DEPARTMENT OF MATHEMATICS & STATISTICS

University of Strathclyde Science

INFORMATION FOR STUDENTS ENTERING YEAR 3 IN ACADEMIC SESSION 2022/23

(Supplement to Student Handbook)

Note: the information contained herein may be subject to change. (For example, the Department reserves the right to alter the list of classes available, the semester in which classes are delivered or degree course structures.)

07 March 2022

GENERAL INFORMATION

Year Co-ordinator: **Dr Matthias Langer**

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DEGREE COURSES

MMath:	**Mathematics**	**(MM)**
	Mathematics and Statistics	**(MMS)**
BSc & BSc Honours:	**Mathematics**	**(M)**
	Mathematics and Statistics	**(MS)**
	Mathematics and Computer Science	**(MCS)**
	Mathematics and Physics	**(MP)**
	Mathematics, Statistics and Accounting	**(MSA)**
	Mathematics, Statistics and Economics	**(MSE)**
	Mathematics, Statistics and Finance	**(MSF)**
	Mathematics, Statistics and Business Analysis	**(MSBA)**
	Mathematics with Teaching/ Mathematics with Teaching (International)	**(MWT)**
	Data Analytics	**(DA)**
BA Joint Degrees:	**Joint with Mathematics and/or Statistics**	**(BA)**

PROVISIONAL REGISTRATION

Student Business asks students to select in advance a provisional curriculum for next session via PEGASUS. The facility to choose your year 3 classes and submit them for approval is now available. Do not wait until the examinations have finished and you know your results.

The deadline for submitting a valid provisional curriculum is **Friday, 25 March 2022**. Any delay in submitting a provisional curriculum may delay your registration for year 3. Your provisional curriculum is not final — you will be able to submit changes to your curriculum before the start of the academic session in September, and even during the first two weeks of each semester.

During the course of the summer, provisional lecture times and rooms for each class will become available at: **www.strath.ac.uk/professionalservices/timetables**

CALENDAR OF DATES (** PROVISIONAL **)

Year 3 classes are taught in two semesters each consisting of a Welcome/Consolidation & Development Week, an 11-week teaching block (including Revision), and a Formal Assessment Period (Examinations):

- **First Semester**

Welcome & Development Week:	Monday, 12 September – Friday, 16 September 2022
Teaching Block:	Monday, 19 September – Friday, 2 December 2022
	(Public holiday: Monday, 26 September 2022)
Formal Assessment:	Monday, 5 December – Friday, 16 December 2022

- **Second Semester**

Consolidation & Development Week:	Monday, 9 January – Friday, 13 January 2023
Teaching Block:	Monday, 16 January – Friday, 31 March 2023
Formal Assessment:	Monday, 17 April – Friday, 19 May 2023

(Normally examinations are held in the diet immediately following the completion of the class.)

- **Resit Examination Diet**

  Wednesday, 26 July – Wednesday, 9 August 2023 (to be confirmed)

  Permission to resit is not automatic, being subject to the approval of the appropriate Board of Examiners.

GENERAL INFORMATION: MATHEMATICS AND STATISTICS

Information for students about all the classes and courses provided by the Department of Mathematics and Statistics can be found via a class on Myplace called:

Mathematics & Statistics: Information for Current Students

All students on our degree courses are automatically enrolled for this class on Myplace.

COURSE REGULATIONS

General and Course Regulations for all BSc, BA (Bachelor) and MMath (Integrated Masters) degrees are published on the University website:

www.strath.ac.uk/studywithus/academicregulations/

Please read the regulations for your degree course carefully, in particular, the information about compulsory and optional classes available to you in Year 3, progress requirements and the award of degrees.

MMath & BSc HONOURS MATHEMATICS AND STATISTICS

The Department offers students on the BSc Honours Mathematics degree course the option of graduating with a BSc Honours degree in Mathematics **and Statistics** at the end of their fourth year. Similarly, MMath Mathematics students may opt for MMath Mathematics **and Statistics**. Generally, students cannot be registered on the Mathematics and Statistics degrees in Years 1 to 3, but have the option of transferring *after they have met the requirements for progress into Year 4.*

The option of graduating in Mathematics and Statistics will be available to students who have completed an appropriate amount of statistics as part of their degree course. (This is in order to achieve Royal Statistical Society accreditation.) For our degree structure, this involves students taking all the statistics classes available to them in Year 3 (MM304, MM307) and Year 4 (currently MM402, MM404, MM407, MM415).

BSc PASS DEGREE COURSES

Most students in Year 2 are currently registered on BSc Honours or MMath degree courses. If you are considering a BSc degree course (sometimes known as a BSc Pass degree) in Year 3, or are transferred down to a BSc degree course by the Faculty Examination Board, you will find the curriculum requirements in the appropriate course regulations. In BSc Pass degrees, your curriculum can include some lower level optional classes and electives in an subject area.

Any Year 2 students currently on a BSc Pass degree course can request to transfer into Honours provided they meet the appropriate criteria (e.g. overall credit total, passes in specific classes, etc.).

If you are transferred down to a BSc Pass degree by the Faculty Examination Board in June or August/September, but still allowed to progress into Year 3, you may have to resubmit a provisional curriculum, especially if you decide to choose low level optional or elective classes.

If you wish to discuss the BSc Pass degree option, or transfer up to BSc Honours, please contact the Year Co-ordinator.

BSc HONOURS MATHEMATICS WITH TEACHING (INCL. MWT INTERNATIONAL)

The Mathematics with Teaching degree involves:

- three 20 credit level 3 classes in semester 1 (with MM302 and MM304 compulsory),
- one 60 credit level 4 class, MM441, in semester 2.

Class MM441 comprises 60 credits chosen from classes MM407, MM408, MM409 and MM415. The 60 credit class will appear on your list of classes on PEGASUS automatically. When choosing your provisional curriculum for approval, you should submit your choice of three individual level 4 classes which will comprise MM441, as well as your choice of level 3 class in semester 1 to accompany the compulsory classes MM302 and MM304.

FACULTY EXAMINATION BOARD & PROGRESS INTO YEAR 4

The Science Faculty Examination Board meets in June, and again in August/September, in order to discuss the performance of every student in Years 1 to 3 on our degree courses. The Examination Board decides whether students are permitted to progress into subsequent years of study. These decisions are based on course regulations and very strict guidelines published by the Department and the University. The Faculty Board of Examiners also takes into account valid Personal Circumstances lodged with Student Business and/or recorded on PEGASUS.

Some of the Examination Board guidelines/course regulations and their implications for student progress are summarized below:

- In order to progress into Year 4 of Mathematics and Statistics BSc Honours degree courses, students require

  360 credits from the course curriculum **AND** ***a credit weighted average mark of at least 40% in their level 3 (and above) classes at their first attempts***.

  The 360 credits must include an appropriate number at level 3 or above (often 120 credits) and any compulsory classes, as published in the University course regulations.

  Progress into Year 4 of MMath degree courses requires a credit weighted average mark of at ***least 60% in level 3 classes at the first attempt***.

- Any student who achieves at least 60 credits from the Year 3 curriculum, but fails to reach a total of 360 credits by September (or the appropriate number of level 3/level 4 credits), will normally be placed in ***Academic Suspension***.

  This means that you do not progress into Year 4 in the following session, nor can you re-attend the Year 3 classes. Instead, you may be permitted to resit the final assessment in any outstanding class the following academic session.

  Note: You are entitled to **two** attempts at a class assessment. While the Exam Board may allow one or two extra attempts, it is not automatic.

- BSc Honours students who fail to obtain an average mark of 40% at the first attempt may be transferred from BSc Honours onto the corresponding BSc Pass degree course (or BSc Mathematics if appropriate).

  MMath students who fail to obtain an average mark of 60% at the first attempt may be transferred from MMath onto the corresponding BSc Honours degree course (if their average is at least 40%) or the appropriate BSc Pass degree (if their average is below 40%).

- Students who fail to achieve at least 60 credits from their Year 3 curriculum by September will be withdrawn from their degree course. (If appropriate, the student may be awarded the Diploma or Certificate of Higher Education.)

FINAL HONOURS ASSESSMENT

BSc Honours degrees (but not Pass degrees) are classified as:

I First, II(i) Upper Second, II(ii) Lower Second, III Third.

Final Honours assessment at the end of Year 4 is based on your **first assessed attempt in compulsory and specified classes at levels 3 and 4.**

The first requirement for the award of an Honours degree is successful completion of Year 4 in order to obtain 120 credits at level 4. On most courses (not MWT), 120 credits are awarded if a student's overall Year 4 average is at least 40%. For the MWT course, credits are obtained from each individual Year 4 class.

Given that this is achieved, the composite mark for Final Honours assessment, ***C***, is calculated using the Faculty of Science Honours assessment algorithm:

$$C = \frac{m \times L3 + 3n \times L4}{m + 3n}$$

where ***L3***, ***L4, m*** and ***n*** are, respectively, the level 3 average mark, level 4 average mark, number of level 3 credits attempted and number of level 4 credits attempted.

In most, **<u>but not all</u>**, of our degree courses, students attempt an equal number of level 3 and level 4 classes. (Normally 120 credits at level 3 in Year 3, 120 credits at level 4 in Year 4.) In this case, the Faculty algorithm simplifies significantly:

$$C = 0.25 \times L3 + 0.75 \times L4$$

The class of Honours awarded is based upon the guidelines in the table below.

C:	<40	40–49	50–59	60–69	≥70
degree:	**Pass**	**III**	**II(ii)**	**II(i)**	**I**

Borderline cases are given very careful consideration.

The 5 year MMath degree is awarded "with Distinction" or "with Merit", based upon performance in Years 4 and 5. Details will be provided to MMath students when provisionally registering for fourth year classes.

<u>Notes</u>

- It is **<u>the first attempts</u>** at examinations that are used for Final Honours assessment. Resits may be required to gain sufficient credits to proceed into Year 4, but normally resit marks will not play a role in Final Honours assessment.
- Under normal circumstances, there are **<u>no resits</u>** of Year 4 examinations. The award of Year 4 credits is based upon your overall year performance (or individual classes for the Mathematics with Teaching degree).
- BA Joint Degree students should consult their Business/HASS Adviser about degree classifications.

CHOICE OF CURRICULUM

To help you choose your Mathematics and Statistics level 3 classes, please consult the Myplace page

Mathematics & Statistics, Information for current students.

All students on our degree courses are registered for this class on Myplace. It contains information about all aspects of our classes, including background details on the topics covered. You can also consult with the lecturer currently in charge of a particular class if you want further information. When choosing your curriculum, please note the following points.

- You should ensure that you have taken or are taking any stated pre-requisites for a class.
- Ensure a reasonable split of classes between Semester 1 and Semester 2.
- Student Business will allow you to amend your curriculum up to the end of the second week in each teaching semester. In other words, any provisional curriculum you submit in March is not final. Indeed, we suggest that you sample the lectures in all optional classes available to you at the start of each semester before making a final decision.
- Once it becomes available, check your provisional timetable to ensure that the lectures for your classes do not clash. (Do not focus on tutorial clashes at this stage — alternatives are often available.)

INFORMATION ON COMPULSORY AND OPTIONAL CLASSES

The tables below contain information about the compulsory and optional classes for each Department of Mathematics and Statistics Honours and MMath degree course in Year 3.

- Normally students are expected to choose a curriculum of 120 credits in Year 3.
- All classes with leading digit "3" (e.g. MM301, MM307, CS308, PH352, …) are at level 3; classes with leading digit "4" are at level 4.
- This information is subject to change — refer to the Course Regulations document for the latest details. Information about all classes can be found at the University's Class Catalogue (or Module Catalogue): **classcat.strath.ac.uk**

Degree course	Compulsory classes	Optional classes
Mathematics (BSc Hons & MMath)	MM300, MM301, MM302	MM303, MM304, MM305, MM306, MM307
Mathematics and Statistics*	MM300, MM301, MM302, MM304, MM307	MM303, MM305, MM306
Mathematics with Teaching	MM302, MM304, MM441	MM301, MM305
MM441 Mathematics with Teaching comprises 60 credits of classes chosen from: MM407, MM408, MM409, MM415 in semester 2		
Mathematics and Computer Science	MM301, MM302, CS308	MM303, MM306, CS310, CS312, CS316, CS317
CS308 Building Software Systems (20 credits, semester 2) CS310 Foundations of Artificial Intelligence (20 credits, semester 2) CS312 Web Applications Development (20 credits, semester 1) CS316 Functional Programming (20 credits, semester 1) CS317 Mobile App Development (20 credits, semester 2)		
Mathematics and Physics	MM300, MM302, PH384, PH386, PH387	MM301, MM305, MM306
PH384 Quantum Physics And Electromagnetism (20 credits, full year) PH386 Condensed Matter Physics (20 credits, full year) PH387 Gases, Liquids and Thermodynamics (20 credits, full year)		

* For students intending to graduate with BSc Honours in Mathematics and Statistics at end of fourth year of study.

<table>
<tr><th>Degree course</th><th>Compulsory classes</th><th>Optional classes</th></tr>
<tr><td>Mathematics, Statistics and Accounting</td><td>MM302, MM304, AG308, AG309, AG310, AG311</td><td>MM300, MM301, MM306, MM307, AG215**, M9117**</td></tr>
<tr><td colspan="3">AG308 Auditing and Assurance (10 credits, semester 2)
AG309 Governance and Accounting Ethics (10 credits, semester 1)
AG310 Contemporary Management Accounting (10 credits, semester 2)
AG311 Advanced Financial Reporting (10 credits, semester 1)
AG215** Business Finance (20 credits, semester 1)
M9117** Business Law (20 credits, semester 1)

** Students seeking professional accreditation in Accounting must take this class if not previously taken.</td></tr>
<tr><td>Mathematics, Statistics and Economics</td><td>MM302, MM304, EC315, EC316</td><td>MM300, MM301, MM306, MM307</td></tr>
<tr><td colspan="3">EC315 Topics in Microeconomics with Cross Section Econometrics (20 credits, semester 1)
EC316 Topics in Macroeconomics with Time Series Econometrics (20 credits, semester 2)</td></tr>
<tr><td>Mathematics, Statistics and Finance</td><td>MM302, MM304, AG312, AG313</td><td>MM300, MM301, MM306, MM307</td></tr>
<tr><td colspan="3">AG312 Advanced Corporate Finance and Financial Markets (20 credits, semester 1)
AG313 Treasury Management and Derivatives (20 credits, semester 2)</td></tr>
<tr><td>Mathematics, Statistics and Business Analysis</td><td>MM302, MM304, MS311, MS361</td><td>MM300, MM301, MM306, MM307</td></tr>
<tr><td colspan="3">MS311 Knowledge and Innovation Management (20 credits, semester 2)
MS361 Understanding and Optimizing Business Systems (20 credits, semester 1)</td></tr>
<tr><td>Data Analytics</td><td>MM304, MM307, CS308</td><td>CS310, CS311, CS312, CS316, CS317,
(+ 20 credit elective)</td></tr>
<tr><td colspan="3">CS308 Building Software Systems (20 credits, semester 2)
CS310 Foundations of Artificial Intelligence (20 credits, semester 2)
CS311 Programming Language Definition and Implementation (20 credits, semester 1)
CS312 Web Applications Development (20 credits, semester 1)
CS316 Functional Programming (20 credits, semester 1)
CS317 Mobile App Development (20 credits, semester 2)

(Students choose 40 credits from the optional classes and a 20 credit elective class.)</td></tr>
</table>

MATHEMATICS AND STATISTICS LEVEL 3 CLASSES

MM300 **Complex Variables and Integral Transforms**
Credits: 20 Semester: 2
Pre-requisites: MM102, MM201, MM202 or equivalent.
Students: Compulsory: M, MM, MS, MMS, MP
Optional: MSA, MSE, MSBA, MSF, MCS, BA

MM301 **Linear Algebra**
Credits: 20 Semester: 1
Pre-requisites: MM201 or equivalent.
Students: Compulsory: M, MM, MS, MMS, MCS
Optional: MP, MWT, MSA, MSE, MSBA, MSF, BA

MM302 **Differential Equations**
Credits: 20 Semester: 1
Pre-requisites: MM201 or equivalent
Students: Compulsory: M, MM, MWT, MP, MS, MSA, MSE, MSBA, MSF, MCS, BA

MM303 **Applicable Analysis 2**
Credits: 20 Semester: 2
Pre-requisites: MM203, MM301 or equivalent
Students: Optional: M, MM, MS, MMS, MCS

MM304 **Inference and Regression Modelling**
Credits: 20 Semester: 1
Pre-requisites: MM204 or equivalent
Students: Compulsory: MWT, MS, MMS, MSA, MSE, MSBA, MSF, DA, BA
Optional: M, MM

MM305 **Mechanics of Rigid Bodies and Fluids**
Credits: 20 Semester: 1
Pre-requisites: MM205 or equivalent
Students: Optional: M, MM, MS, MMS, MWT, MP

MM306 **Numerical Analysis**
Credits: 20 Semester: 2
Pre-requisites: MM206 or equivalent.
Students: Optional: M, MM, MS, MMS, MCS, MP, MSA, MSE, MSBA, MSF, BA

MM307 **Stochastics and Financial Econometrics**
Credits: 20 Semester: 2
Pre-requisites: MM304 or equivalent.
Students: Compulsory: MS, MMS, DA
Optional: M, MM, MSA, MSE, MSBA, MSF, BA

MATHEMATICS AND STATISTICS CLASSES — ADDITIONAL INFORMATION

In addition to the syllabuses provided in the class descriptors in the University's Class Catalogue, classcat.strath.ac.uk, we have produced supplementary information about the content of each of our level 3 and 4 classes**. This is intended to help you choose your Year 3 classes with Year 4 in mind.

**Note: the Department reserves the right to alter the available classes at short notice; therefore the list of available classes and the semester in which they are provided may be subject to change.

MM300 Complex Variables and Integral Transforms

Pre-requisite classes: MM102 Applications of Calculus; MM201 Linear Algebra and Differential Equations; MM202 Advanced Calculus or equivalent.

Students: Compulsory for M, MM, MS, MMS, MP. Optional for MSA, MSE, MSBA, MSF, MCS, BA.

The second semester class MM300 is taught in two parts:

- Complex variables (weeks 1–5);
- Integral transforms (weeks 6–10);

with week 11 being used for revision. There are connections between the two parts, but each part is essentially self-contained.

Part 1: Complex variables

Despite their theoretical background dating back to the sixteenth century, complex numbers today have many real life applications in, for example, fractals, chaos theory, signal analysis, electrical circuits, fluid mechanics, control theory, number theory and structural analysis. It took three hundred years in order to establish the concept of complex variables, yet the emergence of complex analysis as a powerful analytical tool took place over a much shorter time period. However, complex analysis does not necessarily mean complicated analysis. Significantly, complex analysis was the result of generalising (or extending) real analysis from one dimension (the real number line) to two dimensions (the complex plane). For example, the definitions of limits, continuity, differentiation and integration in complex analysis are natural extensions of their real counterparts. Indeed, in some ways functions of a complex variable are simpler than real numbers: polynomials of a complex variable **always** have roots in the complex plane; every differentiable complex function can be differentiated as often as we please and always has a power series expansion. (Neither of these properties is always true in real analysis).

Despite all this discussion about analysis, the complex variables section of MM300 focusses on *methods*. It will begin by re-introducing you to the concept of complex numbers from class MM102 and their many properties. Thereafter, the lectures will extend material on functions of one and more real variables, covered in MM101 and MM202, by examining functions of complex variables and their behaviour. This will include mappings of the complex plane, domain and range of a function, limits, differentiation, power series, residue theory and integration. By the end of the class you will be in a position to apply complex analysis techniques to solve a variety of problems involving the integration of real integrands.

Part 2: Integral transforms

In mathematics there are many classes of problems that are difficult to solve in their original form. An **integral transform** uses integration to “map” an equation from its original “domain” into another domain where manipulating and solving the equation is much easier. The solution is then mapped back to the original domain using the inverse of the integral transform.

The main aims of this part of MM300 are to use **Laplace transforms** and **Fourier transforms** to solve differential and integral equations. We also focus on *methods* in this part of the class. For both type of transform, we start by considering simple functions (like $f(x) = x$, e^x, $\sin(x)$) and then look at ways of extending the scope of functions whose transform we can calculate. (This includes more exotic objects such as the Heaviside function and the Dirac delta function – which is actually a *distribution* and not a function at all!). This builds up a table of transforms and their inverses which we then use to solve various types of differential and integral equations**.**

MM301 Linear Algebra

Pre-requisite classes: MM201 Linear Algebra and Differential Equations or equivalent

Students: Compulsory for M, MM, MS, MMS, MCS. Optional for MP, MWT, MSA, MSE, MSBA, MSF, BA.

In the Mathematics curriculum at Strathclyde, MM301 is the third encounter with linear algebra, after MM103 and MM201.

In MM103, the focus is on the connections between linear algebra and geometry, and in MM201 mostly technical aspects and practical applications of matrix algebra are taught. In MM301, the emphasis is on fundamental aspects of linear algebra. Much of the material is similar to that covered in MM201, but here we aim to develop a coherent picture with a complete set of proofs.

At the start of the class, for about four weeks, abstract algebraic structures are introduced. This approach puts into a wider context things like the properties of real numbers, introduced in MM101 and modular arithmetic.

Many identical structures appear over and over again in different branches of mathematics, and studying these structures in an abstract way, i.e., independent of a specific example, yields information that is valid in a wide range of cases.

This type of abstraction is very useful across virtually all branches of mathematics.

Many of the structures treated at the beginning of the class also appear naturally in the context of linear algebra, which is studied in the second part of the class. This provides ideal opportunities for recognising the structures and for practising what has been learned in the first part.

The material taught in MM301 is useful in many other classes. In particular, the concepts of abstract vector spaces and scalar products are essential for MM303, where algebra and analysis meet.

MM302 Differential Equations

Pre-requisite classes: MM201 Linear Algebra and MM202 Advanced Calculus or equivalent.

Compulsory class for: MM405 Fluids and Waves; MM406 Finite Element Methods for Boundary Value Problems and Approximation; MM408 Mathematical Biology and Marine Population Modelling.

Students: Compulsory for M, MS, MMS, MM, MWT, MP, MSA, MSE, MSBA, MSF, MCS, BA.

The course provides a broad introduction to analytical methods for solving ordinary and partial differential equations and will develop students understanding and technical skills in this area.

The course comprises two parts: ordinary differential equations (ODEs) and partial differential equations (PDEs). This is a methods class, so the main focus is on studying problem-solving techniques for various types of differential equations. Second-order ODEs play a key role in the study of differential equations.

We will look at second order equations in which the coefficients are functions of the independent variable. The principal idea is to assume that a solution exists in the form of a power series and then attempt to find its coefficients. In this context, we consider such examples as the **Legendre equations**, whose solutions reduce to a polynomials, called the **Legendre polynomials**, and the **Bessel's equations**, which we solve using the **Frobenius' Method**. To conclude the ODE part of the course, we look at Systems of Differential Equations. In particular, the **phase plane** analysis is employed to gain a qualitative understanding of the solution behaviour.

In the second part of the course, we study partial differential equations (PDEs). We start with the first-order linear **advection** equation, for which we employ the method of **characteristics**. Afterwards, most of the material is devoted to three important second-order equations from mathematical physics: the **wave** equation, the **Laplace** equation and the **heat** equation, which respectively represent hyperbolic, elliptic and parabolic PDEs. For the wave equation, **D'Alembert's** solution formula is considered. Then the main focus is on the application of the method of **separation of variables** to hyperbolic, elliptic and parabolic PDEs posed in simple domains subject to various boundary conditions. To be able to deal with general boundary conditions, we employ various forms of **Fourier series**. Finally, to be able to apply the method of separation of variables to more general PDEs, we look at certain properties of **Sturm–Liouville eigenvalue** problems.

MM303 Applicable Analysis 2

Pre-requisite classes: MM203 Applicable Analysis 1; MM301 Linear Algebra or equivalent.

Compulsory class for: MM403 Applicable Analysis 3.

Students: Optional for M, MS, MM, MMS, MCS.

Mathematical Analysis provides the rigorous underpinnings for much of the mathematics that is applied to problems arising in the natural sciences. In particular, it is fundamental to differential and integral calculus, numerical analysis, probability theory and optimisation theory. This was partly demonstrated in the second year class MM203 Applicable Analysis, where important theoretical results on functions of a real variable were presented and proved, with some simple numerical applications also discussed. The class MM303 takes the ideas of MM203 a step further. Broadly speaking, in MM303 we shall see how some familiar notions from linear algebra and the theory of functions of a real variable can be generalised and presented within a more abstract setting.

The class begins with the concept of a **metric space** which, in simple terms, can be viewed as a set in which it is possible to measure the distance between elements by means of a distance function (referred to as a **metric**). The prototype is the set of real numbers with distance measured using the modulus function. Definitions of convergence of sequences and continuity of functions on metric spaces can then be obtained by simply taking the corresponding definitions given in MM203 for real sequences and functions of a real variable and replacing the modulus wherever it appears with the appropriate metric. Following this, some of the theory associated with **normed vector spaces** and **Banach spaces** will be presented. In essence, this is where we see a marriage between Analysis and some of the topics from Linear Algebra covered in MM201 and MM301. Specifically, we attach some algebraic structure to the metric space by assuming that the set is actually a vector space. The metric in a normed vector space is defined via a **norm** which can be viewed as a function that gives the magnitude of elements in the vector space (think of the modulus of a real number). As all normed vector spaces are metric spaces all the metric space results hold in a normed vector space setting, but the added algebraic structure enables us to consider additional features such as convergence of infinite series of elements from the vector space and continuity of **linear transformations** (or **linear operators**). Finally, some special types of normed vector spaces known as **inner product spaces** and **Hilbert spaces** will be introduced. These are “special” because the norm on them is defined via something called an **inner product** or **scalar product** (scalar products are also discussed in MM301). An inner product can be regarded as a generalisation of the dot product of vectors in two and three dimensions. The fact that an inner product exists means that some geometrical structure now emerges, particularly the notion of elements in an inner product space being mutually perpendicular (or **orthogonal**) to each other. This has far-reaching implications, some of which will be discussed in the class.

Throughout, the class material will be developed in a similar style to MM203. Fundamental definitions will be given and then used to produce significant results which will be presented as theorems. Proofs of some of these theorems will be worked through in lectures and applications (to justify the use of “Applicable” in the class title) will also be highlighted via numerous examples.

MM304 Inference and Regression Modelling

Pre-requisite classes: MM204 Probability and Statistical Inference or equivalent.

Compulsory class for: MM307 Stochastics and Financial Econometrics.

Students: Compulsory for MWT, MS, MMS, MSA, MSE, MSF, MSBA, DA, BA. Optional for M, MM.

The aim of this class is to give students a better understanding of parameter estimation procedures and hypothesis tests, where they come from, how to use them and what their properties are, as well as introducing regression modelling as an application of both estimation and testing.

The inference part of the class begins with probability distributions and how to work with them, including deriving moments of random variables and using transformations to derive one probability distribution from another, then moves on to estimation procedures and properties of estimators, including maximum likelihood estimation and method of moments estimation, and the last part deals with hypothesis tests, using the classical Neyman-Pearson approach to deriving test statistics and rejection regions but also some computer-based tests. This all builds on the work of MM204 Probability and Statistical Inference, but is much more mathematical and theoretically focussed than that class. To make the most of the class, students need to be comfortable with the work of MM204 but also to have a sound grasp of the 1st and 2nd year mathematics syllabus, to think for themselves and be prepared to put in time to work through plenty of examples themselves to learn how to use the concepts. The material requires working with the natural logarithm and exponential functions, knowing and using results from mathematical analysis, use of differentiation and integration, and algebraic manipulation. The inference part of the class uses tutorial classes to practise using the methods covered in lectures. There are also a few lab classes to illustrate some computer-based procedures that can be useful for investigating properties of estimators and implementing some hypothesis tests. The inference material makes up about two-thirds of the work of the class. The remainder is used to teach regression modelling.

Regression is a much more applied topic than the inference material, but it involves use of both estimation and hypothesis tests. Students will learn the theory of linear regression modelling, beginning with fitting straight line models and moving on to multi-variable models and variable selection, assessing various aspects of the fit of a model, how to fit regression models to data using the R software and how to interpret the results. The lectures on regression will be supplemented by several tutorials at which students will have an opportunity to work through and seek assistance on typical regression-based problems. However, the focus is on regular lab work using R for practical implementation of model fitting.

In lectures for both parts of the class we work though the theory and do examples to show how to use the theory. The assessment for this class involves a group computer project for regression, assessed homeworks for inference and a final exam covering both inference and regression.

MM305 Mechanics of Rigid Bodies and Fluids

Pre-requisite classes: MM201 Linear Algebra and Differential Equations; MM202 Advanced Calculus; MM205 Introduction to Newtonian Mechanics (or equivalents).

Compulsory class for: MM405 Fluids and Waves.

Students: Optional for M, MS, MM, MMS, MWT, MP.

The two parts of MM305 ('Rigid Bodies' and 'Fluids') are loosely related but are taught independently. Both parts use mathematical models to describe and explain complex mechanical phenomena, such as the rolling of a cylinder down an inclined plane or the flow of water in a channel. The approach taken in the class will be similar to previous Mechanics classes, with an emphasis on problem-solving and on interpreting mathematical results to provide physical understanding.

Rigid Bodies

A solid object (e.g. a cylinder or a cube) that deforms relatively little when acted on by forces may be modelled mathematically as a rigid body, that is, a body that retains its shape exactly when subjected to forces. Notions such as the mass, centre of mass, momentum and angular momentum of such a body are introduced, as are the notions of the moment of a force and of a couple. These ideas are then used to study a variety of three-dimensional systems in static equilibrium, and to study a variety of two-dimensional motions (such as the oscillations of a solid pendulum, and the rolling of cylinders).

Fluids

The 'Fluids' part comprises three sections. The first introduces notions such as density and pressure, and then uses these ideas to study fluids (liquids or gases) that are in static equilibrium ("hydrostatics"); examples such as the force on a dam holding back static water are discussed. The second section introduces the kinematic description of flowing fluids (velocity, streamlines, circulation, etc), as well as the concept of conservation of mass. The last section introduces the idea of an inviscid fluid, representing a fluid that has no internal friction (viscosity); the governing equation for such a fluid ("Euler's equation", derived from Newton's Second Law of Motion) is established and is used to investigate a variety of flow problems.

MM306 Numerical Analysis

Pre-requisite classes: MM201 Linear Algebra and Differential Equations; ideally MM206 Mathematical and Statistical Computing or equivalent.

Students: Optional for M, MS, MM, MMS, MCS, MP, MSA, MSE, MSBA, MSF, BA.

The major concern of numerical analysis is to derive practical algorithms to solve mathematical problems (an algorithm is a step-by-step “recipe” of calculations). Even the most sophisticated mathematician can only solve a fraction of real world problems by hand. This may be because of the size of the problem, or the fact that problems can’t be solved analytically. The computer is a necessary tool, for in practice we may encounter problems of enormous size or we may need to solve thousands of intermediate problems to reach our ultimate goal. In this course, which develops topics studied in first and second year classes, we look at a range of problems where numerical analysis is necessary.

We start with interpolation. Here, we are given a discrete set of data points and we attempt to “fill in the gaps” between them. There are many ways of doing this; we focus on polynomial interpolation. There are several very simple techniques for doing this and we analyse the pros and cons of various techniques. Interpolation is closely related to discrete approximation, where we do our best to follow the path of a data set with as simple a representation as possible. We focus on least-squares approximation, which students may have encountered when studying statistics. We generalise to continuous approximation which allows us to use some beautiful results from calculus and analysis to come up with a solution.

Next we look at quadrature, the process of coming up with an approximate value for an integral. You will all appreciate that integration can be very tricky. In fact for many functions it is impossible to do analytically. Starting with the simplest ideas for approximating a definite integral (such as counting squares under a graph) we develop some powerful techniques for integrating very complex functions. It turns out that interpolation and approximation are intimately involved in the design and analysis of good quadrature methods.

We next consider the problem of finding solutions to nonlinear algebraic equations of the form $f(x) = 0$. These problems arise in many areas of application and again they cannot usually be solved using analytical techniques. For a scalar equation, the problem is equivalent to finding where and when the graph $y = f(x)$ cuts the x-axis. We will consider a number of methods to tackle this problem and perform an analysis to determine how quickly the methods converge.

Finally, we consider the numerical solution of first-order ordinary differential equations (ODEs). Although you will have covered analytical techniques for solving ODEs in first and second year, the class of problems that these methods can be applied to is rather narrow. In most real life applications the ODEs of interest are nonlinear and these are difficult if not impossible to solve using analytical methods. We will look at methods based on the local interpolation of previous solution estimates and methods based on Taylor series expansions.

MM306 is not a pre-requisite for any fourth year classes, but will provide useful background for MM406 and MM409.

MM307 Stochastics and Financial Econometrics

Pre-requisite classes: MM204 Probability and Statistical Inference;
MM304 Inference and Regression Modelling.

Students: Compulsory for MS, MMS, DA. Optional for M, MM, MSA, MSE, MSBA, MSF, BA.

This class consists of two parts. The first part is to introduce the basic concepts of random phenomena evolving in time from a probabilistic modelling point of view. It will start with Markov chains, where the key topics include Markov models in finance, the Markov property, n-step transition probabilities, classification of states, the stationary distribution, 1st-step analysis, and simple random walks. It will then discuss Poisson processes as well as their generalization, Renewal processes. The theory will then be applied to study Queueing theory. The queueing systems discussed include the M/M/1 queue, M/M/1 queue with baulking, M/M/ queue, M/M/s queue, where these have the features of Poisson arrivals, exponential service times, single or multiple servers. The properties studied include the probability distribution of queue size and the stationary distribution.

The second part of this class is an introduction to Econometrics. It focuses on models for financial applications, especially time series models. It begins with statistical analysis of financial data, mainly on distributional characteristics of financial returns. Commonly used parametric and non-parametric methods are used for estimation and to test the distribution of financial returns. Application to the calculation of Value-at-Risk is shown as an illustration. The second chapter of this part presents fundamental concepts of time series analysis, such as stationarity, autocorrelation function and tests for serial dependence. Chapter 3 of this part provides simple linear time series models such as AR, MA and ARMA models, and model identification, parameter estimation, model checking and forecasting are discussed. Chapter 4 of this part introduces popular models for financial volatilities, e.g. the autoregressive conditional heteroscedastic (ARCH) model and its extensions. It is shown how to build a model for financial volatility based real data. The lab session uses R-language to practice modelling financial time series. Elementary theory of probability and statistics from MM204 and MM304 is used throughout the class.

MM401 Communicating Mathematics and Statistics

Students: Compulsory for students in 4th year of BSc Honours or MMath in Mathematics, Mathematics and Statistics.
Optional for 4th year students in other BSc Honours Mathematics-based degrees (except Mathematics with Teaching).

This class is comprised of eight hours of lectures on writing and presentation skills, followed by project work with an individual supervisor. You may take this class in either semester (but not both). The format of this class is motivated by the fact that nowadays, many employers place as strong an emphasis on organisational and communication skills as they do on specific advanced subject knowledge. To this end, in this class you will receive specific training in the basic skills of communicating mathematics and statistics while working on an individual project (which will be assessed based on a written report and an oral presentation). Your project will give you a chance to work outwith a lecture course framework on a specific topic. It will allow you to work independently and in depth on a particular subject, give you practice in seeking out and presenting information and help you learn to manage your own time. This type of work is much more aligned to what you are likely to encounter after graduation than attending lectures and sitting examinations!

The bulk of the class will be taken up with a short individual project. MM401 is worth 20 credits, so the time you spend on your project is comparable with that spent on any other Y4 class (including private study and revision time). Although the length of the report will depend on the type of project undertaken, a reasonable guideline is about 20-25 typed pages (about 250 words or equivalent per page). The mathematics or statistics in your project should be at the standard Y4 level of difficulty, and should not overlap significantly with topics taught in other undergraduate lecture modules or with work that you have undertaken for other purposes. You are not required to do original research, but you are expected to present a scholarly description of the chosen topic.

MM402 Modelling and Simulation with Applications to Financial Derivatives

Pre-requisites classes: None.

Students: Compulsory for students in 4th year of BSc or MMath in Mathematics and Statistics.
Optional for 4th year students in other BSc Honours or MMath Mathematics-based degrees (except Data Analytics).

MM402 consists of two parts. The first part of MM402 introduces some general tools for mathematical modelling in both deterministic and stochastic settings. It begins with a brief review of limits and some basic results in calculus (such as big O notation, Mean-Value theorem, Taylor Series, examples and steady state of ODE, linear stability); Finite Difference Methods for PDEs are explained. As for the tools for stochastic modelling, some basic concepts of random variables, distributions and Monte-Carlo methods are reviewed. Then some stochastic processes are introduced such as Birth and Death Processes, and Markov Chains, together with their applications in chemical kinetics, systems biology and population dynamics.

The second part of MM402 focuses on financial derivatives. It begins with the introduction of European call and put (vanilla) options and their payoff diagrams. The "no arbitrage principle", the important underlying principal for pricing an option, is explained. By this "no arbitrage principle", some basic inequalities involving option values are proved. More importantly, by this "no arbitrage principle", the celebrated Black-Scholes PDE is derived, which is followed by the calculation of Greeks. The concept of "Risk neutrality" is discussed and applied in pricing options. Finally, Monte–Carlo Simulation for option valuation and the Binomial Method for European and American option valuation are discussed.

MM403 Applicable Analysis 3

Pre-requisite classes: MM303 Applicable Analysis 2.

Students: Optional for students in 4th year of BSc Honours or MMath Mathematics-based degrees (except Data Analytics).

Many applications of mathematics give rise to differential, integral and stochastic differential equations. One very successful approach to the theoretical and numerical study of such equations is to treat functions (which are the solutions of these equations) as elements of some space, often a vector space. Spaces of functions are usually infinite-dimensional since there are infinitely many degrees of freedom. In contrast to Linear Algebra, where only the algebraic structure (in particular, linearity) is investigated and which is sufficient for finite-dimensional spaces, one adds another structure for infinite-dimensional spaces, namely the concept of convergence and related notions like open and closed sets. This leads to normed spaces, Banach spaces and Hilbert spaces, the main objects studied in Functional Analysis.

In the class MM303 (Applicable Analysis 2) basic properties of these infinite-dimensional spaces and of linear operators in the spaces were studied. In the class MM403 these spaces and operators are investigated more thoroughly. In the first half of the class some of the fundamental theorems of Functional Analysis are discussed and proved.

- The Open Mapping Theorem deals with linear operators between Banach spaces and implies, among other things, that if a linear equation in a Banach space has a unique solution, then this solution depends continuously on the given data.

- The Uniform Boundedness Principle is concerned with sequences of operators and is used, for instance, in the study of numerical quadrature formulae.

- The Hahn–Banach Theorem brings fundamental insight in the geometry of normed spaces.

In the second half of the class Spectral Theory plays an important role. The spectrum of an operator is a generalisation of the set of eigenvalues of a matrix to operators in an infinite-dimensional space. The spectrum is used, for example, for stability investigations and in Quantum Theory, where in the latter application the spectrum is connected with possible outcomes of quantum measurements or possible energy levels. Finally, compact operators are studied, which have many similarities with matrices and appear in connection with differential equations on bounded domains.

MM404 Statistical Modelling and Analysis

Pre-requisite classes: MM204 Probability and Statistical Inference (Essential); MM304 Inference and Regression Modelling (Desirable).

Students: Compulsory for students in 4th year of BSc Honours or MMath in Mathematics and Statistics. Optional for 4th year students in other BSc Honours or MMath Mathematics-based degrees.

This class covers two fundamental statistical concepts, namely Experimental Design and Analysis and Bayesian Analysis. Statistical models and theory are explained in lectures then these concepts are applied in computer lab sessions using real data.

Experimental Design: In a typical experiment, one or more process variables (or factors) are deliberately changed in order to observe the effect the changes have on one or more response variables. The statistical design of experiments (DOE) is an efficient procedure for planning experiments so that the data obtained can be analysed to yield valid and objective conclusions.

This part of the course covers several classical experimental designs, namely, completely randomised designs, randomised block designs, factorial designs, nested designs, repeated measures and cross-over designs. The mathematical modelling concepts build on ideas from MM204 and from the linear regression section of MM304 and these are explained in lectures. Course participants will then have the opportunity to apply these methods using the statistical software package Minitab to some real life data during the lab sessions. The emphasis in computing labs is on learning how to choose the analysis method appropriate to the study design and to validate the fitted model to ensure that accurate estimates are produced. In addition you are expected to further develop your report writing skills and to produce clear and concise, non-technical reports that would be expected of a statistical consultant.

Bayesian Analysis: Almost all of the statistical theory that you are currently familiar with is known as frequentist statistics. An alternative, and now very popular approach, is the Bayesian approach. This approach is based on Bayes theorem, which you learned about in MM104 and MM204. The fundamental concept is the idea of probability being thought of as a measure of belief rather than a frequency in a long run series of trials. The philosophy leads to important concepts such as prior belief, updating belief following collecting of data and a final posterior probability. The methodology is widely used in practical situations and in particular to estimate risks. The application to risk analysis will be considered in detail using the probability theory from MM204 and, to some extent, MM304. Bayesian analysis is computer intensive and a large focus of the class will be laboratory based, using the statistical software package R. In these labs you will learn how to develop Bayesian risk analysis models which are widely used by Government and industry (particularly finance) when making decisions regarding risk. Real world applications are worked on thus giving you an idea how the skills you will develop could be used in your future careers. Similar to the first part of the course, you will further develop your report writing skills, again a skill that is very important in the work-place.

Assessment: The format is continuous assessment and is a mixture of project work, lab based assessment and written class tests, with each contributing to the overall course mark.

MM405 Fluids and Waves

Pre-requisite classes: MM302 (Differential Equations); MM305 (Mechanics of Rigid Bodies and Fluids). Students who have not taken MM305 should discuss the class with the lecturer(s) before choosing it.

MM405 consists of two parts (Fluids and Waves), which are loosely related but are taught independently. Both parts will use **mathematical models** to describe and explain **complex physical phenomena**, such as the flow of air over a wing or the propagation of waves across the ocean. The approach we take will be similar to previous Mechanics classes, with an emphasis on problem-solving and on interpreting mathematical results to provide physical understanding. Another key theme will be the use of appropriate approximations, often informed by the underlying science, to simplify and solve problems.

Fluids. In MM305 we developed the basic ideas of **inviscid fluid mechanics**, which describes the motion of a fluid with no internal friction (viscosity). The Fluids part of MM405 will build on this to develop the theory of the flow of a **viscous** fluid. After deriving the **Navier–Stokes equations** which govern such flows, we will develop exact solutions to flows in progressively more complex situations, starting with **rectilinear flow** in which the velocity vectors at every point are parallel. We will then introduce the idea of **scaling analysis** and use it as a tool to define and explore two interesting flow regimes: slow flow of thin layers of fluid, which can be described by **lubrication theory** and has applications across science and engineering; and rapid flow past a rigid boundary, which can be described by **boundary-layer theory** and is essential to modern aerodynamics.

As well as having some prior experience of fluid mechanics, students should be familiar with the key ideas from **vector calculus** (MM202) and with ordinary and partial **differential equations** (MM302).

Waves. The Waves part of MM305 covers **water waves** and some other systems that exhibit similar mathematical behaviour. It is divided into three chapters. The first two chapters deal with **linear waves**; these waves obey a linear governing equation, so that general solutions can be constructed by superimposing modes corresponding to waves of different wavelength. The first chapter deals with phenomena such as **acoustic waves**, while the second deals with the more complicated behaviour of **dispersive** waves. What makes these waves interesting is that different modes travel at different speeds and so an initial waveform **disperses** over time. We will explore this behaviour and its consequences for various physical systems. The final chapter deals with a category of nonlinear phenomena described by **kinematic wave equations**, which can be solved using a technique called the **method of characteristics**. Particularly important applications of this theory include 'waves' in **traffic flow** and **flood surges** in rivers.

Students should be familiar with ideas of **mass and energy conservation** (e.g. from MM205). This part also requires a knowledge of vector calculus (MM202) and partial differential equations (MM302), and some basic **linear algebra** (MM201) will be useful.

MM406 Finite Element Methods for Boundary Value Problems and Approximation

Pre-requisite classes: MM201 Linear Algebra and Differential Equations; MM202 Advanced Calculus; MM302 Differential Equations.

Students: Optional for students in 4th year of BSc Honours or MMath Mathematics-based degrees (except Data Analytics).

The first semester class MM406 is taught in two parts:

- Approximation theory (weeks 1–5);
- Finite element methods for boundary value problems (weeks 6–10);

with week 11 being used for revision. The two parts are mainly self-contained, but some of the approximation theory results are used in the second part.

Part 1: Approximation theory
This is a mix of mathematical theory and calculations and builds on ideas from linear algebra - in particular it will involve concepts such as linear independence, norms, inner products and orthogonality (all this material will be covered in MM406, but it is helpful to have seen it before). The two main topics are as follows.

- **Finding the element of a linear subspace which is nearest to a given element.** This is a generalisation of the problem: *find the point on the line* $y = 3x - 1$ *which is closest to (2,-1)* to more types of spaces (and higher dimensions). This is easiest to do when the basis of the space is *orthogonal*, and we shall investigate various ways of constructing orthogonal basis elements.

- **Splines.** Given a set of data points (x_k, y_k) for $k = 1, 2, \ldots$, find a piecewise polynomial function which passes through all the data points. The piecewise linear case just involves “joining the dots”, and we begin by looking at this. One of its drawbacks is that it does not give a smooth function (its first derivative jumps at each point x_k) and we investigate the properties of using piecewise cubic polynomials.

Part 2: Finite element methods for boundary value problems
This part of the course is an introduction to a flexible and powerful class of numerical methods for the numerical solution of differential equations called **finite element methods**. The course has two main themes. The first is the derivation of Galerkin finite element methods. The basic idea is to first split up the domain over which the differential equation is to be solved by a set of elements. In one dimension these elements are just segments of the real line and in two dimensions the elements can be triangles or quadrilaterals. On each element it is assumed that the numerical approximation is a polynomial function. A procedure is then devised to calculate the coefficients of the piecewise polynomial approximation of the solution of the differential equation. Concepts required to cope with this part of the course include those above for **Part 1** such as linear independence, approximating subspaces, orthogonality, norms and inner products. The numerical approximation of two-point boundary value problems will be considered first as well as an investigation of mixed boundary conditions. The extension of the finite element method to use higher order basis functions will also be discussed. Towards the end of Part 2, the finite element method will be extended to the solution of two-dimensional partial differential equations, with Poisson’s equation being used as a model elliptic equation.

The second theme is a **theoretical analysis** of the **convergence** and approximation properties of the finite element method. One of the major advantages of the finite element method is its elegant mathematical setting and this helps facilitate detailed analysis of the method. This will rely on optimal approximation properties of the Galerkin approximation as well as estimates in various norms of interpolation errors. Attention will also be given to the derivation of error estimates that can be used in intelligent **solution adaptive algorithms**.

MM407 Applied Statistics in Society

Pre-requisite classes: MM204 Probability and Statistical Inference (Essential); MM206 Mathematical and Statistical Computing (Desirable); MM304 Inference and Regression Modelling (Desirable).

Students: Compulsory for students in 4th year of BSc Honours or MMath in Mathematics and Statistics. Optional for 4th year students in other BSc Honours or MMath Mathematics-based degrees. Optional for 3rd year students, BSc Honours Mathematics with Teaching degrees.

This class is focussed on the application of statistics to real life problems. Several topics are covered each year and taught by staff with professional experience working in specific areas of applied statistics. Although some theory is necessary for a comprehensive understanding of the methodology, the emphasis of the class is on the applications of these statistical methods and students will have the opportunity to work on some real-world data sets using statistical software packages. Examples of topics included in this class are:

Survey Design and Analysis: Surveys are a tool to collect samples of observational data from a population and are used for various different purposes, e.g. market research, opinion polling and pre-election polling.

The lectures present basic concepts of survey sampling, practical issues such as minimising sources of error in survey results, several different approaches to selecting the sample from the population and the corresponding theory of estimation for each of these approaches. For estimation, the concepts of expected value, bias and unbiasedness, variance, covariance, precision and efficiency from MM304 are relevant. Although the derivation of the estimators and their properties is algebraic, the emphasis of the survey design and analysis material is applied and practical.

There will be several practical classes illustrating different aspects of the lecture material and using the SPSS software, students will have the opportunity to analyse some questionnaire data collected at Strathclyde from a series of surveys designed to investigate honey bee colony losses in Scotland. Aspects of these surveys will also be used to illustrate the lectures.

Quantitative Risk Analysis: Most people have an intuitive understanding of what risk is. In this class we aim to formalise this understanding and develop models to quantify risk. Quantification of risk relies on many statistical methods, some of which you will be familiar with. The emphasis in this course is the practical use of such methods, although some new methods will be introduced. In this course you will appreciate the difference between uncertainty and variability, quantify uncertainty using methods such as bootstrapping and Bayesian inference, fit appropriate probability distributions to data, build risk analysis models in R and finally communicate your results as written reports.

All theory will be implemented practically via computing sessions using the statistical software R. You will learn to create bespoke functions in R to implement your models and use summary statistics and plots to communicate your results. This topic builds on the statistical methods outlined in MM204 and the computing techniques covered in MM206.

Assessment: The format is continuous assessment and is a mixture of project work, lab based assessment and written class tests, with each contributing to the overall course mark.

MM408 Mathematical Biology and Marine Population Modelling

Pre-requisite classes: MM302 Differential Equations.

Students: Optional for students in 4th year of BSc Honours or MMath Mathematics-based degrees (except Data Analytics).
Optional for 3rd year students, BSc Honours Mathematics with Teaching degrees.

Mathematical models have become an indispensible tool in understanding how biological systems work. In fact, in many cases the models provide scientists with the only means of making predictions about how biological systems will change or respond to human impacts. Will the population of an invasive non-native species grow or die out? Will a disease outbreak occur or not, and if it does can the disease become endemic? Under what situations can the immune system suppress a growing tumour? How did the leopard get its spots? Are there plenty more fish in the sea, and how do you know? How can marine populations be harvested sustainably? These are some of the biological questions to which you will encounter in this course, and which you can address using your mathematical skills.

The course is delivered in two 5-week parts. Part one will focus on the use of ordinary differential equations, difference equations, and delay differential equations in mathematical biology. You will be introduced to phase-line plots as a graphical means of analyzing systems with one state variable, and phase diagrams for two-dimensional systems. The use of linear stability analysis in understanding the dynamics near steady states will also be explained. You will learn how delay differential equations can be used to represent biological processes, such as breathing or population growth with reproductive delays, and learn how to calculate critical values of a delay required for system stability. Throughout, examples will be drawn from population ecology with particular emphasis on marine populations and ecosystems, including how to obtain maximum sustainable yields from fisheries, and how to analyze the stability of difference equation models for seasonally reproducing fish stocks. Other examples will include bacterial growth, the spread of infectious diseases, and enzyme kinetics.

Part two deals with biological systems which display, not only temporal variation, but also considerable spatial variation. Examples of such systems include the geographical spread of diseases, such as swine flu and foot and mouth disease, the growth and spread of a cancer, and the formation of a fully formed organism from an initial fertilized cell during embryonic development. The usual approach for representing processes of this type is to use partial differential equations. You will be introduced to a class of PDEs known as reaction-diffusion equations. These have been used in mathematical models that have been proposed as possible explanations of why specific spatial patterns occur in the coats of certain animals, such as leopards, zebras, giraffes and, closer to home, belted Galloway cattle. Reaction-diffusion equations also appear in mathematical models of wound healing, nerve signalling and the spatial spread of diseases. In connection with pattern formation, you will learn about a possible explanation that involves a mechanism called diffusion-driven instability. In the case of models of phenomena such as wound healing and cancer growth, you will discover the important role played by travelling wave solutions to reaction-diffusion equations.

The course runs over 10 weeks, with four hours of lectures and one tutorial each week. General feedback on your work will be provided in the tutorials. Additionally, you will have the option of working in small groups in order to submit answers to exam-style questions for grading and feedback. The assessment is in the form of a single three-hour final exam.

MM409 Mathematical Introduction to Networks

Pre-requisite classes: MM201 Linear Algebra and Differential Equations.

Students: Optional for students in 4th year of BSc Honours or MMath Mathematics-based degrees.
Optional for 3rd year students, BSc Honours Mathematics with Teaching degrees.

One cannot ignore the networks we are part of and surrounded by in everyday life. There are our networks of family and friends; the transport networks; the telephone networks; the distribution network that shops use to bring us things to buy; the banking network; it doesn't take much effort to come up with dozens of examples. Analysis of networks, particularly the huge networks that drive the global economy (directly or indirectly) is a vital science; and mathematicians play an important role in the new science of networks.

Some of the mathematical tools we need to perform this analysis may be familiar to you and we start the course by collecting together the key ideas from graph theory and linear algebra that will be useful to us, such as node degree, network connectivity and the spectrum of a matrix. Some of the tools met before in linear algebra will need to be honed in order that they can be applied to networks: for example, you'll need to understand properties of nonnegative matrices and also what a function of a matrix means too. The first half of the class will equip you with the necessary armoury and starts to provide the context for their application.

The second part of the course introduces you to the modern concepts of networks, from the 'small-world' phenomenon to the characterisation of the importance of individuals in a network. We develop combinatorial and algebraic methods for quantifying small fragments in giant networks, to characterise the global structure of a network and to generate random models that emulate real-world networked systems. The course combines the theoretical development of these tools with the practical solution of mathematical problems and the computational methods needed to attack these problems in a real-world environment.

MM415 Medical Statistics

Pre-requisite classes: MM204 Probability and Statistical Inference (Essential); MM304 Inference and Regression Modelling (Desirable).

Students: Optional for 4th year students in BSc Honours or MMath Mathematics-based degrees.
Optional for 3rd year students, BSc Honours Mathematics with Teaching degrees.

Medical Statistics: **Medical statistics is the application of statistical methods to the field of medicine and medical practice.** It is the science of summarising, collecting, presenting, analysing and interpreting data in medical studies, and using this to answer research questions. It also has applications in the pharmaceutical industry which includes design of experiments, analysis of drug trials, and issues of manufacturing a medicine commercially (e.g. efficacy, safely, expiration dates and packaging).

The course is taught as a mix of lectures and computer labs using the statistical analysis packages Minitab and SPSS. Topics covered include survival analysis, experimental design (including clinical trials, cross-over designs and placebo controlled trials), sampling and observational studies (including cohort, cross-sectional and case/control studies), categorical data analysis, power and sample size computations, non-parametric hypothesis, tests, control charts and methods for analysing clinical measurements e.g. ROC curves for diagnostic testing and the construction of reference ranges. Many of the statistical methods build on ideas from MM204 but tailor the methods to suit medical type data which can be less robust than data generated from more controlled scientific studies.

Much of the course deals with the interpretation of statistical results in the context of the medical problem being investigated. This skill is necessary for the application of statistics to medical data which often differs from the traditional, standard interpretation of statistical textbook problems. Lectures will be used to explain the mathematical theory behind the statistical methods and to demonstrate the application of the statistical methods using real life medical examples. In the computer labs, students will apply the statistical methods taught in lectures to real life medical research studies using statistical software – mainly Minitab and SPSS.

The class is assessed by a group project and computer lab test. The computer lab test will be worth 70% of the final mark for the class with the remaining 30% coming from the group project.