

The Stability of Surface Waves on Sheared Currents

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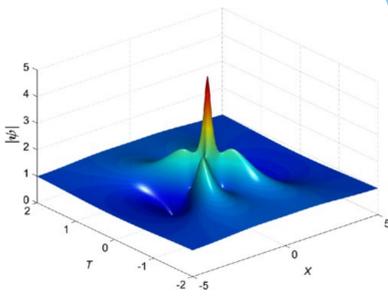
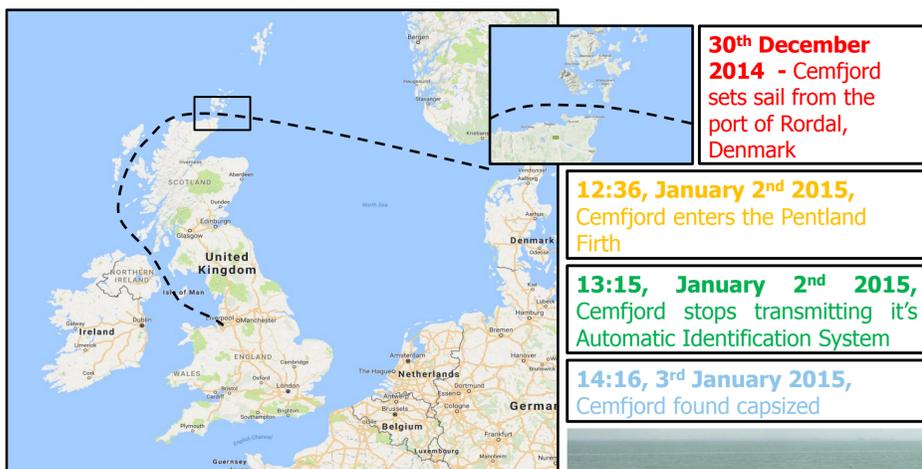


Introduction

Rogue waves

Rogue waves of amplitudes many times their surrounding waves have claimed hundreds of ships in the last century from the Cape of South Africa to the Pentland Firth (Lavrenov, 1998). Many physical mechanisms for rogue wave formation have been suggested, including nonlinear effects such as the Benjamin-Feir instability (Dysthe et al., 2008). Adverse currents can amplify these effects leading to catastrophic shipping accidents (Peregrine & Jonsson, 1983).

Cemfjord's journey through the Pentland Firth



Analytical solution to the nonlinear Schrödinger equation representing the packet amplitude of a rogue wave. Reproduced from (Chabchoub et al., 2012).

Conclusions from the MAIB

"Cemfjord capsized suddenly and rapidly... when it encountered extraordinarily violent, breaking seas in the Pentland Firth..."

"...such conditions were predictable and passage through the Pentland Firth should not have been attempted..."

"The extraordinarily violent sea conditions were created by **gale force winds opposing a strong ebb tidal stream**"

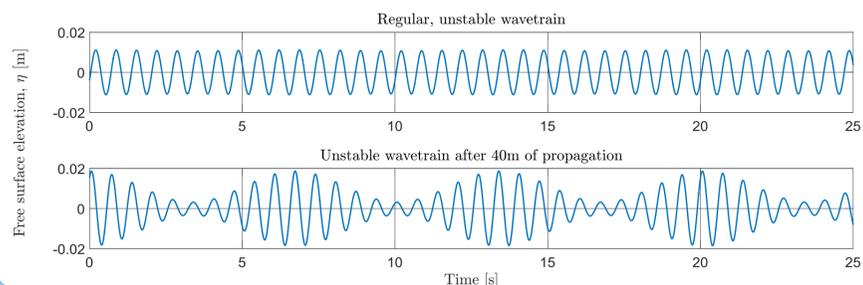


Why are these conditions so treacherous?



The Junction of the Thames and the Medway, J. M. W. Turner

- 1.) Adverse currents increase wave frequency and amplitude thereby increasing steepness
- 2.) Nonlinear terms in steepness are normally neglected in the free surface elevation equation
- 3.) When steepness is increased by currents, the nonlinear terms can no longer be neglected
- 4.) Nonlinear waves are prone to modulation instability - naturally producing pulses of extreme waves from initially regular ones



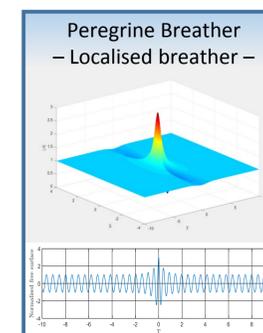
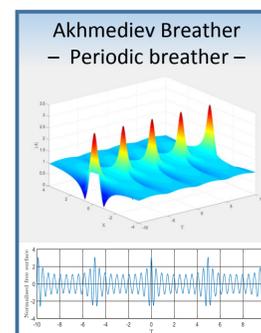
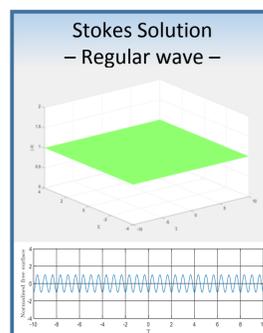
Aim

"To experimentally investigate how vertically and cross-stream horizontally sheared currents alter the stability of surface gravity waves in order to assess the effect of shear on the occurrence of extreme wave events"

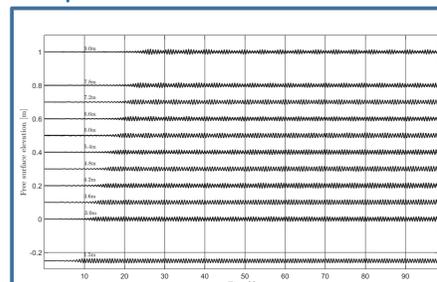
Completed Work

Solutions to the nonlinear Schrödinger equation

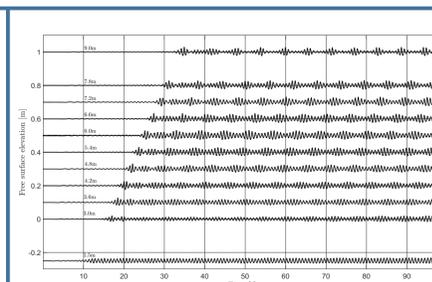
$$i \frac{\partial A}{\partial \tau} + L \frac{\partial^2 A}{\partial X^2} - M |A|^2 A = 0$$



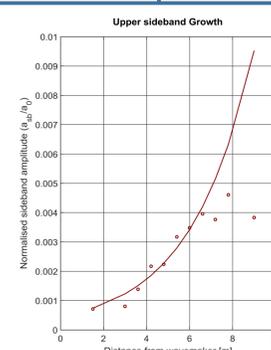
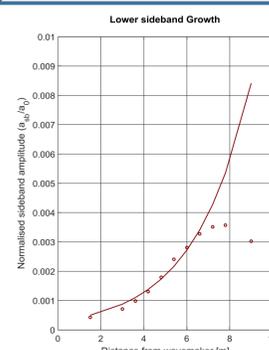
Experimental observations of unstable Stokes waves on sheared current



Wave gauge outputs showing very little growth of an unstable wavetrain on zero current.



Wave gauge outputs showing very high growth of an unstable wavetrain on adverse, sheared current.



Experimental modulation growth for upper and lower sidebands. Obtained in the University College London coastal flume through collaboration with Dr Eugeny Buldakov and Dr Dimitris Stagonas.

Estimation of growth rate

Modulation amplitudes are measured from the spectrum at each position along the flume and plotted with distance from the wavemaker

An exponential curve can then be fitted in the form:

$$\text{amplitude} = \beta e^{\alpha x}$$

Where α is the growth rate

Bibliography

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Dysthe, K., H. E. Krogstad, M. Peter. 2008. Oceanic rogue waves. *Annu. Rev. Fluid Mech*

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