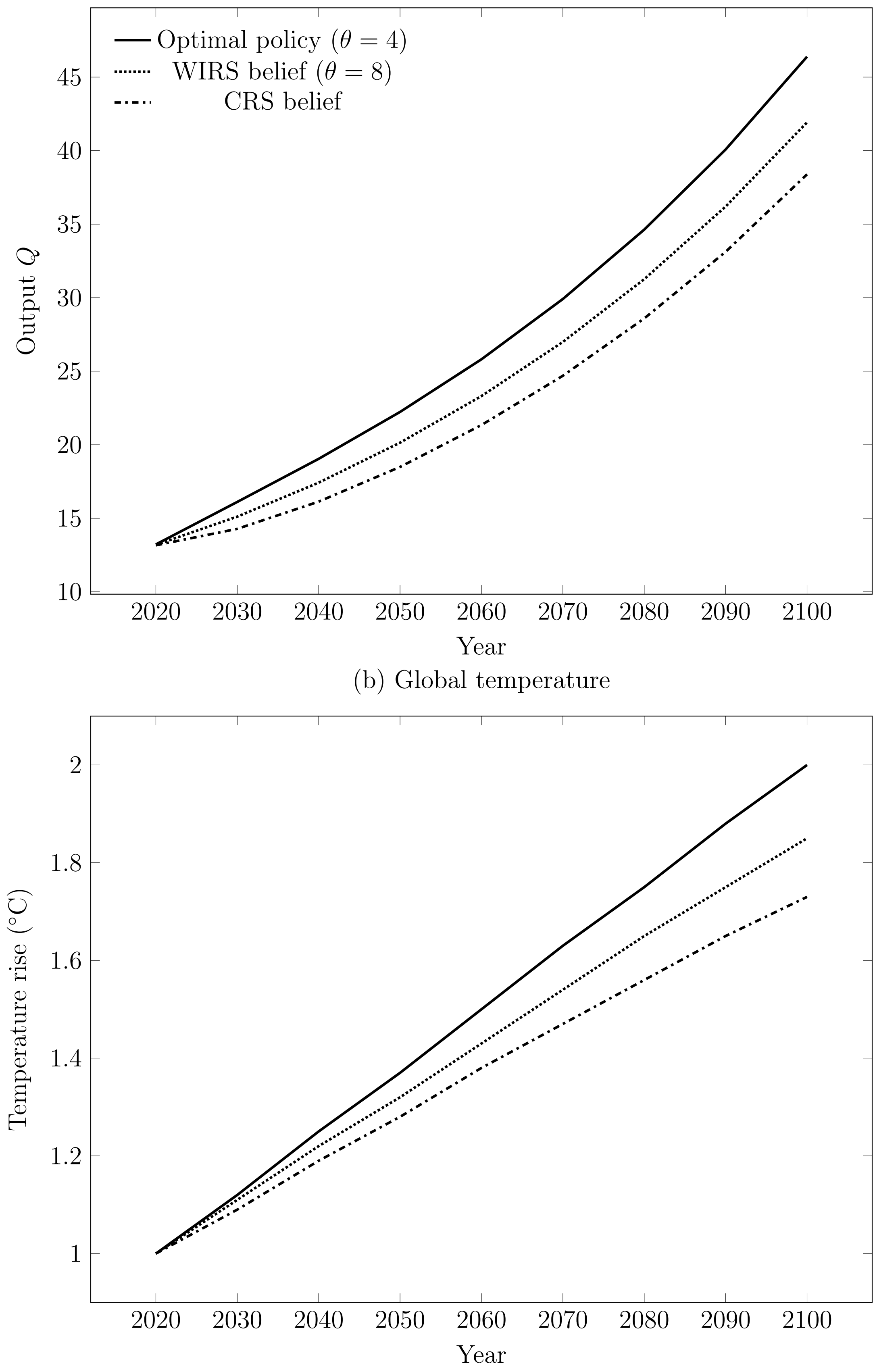


Figure 4: Macroeconomic and climate outcomes under alternative policy beliefs about scale economies. The optimal policymaker correctly accounts for increasing returns to scale ($\theta = 4$), while alternative policymakers assume weaker increasing returns ($\theta = 8$), WIRS, or constant returns, CRS. Note that values of Q are reported in model units.

This first exercise has obvious parallels with policy in, say, the EU relative to policy in China: carbon prices are higher in the EU than in China, and investment subsidies much lower. We can perhaps say, interpreted through the lens of this model, that China is behaving *more optimally* than the EU if it is the case that the macroeconomy exhibits IRS. Furthermore, the success of Chinese policy, with respect to its dominance of green technology manufacturing, and its achievement in plateauing emissions in a fast growing middle-income economy, is perhaps evidence for the existence of aggregate IRS[2].

However, alternatively, we can imagine the *economy experiencing a shock to IRS*. In the results that follow, we consider the economy under its baseline calibration (i.e. $\theta = 4$), which then experiences a time zero *shock* to $\theta = 8$, so the degree of IRS has decreased. There are dependencies in the model however, and so θ cannot be the only change. In our baseline calibration we arbitrarily chose the fixed costs parameter $f = 1$, and in the shock we adjust f in order to keep output Q_0 constant. Changes in θ and f then flow through to changes in A via Eqn (23). The initial capital stock, K_0, is the same in both the $\theta = 4$ and $\theta = 8$ runs.

The carbon taxes chosen by the policymaker (who is behaving entirely optimally) in the economy with weaker returns to scale, are the same as a percentage of GDP as those chosen by the policymaker who (sub-optimally) believes that returns to scale are low, but who actually lives in a high IRS world, i.e. the initial carbon tax chosen is \$89/t as above. Given returns to scale are actually lower, output grows more slowly in the *shocked* economy, as expected.

Full details of the simulated results for each of these economies, plus the WIRS policymaker economy from above for comparison, are shown in Figures 5 and 6.

[2]Note however that our model, combined with the typical trade elasticity parameter suggested by Simonovska and Waugh (2014), would suggest a "CRS policymaker" chooses a carbon price equal to 4/3 that chosen by an optimal policymaker. Recent real world experience suggests a European carbon price ten times that in China (Bruegel – European Think Tank, 2025). Having the EU play the role of the CRS policymaker would therefore suggest that even if China is implementing optimal investment subsidy policy, it is implementing suboptimally low climate policy. Alternatively, it could suggest freakishly large scale economies, equivalent to $\theta \approx 1.1$.

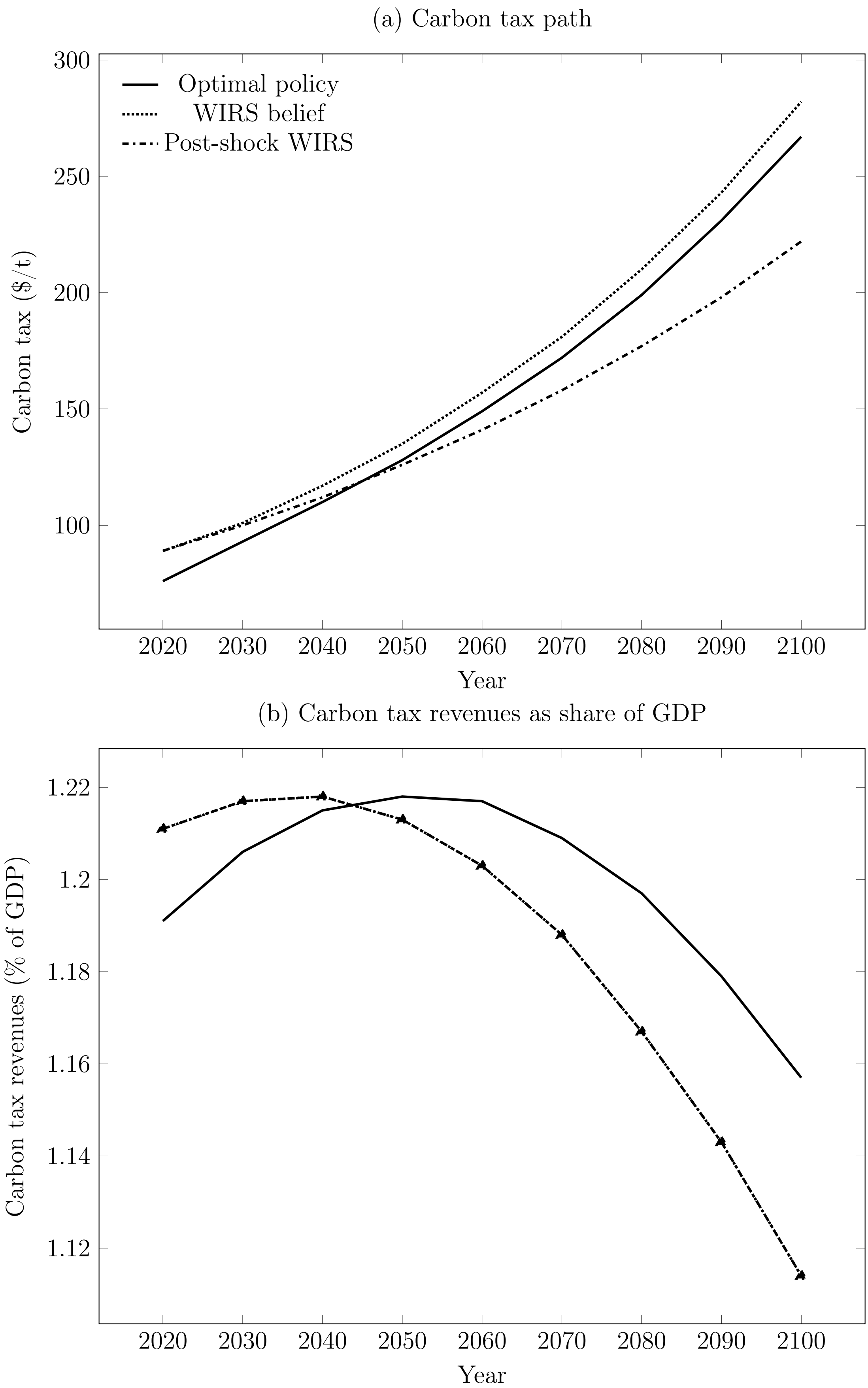


Figure 5: Carbon taxes following a negative shock ($\theta = 4$ changes to $\theta = 8$) to increasing returns to scale juxtaposed with two no-shock counterfactuals: optimal policy ($\theta = 4$) and WIRS belief ($\theta = 8$). Note that post-shock WIRS and WIRS belief exhibit identical paths of carbon tax revenues as share of GDP and are distinguished by markers.

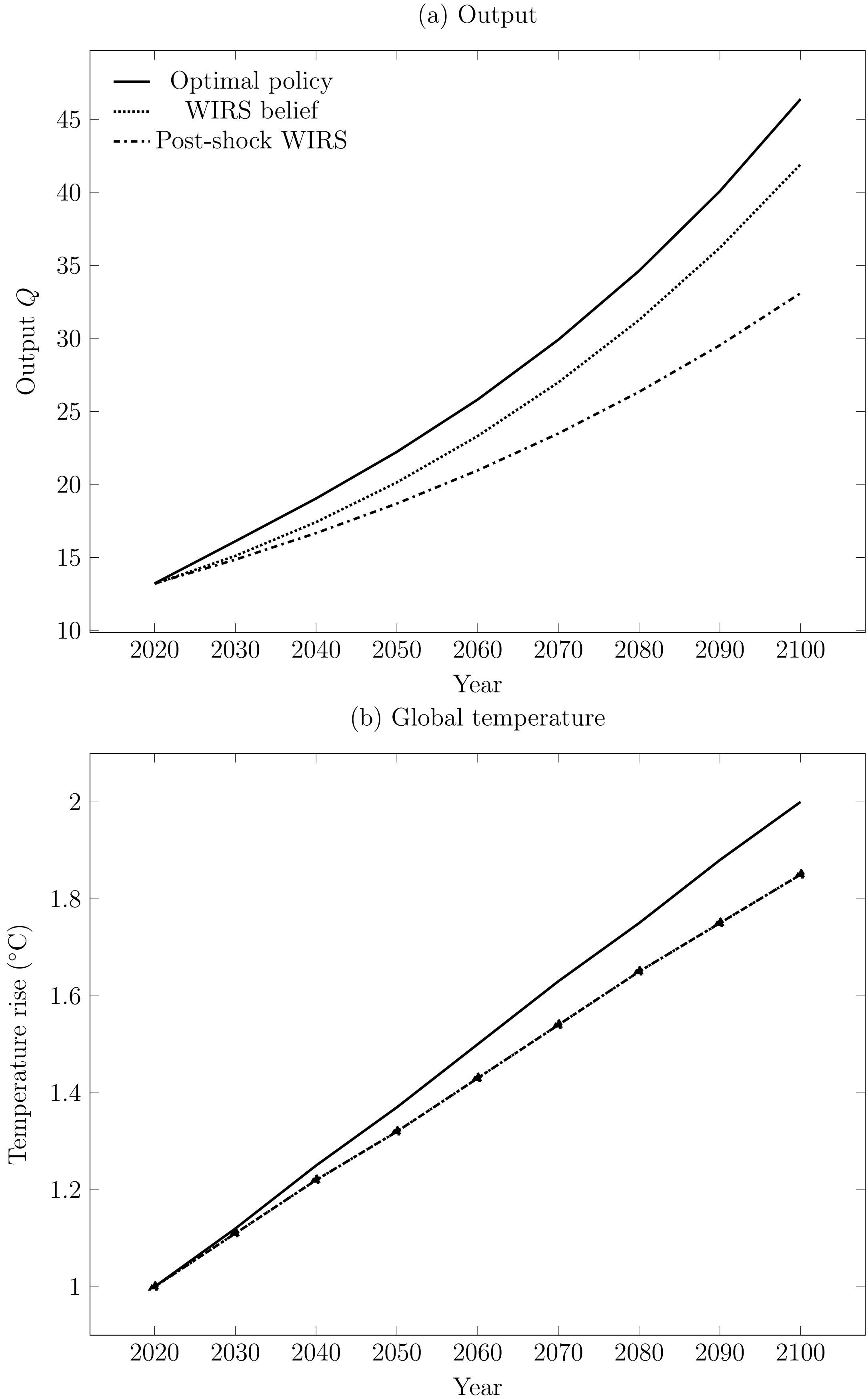


Figure 6: Macroeconomic and climate dynamics following a negative shock ($\theta = 4$ changes to $\theta = 8$) to increasing returns to scale juxtaposed with two no-shock counterfactuals: optimal policy ($\theta = 4$) and WIRS belief ($\theta = 8$). Note that WIRS belief counterfactual and post-shock WIRS exhibit identical temperature rise and are distinguished by markers.

Again, we see that in this second exercise, lower actual levels of IRS in the macroeconomy are associated with higher carbon prices, and again we can perhaps drawn a parallel with policy in the EU relative to policy in China: China has a much more manufacturing intensive economy, and manufacturing is likely where scale economies matter much more than for a more services based economy like that of the EU. Carbon prices are higher in the EU than in China, and investment subsidies much lower. Interpreted through the lens of this model, we could perhaps say that both regions are behaving optimally with respect to their particular industrial structures.

4.4 The Relative Importance of the Policy Mix

Is the carbon tax or the investment subsidy the more *important* component of optimal policy? To answer this question, we return to the central calibrated model of optimal policy.

Firstly, we consider the sign of T_t: if this is negative (i.e. households pay lump sum taxes) then the cost of financing the optimal investment subsidy exceeds the revenues from the optimal carbon tax; and vice versa if this is positive (i.e. households receive lump sum transfers). We find (Table 1) that this is clearly negative: the carbon tax revenues are not enough to finance the required investment subsidies; and this balance decreases (becomes more negative) over time.

Table 1: Carbon tax revenues, investment subsidy charges, and transfer balance

	2020	2030	2040	2050	2060	2070	2080	2090	2100
Carbon tax revenues	0.16	0.19	0.23	0.27	0.31	0.36	0.41	0.47	0.54
Inv. subsidy charge	-1.32	-1.61	-1.90	-2.22	-2.58	-2.99	-3.46	-4.01	-4.64
Transfer balance	-1.16	-1.42	-1.67	-1.95	-2.27	-2.63	-3.05	-3.54	-4.10

Notes: Values report model-implied fiscal outcomes along the transition path from 2020 to 2100. Carbon tax revenues and investment subsidy charges are reported in model units. The transfer balance is defined as carbon tax revenues net of investment subsidy charges.

On this measure, the optimal investment subsidy is certainly the more *fiscally* important.

In welfare terms however, the carbon tax appears to be the more important policy instrument. We compare social welfare (the household lifetime utility function evaluated using actual consumption path along the projection out to 2420) in the case of optimal policy (OP), optimal carbon tax only (CT), optimal investment subsidy only (IS), and laissez faire (LF). The results are in Table 2, and while these social welfare figures have no meaning in a cardinal sense, their ordering allows us to rank outcomes.

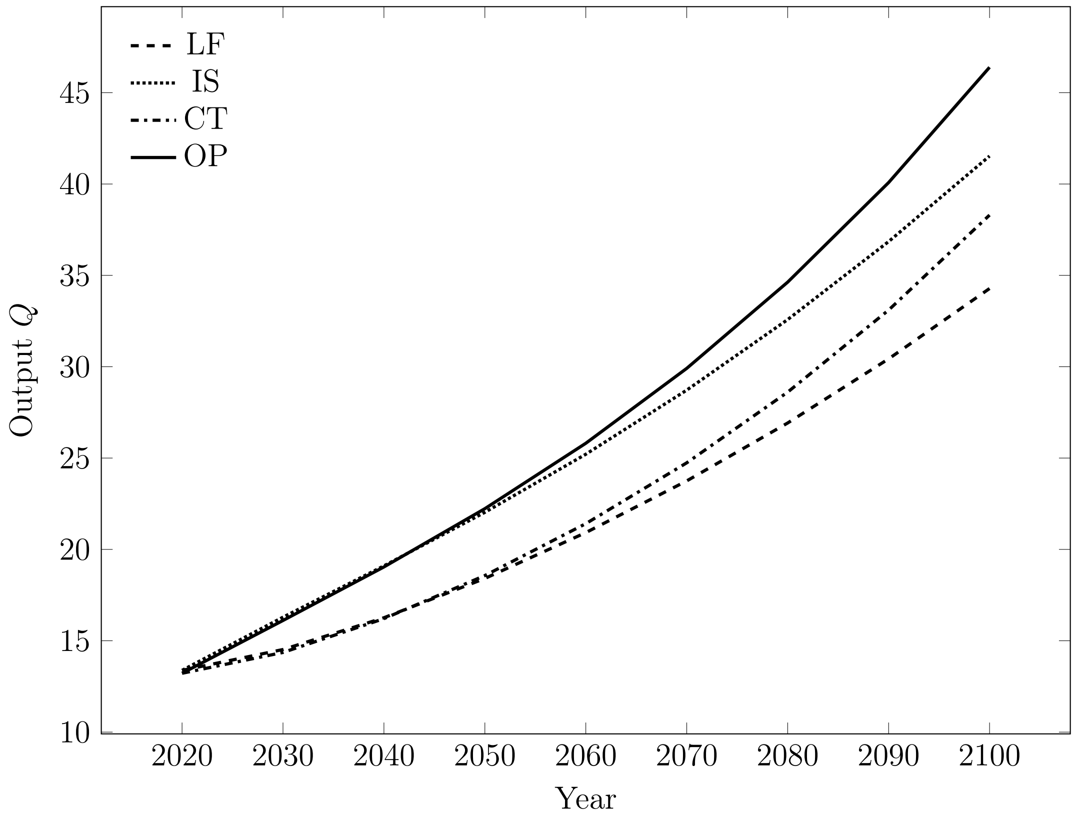
Table 2: Social welfare under alternative policy scenarios

Scenario	Social Welfare
LF	15.58
IS	15.72
CT	16.07
OP	16.21

Notes: LF denotes the laissez-faire scenario, IS denotes the optimal investment subsidy policy only, CT denotes the optimal carbon tax policy only, and OP denotes full optimal policy. Social welfare is a sum of discounted log-transformed consumption flows up to 2420 (i.e. we simulate the model by 400 years which brings as close enough to the infinite horizon considered in the utility function).

We see from this ordering that carbon taxes are more welfare enhancing than are investment subsidies. The reason for this can then be seen in Figure 7: eventually climate damages dominate the growth effects of enhanced capital accumulation.

(a) Output



(b) Global temperature

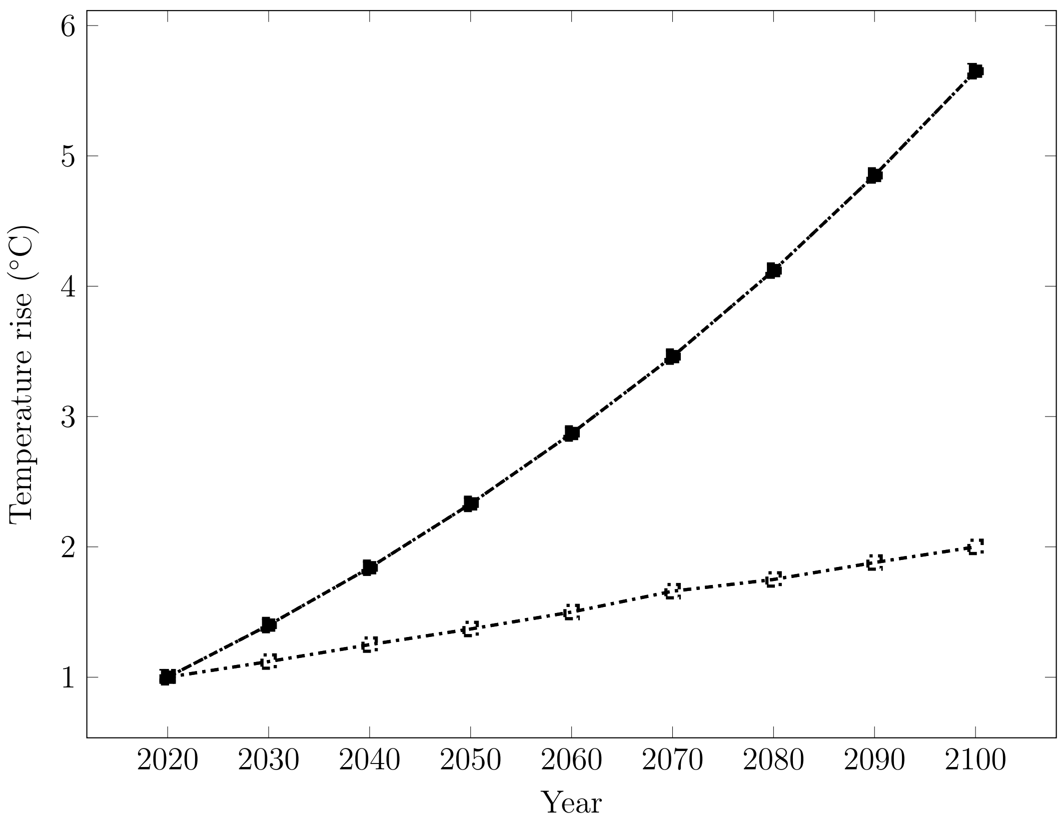


Figure 7: Output and temperature outcomes under alternative policy scenarios. LF denotes laissez-faire, IS investment subsidies only, CT carbon taxation only, and OP the full optimal policy. LF and IS temperature paths coincide exactly and are distinguished using markers; the same applies for CT and OP.

5 Conclusion

In this paper we present an analytic integrated assessment climate-economy model that features aggregate increasing returns to scale. Optimal policy then, clearly, also involves an investment subsidy that is increasing in the strength of increasing returns, but our main result is that the optimal social cost of carbon is now decreasing in the strength of increasing returns.

Our model therefore shows formally that a deliberate "overcapacity/abundance" policy may be part of optimal climate policy. We use this model as a lens to consider the case of China. China is now the pre-eminent global manufacturer, and even more so when we consider those sectors necessary for the Climate Transition. China has achieved this in part via a deliberate programme of state-backed finance and industrial subsidies. China is therefore a poster child for the policy prescriptions suggested by our model.

Looking in more detail at China as an exemplar of our model's prescription of optimal policy, against, say, the European Union, as an exemplar of an entity following the prescriptions of models without increasing returns to scale, we see that China both implements investment subsidies and has a lower carbon price than the EU - exactly as our model would suggest. We can therefore view China and the EU through the lens of the model and state that either China appears to be behaving *more optimally*, or that both are optimal policymakers with respect to their own specific economies - the higher manufacturing intensity of the Chinese economy implying a higher degree of scale economies. Alternatively, we can look at the real world success of China in developing an overwhelmingly dominant position in green technology manufacturing, while plateauing its emissions, as evidence for the validity of this model and the existence of aggregate increasing returns to scale.

Finally, we can use the model to evaluate the relative importance of these two policy instruments under the assumption that the model is valid: the investment subsidy is by far the most fiscally relevant instrument, with costs far exceeding the revenues brought in by carbon pricing. But, in welfare terms, carbon pricing remains the more important instrument: without carbon pricing shifting the economy towards a low carbon path, the negative impacts of climate damages on GDP dominate even the positive impacts of subsidised capital accumulation.

References

Acemoglu, D., Aghion, P., Bursztyn, L. and Hemous, D. (2012), 'The environment and directed technical change', *American Economic Review* **102**(1), 131–166.

Anguo, L., Ge, G. and Kaizhong, Y. (2011), 'Returns to scale in the production of selected manufacturing sectors in china', *Energy Procedia* **5**, 604–612.
URL: *https://doi.org/10.1016/j.egypro.2011.03.106*

Arkolakis, C., Costinot, A. and Rodríguez-Clare, A. (2012), 'New trade models, same old gains?', *American Economic Review* **102**(1), 94–130.

Basu, S. and Fernald, J. G. (1997), 'Returns to scale in U.S. production: Estimates and implications', *Journal of Political Economy* **105**(2), 249–283.

Baxter, M. and King, R. G. (1991), 'Productive externalities and business cycles', *Journal of Monetary Economics* **27**(1), 1–31.

Bruegel – European Think Tank (2025), 'Different routes to decarbonisation: EU carbon pricing versus china's green tech', `https://www.bruegel.org/newsletter/different-routes-decarbonisation-eu-carbon-pricing-versus-chinas-green-tech`. Bruegel newsletter; Europe's and China's decarbonisation strategies compared, with EU carbon prices around €78 per tonne and China's focus on green technology.
URL: *https://www.bruegel.org/newsletter/different-routes-decarbonisation-eu-carbon-pricing-versus-chinas-green-tech*

Caballero, R. J. and Lyons, R. K. (1992), 'External effects in U.S. procyclical productivity', *Journal of Monetary Economics* **29**(2), 209–225.

Campiglio, E., Spiganti, A. and Wiskich, A. (2024), 'Clean innovation, heterogeneous financing costs, and the optimal climate policy mix', *Journal of Environmental Economics and Management* **128**, 103071.
URL: *https://doi.org/10.1016/j.jeem.2024.103071*

Catalano, M., Forni, L. and Pezzolla, E. (2020), 'Climate-change adaptation: The role of fiscal policy', *Resource and Energy Economics* **59**, 100919.
URL: *https://doi.org/10.1016/j.reseneeco.2019.07.005*

Conte, B., Desmet, K. and Rossi-Hansberg, E. (2025), 'On the geographic implications of carbon taxes', *The Economic Journal* .
URL: *https://academic.oup.com/ej/advance-article/doi/10.1093/ej/ueaf027/8113994*

Cuñat, A. and Zymek, R. (2021), 'International value-added linkages in development accounting', *Journal of International Economics* **131**, 103475.

Cuñat, A. and Zymek, R. (2025), Unbalanced trade 2.0, Technical Report 2025/191, International Monetary Fund.
URL: *https://www.imf.org/en/publications/wp/issues/2025/09/26/unbalanced-trade-2-570525*

Dhami, S. and Zeppini, P. (2025), 'Green technology adoption under uncertainty, increasing returns, and complex adaptive dynamics', *Journal of Economic Behavior Organization* **233**, 106953.
URL: *https://cnrs.hal.science/hal-04998831v1/file/Dhami$_Z$eppini$_2$025.pdf*

Ding, S., Guariglia, A. and Harris, R. (2016), 'The determinants of productivity in chinese large and medium-sized industrial firms, 1998-2007', *Journal of Productivity Analysis* **45**(2), 131–155.
URL: *https://link.springer.com/article/10.1007/s11123-015-0460-0*

Eaton, J. and Kortum, S. (2002), 'Technology, geography, and trade', *Econometrica* **70**(5), 1741–1779.

Farrokhi, F. and Lashkaripour, A. (2025), 'Can trade policy mitigate climate change?', *Econometrica* **93**(5), 1561–1599.
URL: *https://doi.org/10.3982/ECTA20153*

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A. (2014), 'Optimal taxes on fossil fuel in general equilibrium', *Econometrica* **82**(1), 41–88.

Goulder, L. H. and Mathai, K. (2000), 'Optimal CO_2 abatement in the presence of induced technological change', *Journal of Environmental Economics and Management* **39**(1), 1–38.

Grübler, A. and Gritsevskyi, A. (1999), 'A model of endogenous technological change through learning-by-doing', *Energy Policy* **27**(10), 545–554.

Hart, R. (2019), 'To everything there is a season: Carbon pricing, research subsidies, and the transition to fossil-free energy', *Journal of the Association of Environmental and Resource Economists* **6**(2), 349–389.
URL: *https://doi.org/10.1086/701805*

Jin, W., van der Ploeg, F. and Zhang, L. (2024), 'How clean capital slows down disinvestment of carbon-intensive capital in the low-carbon transition', *Journal of Economic Dynamics Control* **162**, 104852.
URL: *https://www.sciencedirect.com/science/article/pii/S0165188924000496*

Koomey, J., Hausker, K., Schmidt, Z. and Lashof, D. (2023), 'Abandon the idea of an "optimal economic path" for climate policy', *WIREs Climate Change* **14**(6), e850.
URL: *https://doi.org/10.1002/wcc.8508*

Krugman, P. (1980), 'Scale economies, product differentiation, and the pattern of trade', *American Economic Review* **70**(5), 950–959.
URL: *https://www.jstor.org/stable/1805774*

Krugman, P. (1991), 'Increasing returns and economic geography', *Journal of Political Economy* **99**(3), 483–499.

Kunce, M. (2023), 'Decentralized environmental regulation in a model with increasing-returns-to-scale', *Theoretical Mathematics Applications* **13**(2), 1–15.
URL: *https://doi.org/10.47260/tma/1321*

Lipsey, R. G. and Lancaster, K. (1956), 'The general theory of second best', *Review of Economic Studies* **24**(1), 11–32.
URL: *https://www.jstor.org/stable/2296233*

Mathiesen, B. V. et al. (2023), 'The green transition of industry: Toward integrated strategies for energy efficiency, electrification, and renewable energy', *Journal Title (from ScienceDirect)* . Available via ScienceDirect.

Melitz, M. J. (2003), 'The impact of trade on intra-industry reallocations and aggregate industry productivity', *Econometrica* **71**(6), 1695–1725.

Nordhaus, W. D. (1991), 'To slow or not to slow: The economics of the greenhouse effect', *The Economic Journal* **101**(407), 920–937.
URL: *https://www.jstor.org/stable/2233864*

Nordhaus, W. D. (2002), 'The question of climate change: The role of *r&dice* and induced technological change', *Proceedings of the National Academy of Sciences* **99**(suppl. 1), 13980–13986.

Organisation for Economic Co-operation and Development (2025+), *Environment at a Glance Indicators*, OECD Publishing.
URL: *https://www.oecd.org/en/publications/environment-at-a-glance-indicators*$_a c4b8b89 - en/$

Our World in Data (Global Change Data Lab) (2025), 'Solar photovoltaic module price [dataset]', `https://ourworldindata.org/grapher/solar-pv-prices`. Data adapted from IRENA (2025), Nemet (2009), and Farmer & Lafond (2016); retrieved January 14, 2026.
URL: *https://ourworldindata.org/grapher/solar-pv-prices*

Our World in Data – Global Change Data Lab (2026), 'Share of primary energy consumption from low-carbon sources [dataset]', `https://ourworldindata.org/grapher/low-carbon-share-energy`. Data adapted from Energy Institute – Statistical Review of World Energy (2025), retrieved January 15, 2026; low-carbon energy is defined as the sum of nuclear and renewable sources using the substitution method.
URL: *https://ourworldindata.org/grapher/low-carbon-share-energy*

Papageorgiou, C., Saam, M. and Schulte, P. (2017), 'Substitution between clean and dirty energy inputs: A macroeconomic perspective', *Review of Economics and Statistics* **99**(2), 281–290.
URL: *https://doi.org/10.1162/REST*$_{a0}$0613

Ritchie, H. (2023), 'How much co_2 can the world emit while keeping warming below 1.5 °c and 2 °c?', *Our World in Data* . `https://ourworldindata.org/how-much-co2-can-the-world-emit-while-keeping-warming-below-15c-and-2c`.
URL: *https://ourworldindata.org/how-much-co2-can-the-world-emit-while-keeping-warming-below-15c-and-2c*

Rotunno, L. and Ruta, M. (2024), Trade implications of china's subsidies, Working paper no. 2024/180, International Monetary Fund. Prepared August 23 2024; 43 pages.
URL: *https://www.imf.org/en/Publications/WP/Issues/2024/08/15/Trade-Implications-of-China-s-Subsidies-552506*

Simonovska, I. and Waugh, M. E. (2014), 'The elasticity of trade: Estimates and evidence', *American Economic Review* **104**(5), 258–262.

Stern, D. I. (2012), 'Interfuel substitution: A meta-analysis', *Journal of Economic Surveys* **26**(2), 307–331.
URL: *https://doi.org/10.1111/j.1467-6419.2011.00683.x*

Stern, N. and Stiglitz, J. E. (2023), 'Climate change and growth', *Industrial and Corporate Change* **32**(2), 277–303.
URL: *https://doi.org/10.1093/icc/dtad008*

Stern, N., Stiglitz, J. E. and Taylor, C. (2022*a*), 'The economics of immense risk, urgent action and radical change: Towards new approaches to the economics of climate change', *Journal of Economic Methodology* **29**(3), 181–216.

Stern, N., Stiglitz, J. and Taylor, C. (2022*b*), 'The economics of immense risk, urgent action and radical change: towards new approaches to the economics of climate change', *Journal of Economic Methodology* **29**(3), 181–216.
URL: *https://www.tandfonline.com/doi/full/10.1080/1350178X.2022.2040740*

Syverson, C. (2011), 'What determines productivity?', *Journal of Economic Literature* **49**(2), 326–365.

Tol, R. S. J. (2023), 'Social cost of carbon estimates have increased over time', *Nature Climate Change* **13**, 532–536.
URL: *https://www.nature.com/articles/s41558-023-01680-x*

Tol, R. S. J. (2024), 'Database for the meta-analysis of the social cost of carbon (v2025.1)', `https://arxiv.org/html/2402.09125v3`. arXiv:2402.09125v3 [econ.EM].
URL: *https://arxiv.org/html/2402.09125v3*

van der Ploeg, F. and Rezai, A. (2021), 'Optimal carbon pricing in general equilibrium: Temperature caps and stranded assets in an extended annual dsge model', *Journal of Environmental Economics and Management* **110**, 102522.
URL: *https://doi.org/10.1016/j.jeem.2021.102522*

World Bank (2026), 'Total greenhouse gas emissions including lulucf (mt co_2e): World development indicators', `https://data.worldbank.org/indicator/EN.GHG.ALL.LU.MT.CE.AR5`. Accessed January 15, 2026; data compiled from the World Development Indicators, based on the EDGAR (Emissions Database for Global Atmospheric Research) Community GHG Database.
URL: *https://data.worldbank.org/indicator/EN.GHG.ALL.LU.MT.CE.AR5*

Yang, Z. (2019), 'Increasing returns to scale in energy-intensive sectors and its implications on climate change modelling', *Energy Economics* **83**, 208–216.
URL: *https://doi.org/10.1016/j.eneco.2019.06.011*

A Appendix:

A.1 Households

- Within period, households maximise

$$u = Q = \left[\int_0^n q_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

subject to

$$PQ = \int_0^n p_i q_i di$$

- i.e.

$$\begin{aligned} 0 &= \frac{\partial}{\partial q_i} \left[\left(\int_0^n q_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \lambda \left(\int_0^n p_i q_i di - P_t Q_t \right) \right] \\ &= Q^{\frac{1}{\theta}} q_i^{-\frac{1}{\theta}} - \lambda p_i \end{aligned}$$

so

$$q_i = \lambda^{-\theta} p_i^{-\theta} Q$$

is our demand for good i, but need to eliminate the Langrange Multiplier λ

- Have

$$\begin{aligned} PQ &= \int_0^n p_i q_i di = \int_0^n \lambda^{-\theta} p_i^{1-\theta} Q di \\ \text{i.e. } P &= \lambda^{-\theta} \int_0^n p_i^{1-\theta} di \end{aligned}$$

And

$$\begin{aligned} Q &= \left[\int_0^n q_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = \left[\int_0^n \left(\lambda^{-\theta} p_i^{-\theta} Q \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \\ \text{i.e. } \lambda^{-\theta} &= \left[\int_0^n p_i^{1-\theta} di \right]^{-\frac{\theta}{\theta-1}} \\ \text{i.e. } \lambda &= \left[\int_0^n p_i^{1-\theta} di \right]^{\frac{1}{\theta-1}} \end{aligned}$$

So

$$P = \lambda^{-\theta} \int_0^n p_i^{1-\theta} di = \left[\int_0^n p_i^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

i.e.

$$\lambda = P^{-1}$$
$$\text{and } P = \left[\int_0^n p_i^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

- So demand for good i is
$$q_i = \left(\frac{p_i}{P}\right)^{-\theta} Q$$

A.2 Firms

- Infinitesimal, differentiated, monopolistically competitive firms take demand for their product as given and choose prices to maximise profits
- Produce according to
$$q_i = \phi K_i^\alpha \left(\kappa L_{if}^\rho + L_{ir}^\rho\right)^{\frac{\nu}{\rho}} (L_{iF})^{1-\alpha-\nu} - f$$
- First of all, for a given quantity q_i produced, firms choose inputs to minimise costs $rK_i + w_r L_{ir} + w_f L_{if} + w_F L_{iF}$ i.e.

$$\begin{aligned}
0 &= \frac{\partial}{\partial K_i}\left[rK_i + w_r L_{ir} + w_f L_{if} + w_F L_{iF} - \mu\left(\phi K_i^\alpha \left(\kappa L_{if}^\rho + L_{ir}^\rho\right)^{\frac{\nu}{\rho}} (L_{iF})^{1-\alpha-\nu} - f - q_i\right)\right] \\
&= r - \mu\alpha\phi K_i^{\alpha-1} \left(\kappa L_{if}^\rho + L_{ir}^\rho\right)^{\frac{\nu}{\rho}} (L_{iF})^{1-\alpha-\nu} = r - \mu\alpha\frac{q_i + f}{K_i} \\
\text{so } r &= \frac{\alpha}{K_i}\mu(q_i + f) \\
\text{i.e. } \mu &= \frac{rK_i}{\alpha}\frac{1}{q_i + f}
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{\partial}{\partial L_{iF}}\left[rK_i + w_r L_{ir} + w_f L_{if} + w_F L_{iF} - \mu\left(\phi K_i^\alpha \left(\kappa L_{if}^\rho + L_{ir}^\rho\right)^{\frac{\nu}{\rho}} (L_{iF})^{1-\alpha-\nu} - f - q_i\right)\right] \\
&= w_F - \mu(1-\alpha-\nu)\phi K_i^\alpha \left(\kappa L_{if}^\rho + L_{ir}^\rho\right)^{\frac{\nu}{\rho}} (L_{iF})^{-\alpha-\nu} = w_F - \mu(1-\alpha-\nu)\frac{q_i + f}{L_{iF}} \\
\text{so } w_F &= \left(\frac{1-\alpha-\nu}{L_{iF}}\right)\mu(q_i + f) \\
\text{i.e. } L_{iF} &= \frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}K_i
\end{aligned}$$

$$
\begin{aligned}
0 &= \frac{\partial}{\partial L_{if}} \left[\begin{array}{c} rK_i + w_r L_{ir} + w_f L_{if} + w_F L_{iF} \\ -\mu \left(\phi K_i^{\alpha} \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right)^{\frac{\nu}{\rho}} \left(L_{iF} \right)^{1-\alpha-\nu} - f - q_i \right) \end{array} \right] \\
&= w_f - \mu \left(q_i + f \right) \frac{\nu \kappa L_{if}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \\
\text{so } w_f &= \frac{\nu \kappa L_{if}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \mu \left(q_i + f \right) \\
\text{i.e. } \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right) L_{if}^{1-\rho} &= \frac{\nu \kappa}{\alpha} \frac{r}{w_f} K_i
\end{aligned}
$$

$$
\begin{aligned}
0 &= \frac{\partial}{\partial L_{ir}} \left[\begin{array}{c} rK_i + w_r L_{ir} + w_f L_{if} + w_F L_{iF} \\ -\mu \left(\phi K_i^{\alpha} \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right)^{\frac{\nu}{\rho}} \left(L_{iF} \right)^{1-\alpha-\nu} - f - q_i \right) \end{array} \right] \\
&= w_r - \mu \left(q_i + f \right) \frac{\nu L_{ir}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \\
\text{so } w_r &= \frac{\nu L_{ir}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \mu \left(q_i + f \right) \\
\text{i.e. } \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right) L_{ir}^{1-\rho} &= \frac{\nu}{\alpha} \frac{r}{w_r} K_i
\end{aligned}
$$

So

$$
\begin{aligned}
r &= \frac{\alpha}{K_i} \mu \left(q_i + f \right) \\
w_F &= \left(\frac{1-\alpha-\nu}{L_{iF}} \right) \mu \left(q_i + f \right) \\
w_f &= \frac{\nu \kappa L_{if}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \mu \left(q_i + f \right) \\
w_r &= \frac{\nu L_{ir}^{\rho-1}}{\kappa L_{if}^{\rho} + L_{ir}^{\rho}} \mu \left(q_i + f \right)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\nu}{\alpha} \frac{r}{w_r} K_i &= \frac{1}{\kappa} \frac{w_f}{w_r} \frac{\nu \kappa}{\alpha} \frac{r}{w_f} K_i \\
\left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right) L_{ir}^{1-\rho} &= \frac{1}{\kappa} \frac{w_f}{w_r} \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho} \right) L_{if}^{1-\rho} \\
\text{i.e. } L_{ir} &= \left(\frac{1}{\kappa} \frac{w_f}{w_r} \right)^{\frac{1}{1-\rho}} L_{if}
\end{aligned}
$$

and

$$\frac{\nu\kappa}{\alpha}\frac{r}{w_f}K_i = \left(\kappa L_{if}^{\rho} + L_{ir}^{\rho}\right) L_{if}^{1-\rho} = \left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right) L_{if}$$

$$\text{i.e. } L_{if} = \frac{\nu\kappa}{\alpha}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i$$

$$\text{and } L_{ir} = \frac{\nu\kappa}{\alpha}\left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{1}{1-\rho}}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i$$

- Therefore

$$\begin{aligned} q_i + f &= \phi K_i^{\alpha}\left(\kappa L_{if}^{\rho} + L_{ir}^{\rho}\right)^{\frac{\nu}{\rho}}(L_{iF})^{1-\alpha-\nu} \\ &= \phi K_i^{\alpha} \times \\ &\left(\begin{array}{c} \kappa\left(\frac{\nu\kappa}{\alpha}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i\right)^{\rho} \\ +\left(\frac{\nu\kappa}{\alpha}\left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{1}{1-\rho}}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i\right)^{\rho} \end{array}\right)^{\frac{\nu}{\rho}} \\ &\times\left(\frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}K_i\right)^{1-\alpha-\nu} \\ &= \phi\left(\frac{\nu\kappa}{\alpha}\right)^{\nu}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{\nu\frac{1-\rho}{\rho}}\left(\frac{r}{w_f}\right)^{\nu}\left(\frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}\right)^{1-\alpha-\nu} K_i \end{aligned}$$

i.e.

$$q_i + f = WK_i$$

where

$$W = \phi\left(\frac{\nu\kappa}{\alpha}\frac{r}{w_f}\right)^{\nu}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{\nu\frac{1-\rho}{\rho}}\left(\frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}\right)^{1-\alpha-\nu}$$

- Firms take demand as given and choose prices to maximise profits i.e.

$$
\begin{aligned}
0 &= \frac{\partial}{\partial p_i}\left[p_i q_i - rK_i - w_F L_{iF} - w_f L_{if} - w_r L_{ir}\right] \\
&= \frac{\partial}{\partial p_i}\left[\begin{array}{c} p_i q_i - rK_i - w_F \frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}K_i - w_f \frac{\nu\kappa}{\alpha}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i \\ -w_r \frac{\nu\kappa}{\alpha}\left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{1}{1-\rho}}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\frac{r}{w_f}K_i \end{array}\right] \\
&= \frac{\partial}{\partial p_i}\left[p_i q_i - rK_i\left(1 + \frac{1-\alpha-\nu}{\alpha} + \frac{\nu}{\alpha}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{-1}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)\right)\right] \\
&= \frac{\partial}{\partial p_i}\left[p_i q_i - \frac{r}{\alpha}K_i\right] = \frac{\partial}{\partial p_i}\left[p_i q_i - \frac{r}{\alpha W}q_i - \frac{rf}{\alpha W}\right] \\
&= \frac{\partial}{\partial p_i}\left[P^\theta Q p_i^{1-\theta} - \frac{r}{\alpha W}P^\theta Q p_i^{-\theta} - \frac{rf}{\alpha W}\right] \\
&= (1-\theta)\,P^\theta Q p_i^{-\theta} + \theta\frac{r}{\alpha W}P^\theta Q p_i^{-\theta-1}
\end{aligned}
$$

i.e.

$$p_i = \frac{\theta}{\theta-1}\frac{r}{\alpha W}$$

- Free entry implies zero profits

$$
\begin{aligned}
0 &= \frac{\theta}{\theta-1}\frac{r}{\alpha W}q_i - \frac{r}{\alpha W}q_i - \frac{rf}{\alpha W} \\
\text{i.e. } q_i &= f(\theta-1) \\
\text{so } q_i + f &= f\theta
\end{aligned}
$$

- Aggregate production

$$
\begin{aligned}
Q &= \left(\int_0^n q_i^{\frac{\theta-1}{\theta}}\,di\right)^{\frac{\theta}{\theta-1}} = \left(\int_0^n (f(\theta-1))^{\frac{\theta-1}{\theta}}\,di\right)^{\frac{\theta}{\theta-1}} \\
&= f(\theta-1)\,n^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

And given

$$
\begin{aligned}
K &= nK_i \\
\text{have } K &= n\frac{q_i+f}{W} \\
\text{i.e. } n &= \frac{WK}{f\theta}
\end{aligned}
$$

So

$$Q = f(\theta-1)\left(\frac{W}{f\theta}\right)^{\frac{\theta}{\theta-1}} K^{\frac{\theta}{\theta-1}}$$

- Circular flow of income i.e.

$$\begin{aligned} PQ &= \int_0^n p_i q_i di = \int_0^n f\theta \frac{r}{\alpha W} di \\ &= f\theta \frac{r}{\alpha W} n = \frac{r}{\alpha} K \end{aligned}$$

i.e.

$$r = \frac{\alpha PQ}{K}$$

we normalise the aggregate price level i.e.

$$P = 1$$

so that

$$r = \alpha f\left(\theta - 1\right)\left(f\theta\right)^{\frac{\theta}{1-\theta}}\left(W\right)^{\frac{\theta}{\theta-1}} K^{\frac{1}{\theta-1}}$$

- Normalisation also gives (actually just the same condition as above):

$$\begin{aligned} 1 &= \left[\int_0^n p_i^{1-\theta} di\right]^{\frac{1}{1-\theta}} = \left[\int_0^n \left(\frac{\theta}{\theta-1}\frac{r}{\alpha W}\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \\ &= \frac{\theta}{\theta-1}\frac{r}{\alpha W} n^{\frac{1}{1-\theta}} \\ &= \frac{\theta}{\theta-1}\frac{r}{\alpha}\left(\frac{K}{f\theta}\right)^{\frac{1}{1-\theta}} W^{\frac{\theta}{1-\theta}} \\ \text{i.e. } W &= \left(\frac{\theta-1}{\theta}\frac{\alpha}{r}\right)^{\frac{1-\theta}{\theta}}\left(\frac{f\theta}{K}\right)^{\frac{1}{\theta}} \end{aligned}$$

-

$$\begin{aligned} K &= nK_i \\ L_F &= nL_{iF} \\ L_f &= nL_{if} \\ L_r &= nL_{iF} \\ r &= \frac{\alpha PQ}{K} \end{aligned}$$

$$\begin{aligned} r &= \frac{\alpha}{K} n\mu\left(q_i + f\right) = \frac{\alpha}{K} Q \\ w_F &= \left(\frac{1-\alpha-\nu}{L_F}\right) Q \\ w_f &= \frac{\nu\kappa L_f^{\rho-1}}{\kappa L_f^{\rho} + L_r^{\rho}} Q \\ w_r &= \frac{\nu L_r^{\rho-1}}{\kappa L_f^{\rho} + L_r^{\rho}} Q \end{aligned}$$

- To solve:
  - 1. Have values for K, L_F, L_f, L_r. Guess value for r. This gives value for

$$W = \left(\frac{\theta - 1}{\theta}\frac{\alpha}{r}\right)^{\frac{1-\theta}{\theta}}\left(\frac{f\theta}{K}\right)^{\frac{1}{\theta}}$$

    2. Then use

$$\begin{aligned} w_F &= \frac{1-\alpha-\nu}{\alpha}\frac{K}{L_F}r \\ w_f &= \frac{\nu\kappa}{\alpha}\left(\kappa + \left(\frac{L_r}{L_f}\right)^{\rho}\right)^{-1}\frac{K}{L_f}r \\ w_r &= \frac{1}{\kappa}\left(\frac{L_f}{L_r}\right)^{1-\rho} w_f \end{aligned}$$

    to calculate

$$W = \phi\left(\frac{\nu\kappa}{\alpha}\frac{r}{w_f}\right)^{\nu}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{\nu\frac{1-\rho}{\rho}}\left(\frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}\right)^{1-\alpha-\nu}$$

    3. Iterate guess of r until values for W agree
    4. Then finally, have

$$Q = f(\theta - 1)\left(\frac{W}{f\theta}\right)^{\frac{\theta}{\theta-1}} K^{\frac{\theta}{\theta-1}}$$

A.3 Dynamics

- There are N persons in this economy. Let N be a quantity with units of "persons" and let N' be a dimensionless scalar with the numerical magnitude of N. Further assume that these persons are divided into single person households. Over time, the number of households stays constant, but the labour inputs that each household inelastically supplies grows at rate ω (whether this is due to growth in household size or growth in labour productivity is irrelevant)

- Households make consumption savings decisions by maximising

$$U_t = \sum_{s=0}^{\infty}\beta^s N' \ln\left(\frac{C_{t+s}}{N'}\right)$$

  subject to

$$K_{t+1} = p_t^K K_t + \max\{w_{Ft}, w_{ft}, w_{rt}\} N\omega^t + T_t - C_t$$

  (note that utility is dimensionless, and capital, capital income, labour income, lump sum tax/transfer income, and consumption, all have units of output)

- In equilibrium, must have $w_{Ft} = w_{ft} = w_{rt}$ otherwise identical households will all allocate all their labour to the same sector. So let $w_t = w_{Ft} = w_{ft} = w_{rt}$ then

$$V_t(K_t) = N' \ln C_t + \beta V_{t+1}(K_{t=1})$$

F.O.C.

$$0 = \frac{N'}{C_t} + \beta \frac{\partial V_{t+1}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial C_{t+1}} = \frac{N'}{C_t} - \beta \frac{\partial V_{t+1}}{\partial K_{t+1}}$$

$$\text{i.e. } \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{N'}{\beta C_t}$$

E.T.

$$\frac{\partial V_t}{\partial K_t} = \beta \frac{\partial V_{t+1}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial K_t} = \beta p_t^K \frac{\partial V_{t+1}}{\partial K_{t+1}}$$

- So, obtain Euler Equation by iterating forward

$$\frac{C_{t+1}}{C_t} = \beta p_{t+1}^K$$

households choose consumption to satisfy:

$$K_{t+1} = p_t^K K_t + w_t N' \omega^t + T_t - C_t = (1-\lambda) Q_t$$
$$C_{t+1} = \beta p_{t+1}^K C_t$$

where guess of C_0 is determined such that, in the absence of climate damages,

$$\lim_{t \to \infty} p_t^K = \frac{\omega}{\beta}$$

(I think, since in absence of climate damages there is a balanced growth path)

- In fact, we have

$$p_t^K = r_t = \alpha \frac{Q_t}{K_t}$$
$$K_{t+1} = \alpha \beta Q_t$$
$$C_t = (1 - \alpha\beta) Q_t$$

This does indeed satisfy the Euler Equation i.e.

$$\frac{C_{t+1}}{C_t} = \frac{Q_{t+1}}{Q_t} = \beta p_{t+1}^K$$
$$\text{i.e. } \frac{Q_{t+1}}{Q_t} = \beta \alpha \frac{Q_{t+1}}{K_{t+1}}$$
$$\text{rearranging: } K_{t+1} = \alpha \beta Q_t \text{ , as required}$$

A.4 Social Planner's Solution

- In actual fact, the economy evolves according to

$$\begin{aligned} K_{t+1} &= Q_t - C_t \\ &= Ae^{-\chi E_t}\left(K_t^{\alpha}\left(\kappa L_{ft}^{\rho} + L_{rt}^{\rho}\right)^{\frac{\nu}{\rho}}\left(N\omega^t - L_{ft} - L_{rt}\right)^{1-\alpha-\nu}\right)^{\varepsilon} - C_t \end{aligned}$$

  and

$$E_{t+1} = E_t + L_{ft}$$

- Note that cumulated emissions are in person units (there really should be an emissions per person coefficient multiplying L_{ft} in the equation of motion for E_t but since the choice of scale for E_t is arbitrary, we can choose this coefficient equal to 1. Even in this case though, it should be a coefficient of '1 unit of emissions per worker', but we drop for simplicity and just record units of emissions as the same as the units of workers i.e. "persons".) Note further that the constant χ therefore has "per person" units.

- We know that

$$K_t^{\alpha}\left(\kappa L_{ft}^{\rho} + L_{rt}^{\rho}\right)^{\frac{\nu}{\rho}}\left(N\omega^t - L_{ft} - L_{rt}\right)^{1-\alpha-\nu} = \frac{W_t}{\phi}K_t$$

  where

$$W_t = \left(\frac{\theta-1}{\theta}\frac{\alpha}{r_t}\right)^{\frac{1-\theta}{\theta}}\left(\frac{f\theta}{K_t}\right)^{\frac{1}{\theta}}$$

  i.e.

$$Q_t = f(\theta-1)\left(\frac{W_t}{f\theta}\right)^{\frac{\theta}{\theta-1}}K_t^{\frac{\theta}{\theta-1}} = Ae^{-\chi E_t}\left(\frac{W_t}{\phi_t}K_t\right)^{\varepsilon}$$

$$\varepsilon = \frac{\theta}{\theta-1}$$

  and

$$f(\theta-1)\left(\frac{1}{f\theta}\right)^{\frac{\theta}{\theta-1}} = Ae^{-\chi E_t}\left(\frac{1}{\phi_t}\right)^{\frac{\theta}{\theta-1}}$$

  i.e.

$$\begin{aligned} A &= f(\theta-1)\left(\frac{1}{f\theta}\right)^{\frac{\theta}{\theta-1}} \\ \phi_t &= e^{-\chi\frac{\theta-1}{\theta}E_t} \end{aligned}$$

- Bellman equation:

$$V_t(K_t, E_t) = N'\ln C_t + \beta V_{t+1}(K_{t+1}, E_{t+1})$$

s.t.

$$K_{t+1} = Ae^{-\chi E_t}\left(K_t^{\alpha}\left(\kappa L_{ft}^{\rho} + L_{rt}^{\rho}\right)^{\frac{\nu}{\rho}}\left(N\omega^t - L_{ft} - L_{rt}\right)^{1-\alpha-\nu}\right)^{\varepsilon} - C_t$$
$$E_{t+1} = E_t + L_{ft}$$

F.O.C.s

$$0 = \frac{N'}{C_t} - \beta\frac{\partial V_{t+1}}{\partial K_{t+1}}$$
$$\text{i.e. } \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{N'}{\beta C_t}$$

$$0 = \beta\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial L_{rt}}\right]$$
$$\text{i.e. } 0 = \frac{\partial K_{t+1}}{\partial L_{rt}} = \frac{\partial Q_t}{\partial L_{rt}} + \frac{\partial Q_t}{\partial L_{Ft}}\frac{\partial L_{Ft}}{\partial L_{rt}} = \frac{\partial Q_t}{\partial L_{rt}} - \frac{\partial Q_t}{\partial L_{Ft}}$$
$$\text{i.e. } \frac{\partial Q_t}{\partial L_{rt}} = \frac{\partial Q_t}{\partial L_{Ft}}$$

NB

$$\frac{\partial Q_t}{\partial L_{Ft}} = \varepsilon\left(\frac{1-\alpha-\nu}{L_{Ft}}\right)Q_t$$
$$\text{and } \frac{\partial Q_t}{\partial L_{rt}} = \varepsilon\frac{\nu L_{rt}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}}Q_t$$
$$\text{so if } w_t = w_F(t) = w_r(t) \text{ given by firm behaviour}$$
$$\text{then } w_t = \frac{1}{\varepsilon}\frac{\partial Q_t}{\partial L_{rt}} = \frac{1}{\varepsilon}\frac{\partial Q_t}{\partial L_{Ft}}$$

$$0 = \beta\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial L_{ft}} + \frac{\partial V_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial L_{ft}}\right]$$
$$\text{i.e. } 0 = \frac{N'}{\beta C_t}\frac{\partial K_{t+1}}{\partial L_{ft}} + \frac{\partial V_{t+1}}{\partial E_{t+1}} = \frac{N'}{\beta C_t}\left(\frac{\partial Q_t}{\partial L_{ft}} - \frac{\partial Q_t}{\partial L_{Ft}}\right) + \frac{\partial V_{t+1}}{\partial E_{t+1}}$$
$$\text{i.e. } \frac{\partial Q_t}{\partial L_{ft}} = \frac{\partial Q_t}{\partial L_{Ft}} - \frac{\beta C_t}{N'}\frac{\partial V_{t+1}}{\partial E_{t+1}}$$

E.T.

$$\frac{\partial V_t}{\partial K_t} = \beta\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial K_t}\right] = \beta\frac{\partial V_{t+1}}{\partial K_{t+1}}\alpha\varepsilon\frac{Q_t}{K_t}$$
$$\text{i.e. } \frac{\partial V_{t+1}}{\partial K_{t+1}} = \alpha\beta\varepsilon\frac{Q_{t+1}}{K_{t+1}}\frac{\partial V_{t+2}}{\partial K_{t+2}}$$

$$\frac{\partial V_t}{\partial E_t} = \beta\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial E_t} + \frac{\partial V_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial E_t}\right] = \beta\left[\frac{\partial V_{t+1}}{\partial E_{t+1}} - \chi Q_t\frac{\partial V_{t+1}}{\partial K_{t+1}}\right]$$

Combining

$$\begin{aligned}\frac{C_{t+1}}{C_t} &= \alpha\beta\varepsilon\frac{Q_{t+1}}{K_{t+1}}\\ w_t &= \frac{1}{\varepsilon}\frac{\partial Q_t}{\partial L_{rt}} = \frac{1}{\varepsilon}\frac{\partial Q_t}{\partial L_{Ft}}\\ \frac{\partial Q_t}{\partial L_{ft}} &= \varepsilon w_t - \frac{\beta C_t}{N'}\frac{\partial V_{t+1}}{\partial E_{t+1}}\\ \frac{\partial V_{t+1}}{\partial E_{t+1}} &= \beta\left[\frac{\partial V_{t+2}}{\partial E_{t+2}} - \frac{\chi N'}{\beta}\frac{Q_{t+1}}{C_{t+1}}\right]\end{aligned}$$

Guess that $K_{t+1} = \lambda Q_t$ so that $C_t = (1-\lambda)\,Q_t$ and $C_{t+1} = (1-\lambda)\,Q_{t+1}$ where λ is a dimensionless constant. Then

$$\begin{aligned}\frac{C_{t+1}}{C_t} &= \alpha\beta\varepsilon\frac{Q_{t+1}}{K_{t+1}}\\ \frac{(1-\lambda)\,Q_{t+1}}{(1-\lambda)\,Q_t} &= \alpha\beta\varepsilon\frac{Q_{t+1}}{\lambda Q_t}\\ &\text{i.e. } \lambda = \alpha\beta\varepsilon \text{ confirming our guess}\end{aligned}$$

Further guess that $\tau_t = \zeta\frac{Q_t}{N}$ where ζ is a dimensionless constant (so that τ_t has units of output per person, consistent with the wage to which it is added), and where

$$\tau_t = -\frac{\beta C_t}{N'}\frac{\partial V_{t+1}}{\partial E_{t+1}}$$

then

$$\begin{aligned}\frac{\partial Q_t}{\partial L_{ft}} &= \varepsilon w_t + \tau_t\\ \frac{\nu L_{rt}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}}Q_t &= w_t + \frac{\tau_t}{\varepsilon}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial V_{t+1}}{\partial E_{t+1}} &= \beta\left[\frac{\partial V_{t+2}}{\partial E_{t+2}} - \frac{\chi N'}{\beta}\frac{Q_{t+1}}{C_{t+1}}\right]\\ -\frac{\beta C_t}{N'}\frac{\partial V_{t+1}}{\partial E_{t+1}} &= \beta\left[-\frac{\beta C_{t+1}}{N'}\frac{\partial V_{t+2}}{\partial E_{t+2}}\frac{C_t}{C_{t+1}} + \chi\frac{Q_{t+1}}{C_{t+1}}C_t\right]\\ \tau_t &= \beta\left[\tau_{t+1}\frac{C_t}{C_{t+1}} + \frac{\chi}{(1-\alpha\beta\varepsilon)}C_t\right]\\ \zeta\frac{Q_t}{N} &= \beta\left[\zeta\frac{Q_{t+1}}{N}\frac{C_t}{C_{t+1}} + \frac{\chi}{(1-\alpha\beta\varepsilon)}C_t\right]\\ \zeta &= \beta\,(\zeta + \chi N)\\ \text{i.e. } \zeta &= \frac{\beta\chi N}{1-\beta} \text{ dimensionless, confirming our guess}\end{aligned}$$

- So the Social Planner's Solution is to set

$$\begin{aligned}
K_{t+1} &= \alpha\beta\varepsilon Q_t \\
C_t &= (1-\alpha\beta\varepsilon)\, Q_t \\
w_F(t) = w_t &= \left(\frac{1-\alpha-\nu}{L_{Ft}}\right) Q_t \\
w_r(t) = w_t &= \frac{\nu L_{rt}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}} Q_t \\
w_f(t) &= \frac{\nu\kappa L_{ft}^{\rho-1}}{\kappa L_{ft}^{\rho} + L_{rt}^{\rho}} Q_t = w_t + \frac{\beta\chi}{\varepsilon(1-\beta)} Q_t
\end{aligned}$$

- The Social Planner's Solution can be decentralised by
  - taking aggregate actions:
    1. Paying an investment subsidy that raises the return on capital from $\alpha\frac{Q_t}{K_t}$ (which implies $K_{t+1} = \alpha\beta Q_t$) to $\alpha\varepsilon\frac{Q_t}{K_t}$ (which implies $K_{t+1} = \alpha\beta\varepsilon Q_t$)
    2. Charging a total carbon tax amount to the fossil sector of $\frac{\beta\chi}{\varepsilon(1-\beta)} Q_t L_{ft}$
    3. Paying/charging the balance of these to households as a lump sum transfer i.e.

$$T_t = \frac{\beta\chi}{\varepsilon(1-\beta)} Q_t L_{ft} - (\varepsilon-1)\,\alpha Q_t$$

  - i.e. in terms of what households see
    1. Household actions satisfy

$$\begin{aligned}
K_{t+1} &= p_t^K K_t + w_t N'\omega^t + T_t - C_t \\
C_{t+1} &= \beta p_{t+1}^K C_t
\end{aligned}$$

    2. Where

$$\begin{aligned}
p_t^K &= \alpha\varepsilon\frac{Q_t}{K_t} = \varepsilon r_t \\
K_{t+1} &= \alpha\beta\varepsilon Q_t \\
C_t &= (1-\alpha\beta\varepsilon)\, Q_t
\end{aligned}$$

    3. This does indeed satisfy the Euler Equation i.e.

$$\begin{aligned}
\frac{C_{t+1}}{C_t} &= \frac{Q_{t+1}}{Q_t} = \beta p_{t+1}^K \\
\text{i.e. } \frac{Q_{t+1}}{Q_t} &= \beta\alpha\varepsilon\frac{Q_{t+1}}{K_{t+1}} \\
\text{rearranging: } K_{t+1} &= \alpha\beta\varepsilon Q_t \text{ , as required}
\end{aligned}$$

A.5 Algorithmic Implementation

- Arrive in period t with K_t and E_t, as well as total stock of effect labour, $N_t = N_0 \omega^t$, known. Know all the parameter values, and also have:

$$\phi_t = e^{-\frac{\chi}{\varepsilon} E_t}$$

- Want to implement some share $n \geq 0$ of optimal climate policy, and $m \geq 0$ of optimal investment policy. If $m = n = 1$ then we have the Social Planner's Solution. If $m = n = 0$ we have laissez-faire. And other values are possible too e.g. $m, n > 1$ for sub-optimally interventionist policy

- Dropping time subscripts, define

$$Q(L_f, L_r) = Ae^{-\chi E}(K^{\alpha}(\kappa L_f^{\rho} + L_r^{\rho})^{\frac{\nu}{\rho}}(N - L_f - Lr)^{1-\alpha-\nu})^{\varepsilon}$$

- Guess a value for r

- Then have

$$W1 = \left(\frac{\theta - 1}{\theta}\frac{\alpha}{r}\right)^{\frac{1-\theta}{\theta}}\left(\frac{f\theta}{K}\right)^{\frac{1}{\theta}}$$

- Choose L_f and L_r such that

$$\begin{aligned}
w_F &= \frac{1-\alpha-\nu}{\alpha}\frac{K}{N - L_f - Lr}r \\
w_f &= \frac{\nu\kappa}{\alpha}\left(\kappa + \left(\frac{L_r}{L_f}\right)^{\rho}\right)^{-1}\frac{K}{L_f}r \\
w_r &= \frac{1}{\kappa}\left(\frac{L_f}{L_r}\right)^{1-\rho} w_f
\end{aligned}$$

where equilibrium in the labour market demands that

$$\begin{aligned}
w_F &= w \\
w_r &= w \\
w_f &= w + n \times \frac{\beta\chi}{\varepsilon(1-\beta)}Q(L_f, L_r)
\end{aligned}$$

- Now calculate

$$W2 = \phi\left(\frac{\nu\kappa}{\alpha}\frac{r}{w_f}\right)^{\nu}\left(\kappa + \left(\frac{1}{\kappa}\frac{w_f}{w_r}\right)^{\frac{\rho}{1-\rho}}\right)^{\nu\frac{1-\rho}{\rho}}\left(\frac{1-\alpha-\nu}{\alpha}\frac{r}{w_F}\right)^{1-\alpha-\nu}$$

guess of r until $W1 = W2 \equiv W$ and check that resulting r satisfies

$$r = \frac{\alpha}{K}Q(L_f, L_r)$$

together with

$$Q(L_f, L_r) = f\,(\theta - 1)\left(\frac{W}{f\theta}\right)^{\frac{\theta}{\theta-1}} K^{\frac{\theta}{\theta-1}}$$

- Policymaker subsidises investment income so that $p^K = (1+m\times(\varepsilon-1))\times r$, funded by the carbon tax and a lump sum transfer

$$T = -m(\varepsilon - 1)rK + n\frac{\beta\chi}{\varepsilon\,(1-\beta)}Q(L_f, L_r)L_f$$

- Check $Q(L_f, L_r) = wN + p^K K + T$
- To iterate system forward:

$$\begin{aligned} K_{t+1} &= \alpha\beta(1 + m(\varepsilon - 1))Q(L_f, L_r) \\ E_{t+1} &= E + L_f \\ N_{t+1} &= N\omega \end{aligned}$$

- It remains to check that the Euler Equation is satisfied: given $C_t = Q_t - K_{t+1}$, must have:

$$\frac{C_{t+1}}{C_t} = \beta p^K_{t+1}$$

B Independence of Optimal Carbon Tax and Degree of IRS under full re-calibration

In this Appendix we show the independence of the initial optimal carbon tax to variations in θ (and hence to variations in the degree of scale economies) *if we redo the entire calibration procedure.*

Firstly, note that the carbon budget is independent of the degree of scale economies: initial levels of capital and effective labour supply are unchanged from our varying of θ, and the 2020 level of emissions is then determined by the value of L_{f0} under the assumption that no policy is imposed. This value will therefore be independent of the degree of scale economies as it is determined by the need to match the 80:20 fossil:low carbon energy mix seen in the data. If this L_{f0} is independent of the degree of scale economies, then so too is the overall carbon budget to hit the 2^oC temperature rise limit, since this is simply 2.5 times this L_{f0} value.

Next we show some simulated results from various runs involving different values of $\theta = 4, 6, 8$ each with full recalibration.

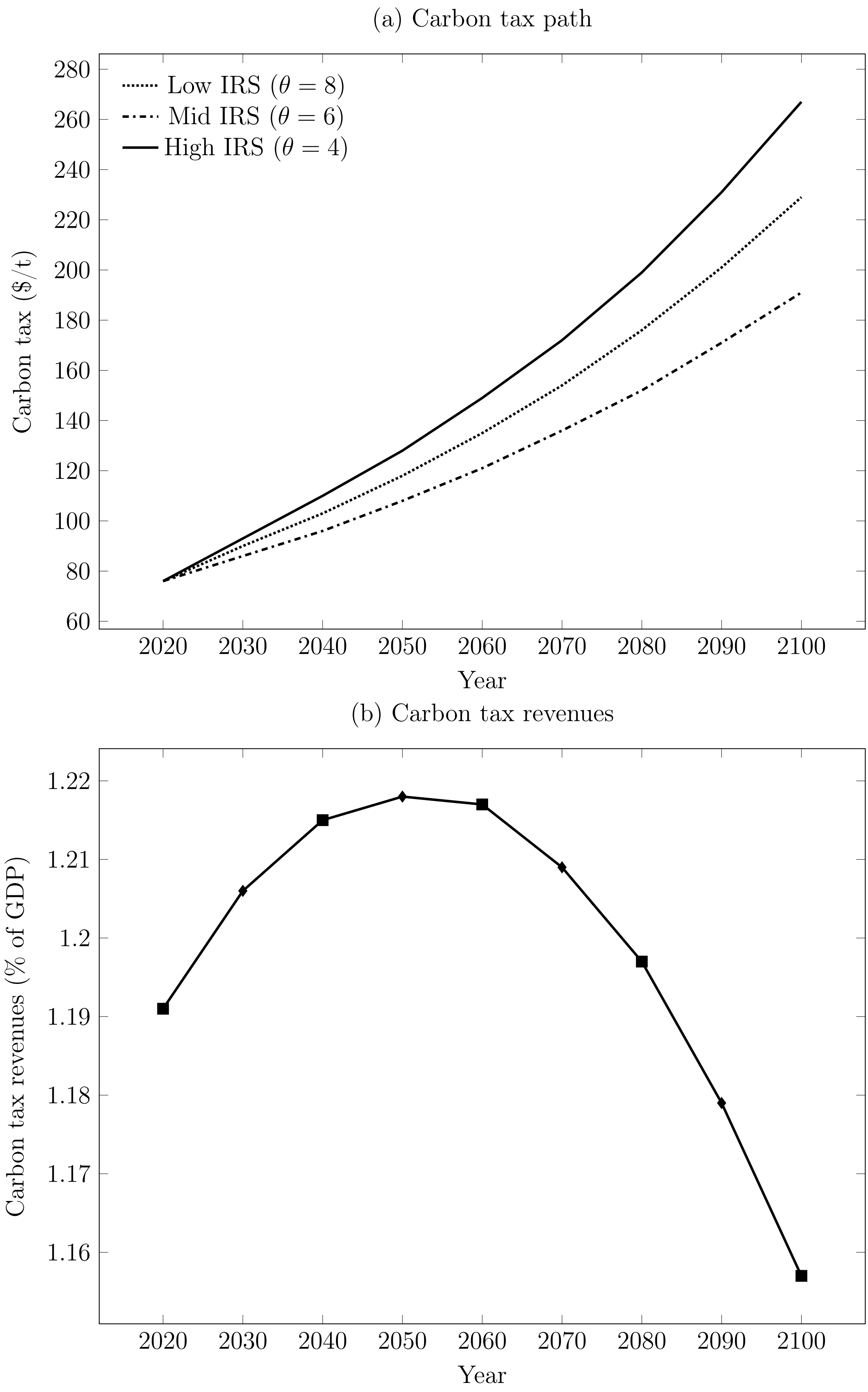
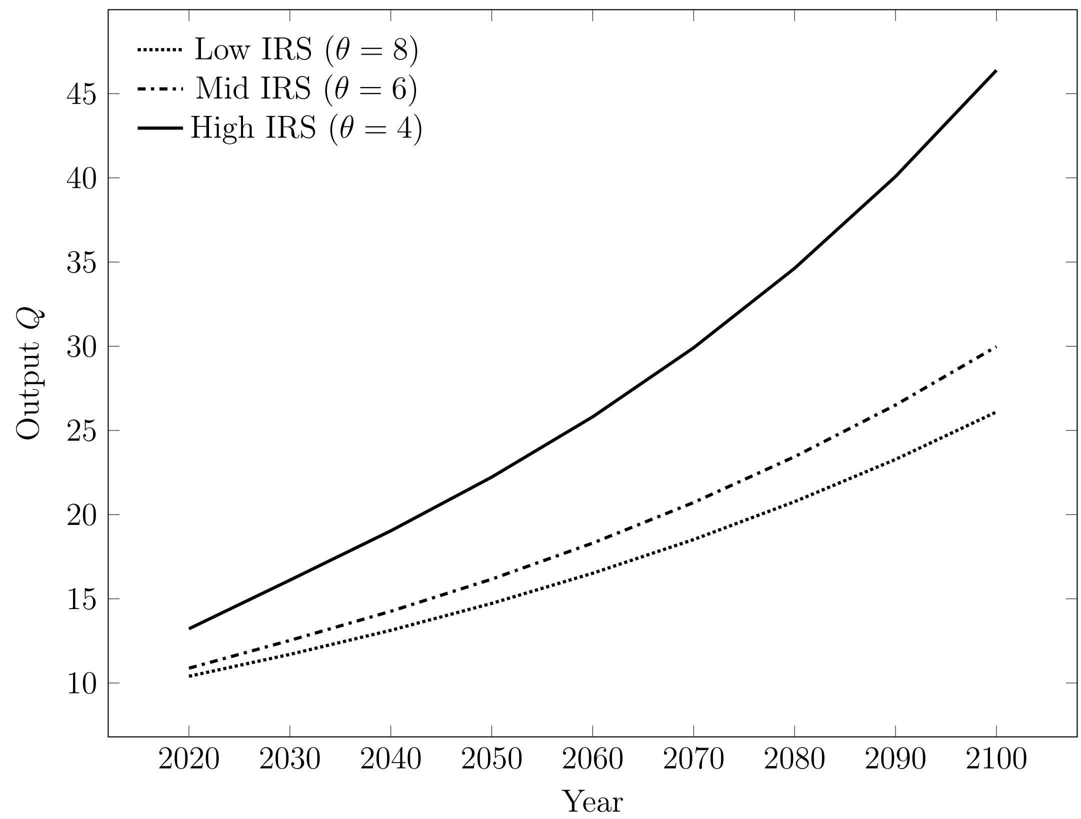


Figure B.1: Carbon taxes under alternative degrees of increasing returns to scale in a recalibrated model. Note that carbon tax revenues as a share of GDP coincide exactly; staggered markers in panel (b) indicate these overlapping scenarios.

(a) Output



(b) Global temperature

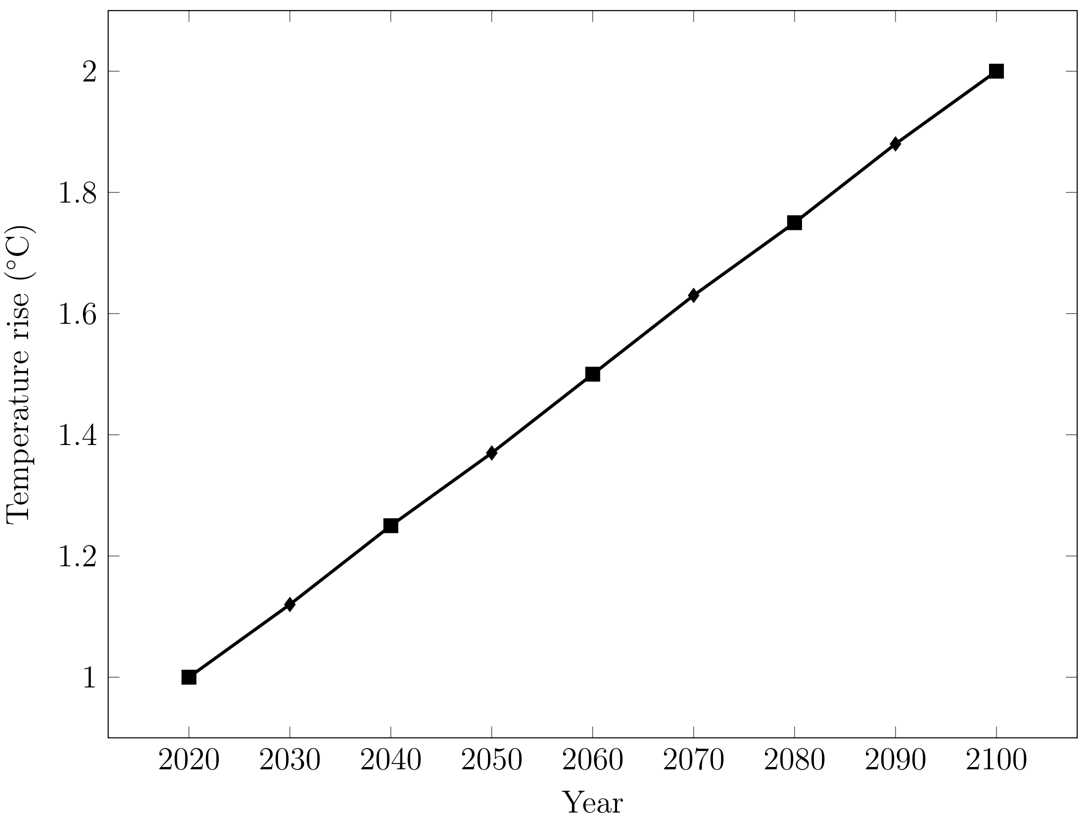


Figure B.2: Macroeconomic and climate outcomes under alternative degrees of increasing returns to scale in a recalibrated model. Note that temperature paths coincide exactly; staggered markers in panel (b) indicate these overlapping scenarios.

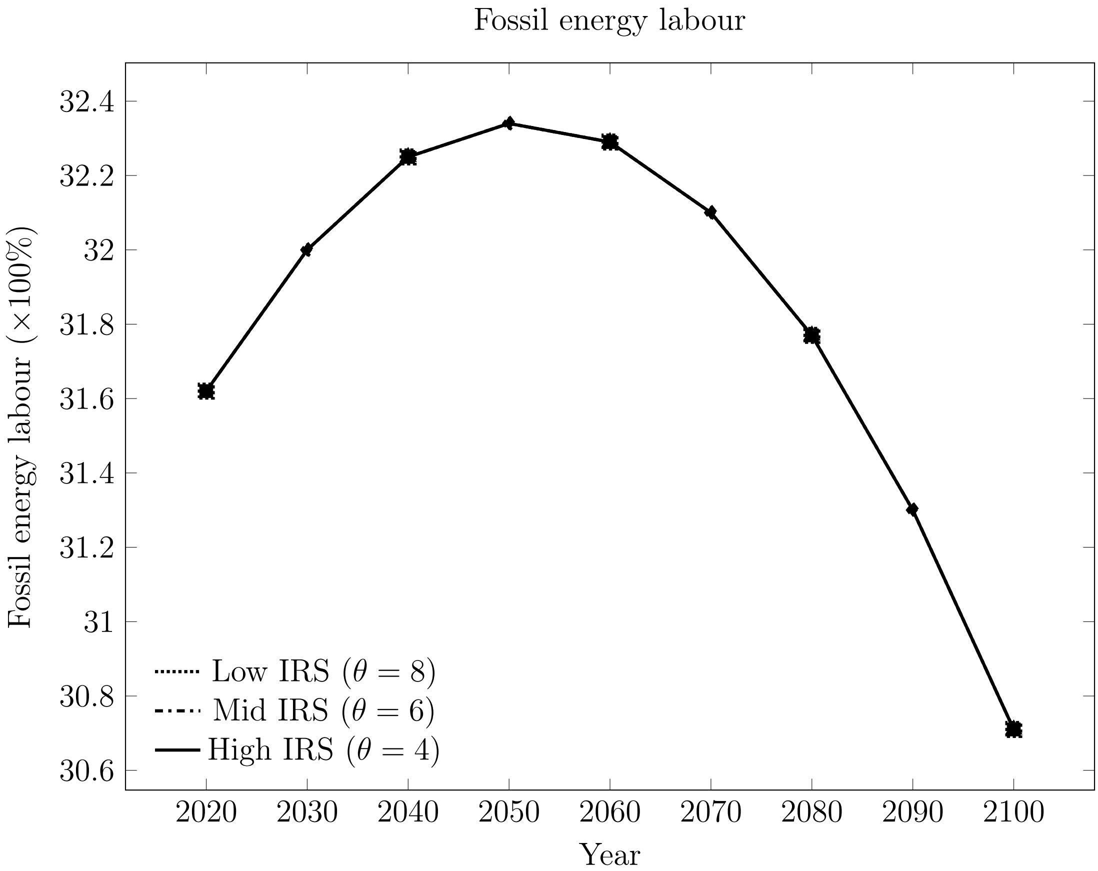


Figure B.3: Fossil energy labour across alternative degrees of increasing returns to scale in a recalibrated model. Reported values are multiplied by 100% to improve readability. All scenarios coincide exactly; staggered markers indicate overlapping paths.

These simulations show that, not only do we have the same carbon budget in each run, but this carbon budget is met using the same timepath of emissions (L_{ft}) and carbon tax revenues as a share of GDP. Equation (31) then says that:

$$\text{Carbon tax revenue share} \equiv share_t = \frac{\beta\chi(\theta-1)}{\theta\,(1-\beta)} L_{ft}$$

So equal shares of carbon tax revenues and equal emissions (L_{ft}) across runs with different θ implies that:

$$\frac{\chi(\theta-1)}{\theta} = \frac{share_t}{L_{ft}} \frac{1-\beta}{\beta} = const$$

where this constant is independent of the degree of scale economies.

This says that variation in θ (and hence the degree of scale economies) will be entirely offset by offsetting variation in χ, the climate damages parameter, leading to an optimal carbon tax that is independent of θ. As we state in the main text though, this is not what we mean by the dependence of the optimal carbon tax upon the degree of scale economies in the aggregate economy. We are simply using this calibration method to calibrate our damages parameter χ. Once χ is calibrated, the optimal carbon tax does indeed depend upon the degree of scale economies in the aggregate economy.